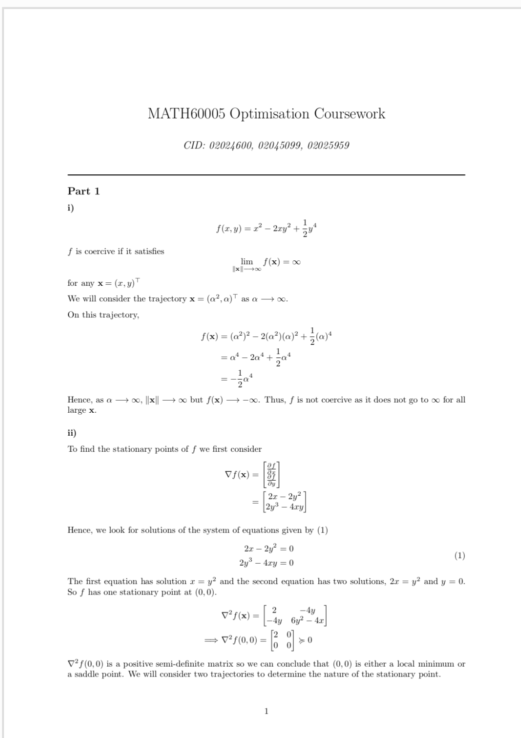
Digital Receipt

This receipt acknowledges that Turnitin received your paper. Below you will find the receipt information regarding your submission.

The first page of your submissions is displayed below.

Submission author:	Max Fricker
Assignment title:	Coursework 1 Drop Box Autumn 23
Submission title:	02024600_Coursework1
File name:	02024600_Coursework1.pdf
File size:	253.32K
Page count:	5
Word count:	1,076
Character count:	3,815
Submission date:	17-Nov-2023 10:28AM (UTC+0000)
Submission ID:	216053101


MATH60005 Optimisation Coursework

CID: 02024600, 02045099, 02025959

Part 1

i)

$$f(x,y) = x^2 - 2xy^2 + \frac{1}{2}y^4$$

f is coercive if it satisfies

$$\lim_{\|\mathbf{x}\| \longrightarrow \infty} f(\mathbf{x}) = \infty$$

for any $\mathbf{x} = (x,y)^\top$

We will consider the trajectory $\mathbf{x} = (\alpha^2, \alpha)^\top$ as $\alpha \longrightarrow \infty$.

On this trajectory,

$$\begin{aligned} f(\mathbf{x}) &= (\alpha^2)^2 - 2(\alpha^2)(\alpha)^2 + \frac{1}{2}(\alpha)^4 \\ &= \alpha^4 - 2\alpha^4 + \frac{1}{2}\alpha^4 \\ &= -\frac{1}{2}\alpha^4 \end{aligned}$$

Hence, as $\alpha \longrightarrow \infty$, $\|\mathbf{x}\| \longrightarrow \infty$ but $f(\mathbf{x}) \longrightarrow -\infty$. Thus, f is not coercive as it does not go to ∞ for all large $\mathbf{x}$.

ii)

To find the stationary points of f we first consider

$$\begin{aligned} \nabla f(\mathbf{x}) &= \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \\ &= \begin{bmatrix} 2x - 2y^2 \\ 2y^3 - 4xy \end{bmatrix} \end{aligned}$$

Hence, we look for solutions of the system of equations given by (1)

$$\begin{aligned} 2x - 2y^2 &= 0 \\ 2y^3 - 4xy &= 0 \end{aligned} \tag{1}$$

The first equation has solution $x = y^2$ and the second equation has two solutions, $2x = y^2$ and $y = 0$. So f has one stationary point at $(0,0)$.

$$\begin{aligned} \nabla^2 f(\mathbf{x}) &= \begin{bmatrix} 2 & -4y \\ -4y & 6y^2 - 4x \end{bmatrix} \\ \implies \nabla^2 f(0,0) &= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \succcurlyeq 0 \end{aligned}$$

$\nabla^2 f(0,0)$ is a positive semi-definite matrix so we can conclude that $(0,0)$ is either a local minimum or a saddle point. We will consider two trajectories to determine the nature of the stationary point.

1