

Math 307: Homework 06

Due Wednesday, October 15 (at the beginning of class). This homework is mostly based on sections 2.3 and 2.4 in the textbook.

Problem 1. Is $\begin{bmatrix} 3 & 5 \\ 3 & 4 \end{bmatrix}$ in $\text{Span} \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$?

Problem 2. Find the determinant of the matrix $\begin{bmatrix} 4 & 3 & 2 & 1 \\ -2 & 5 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & -2 \end{bmatrix}$

Problem 3.

(a) Find the inverse of the matrix $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 4 & -1 \\ 2 & 1 & 2 \end{bmatrix}$ using row reduction.

(b) Use the inverse matrix from part (a) to solve the linear system

$$\begin{cases} -2y + z = 2 \\ 2x + 4y - z = -1 \\ 2x + y + 2z = 5 \end{cases}$$

Problem 4. Show that $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$ form a basis for \mathbb{R}^3 .

Problem 5. For each part, determine whether the vectors form a basis for \mathbb{R}^3 .

(a) $\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 8 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -7 \\ -9 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

Instructions for problems 3-6:

- (a) Find a basis for the nullspace of the matrix
- (b) Find a basis for the row space of the matrix
- (c) Find a basis for the column space of the matrix
- (d) Determine the rank of the matrix
- (e) Determine whether the matrix is invertible

Note that parts (a)-(c) do not have unique answers.

Problem 6. $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$

Problem 7. $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

Problem 8. $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

Problem 9. $\begin{bmatrix} 1 & 1 & 0 & 3 \\ 1 & 1 & 1 & -2 \\ 3 & 3 & 2 & -1 \end{bmatrix}$

Problem 10. Determine whether the given vectors are linearly dependent or linearly independent.

(a) $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $x^3 - 1, x^2 - 1, x - 1, 1.$