# Math 307: Homework 05

*Due Wednesday, October 8 (at the beginning of class). This homework is based on sections 2.2 and 2.3*

**Problem 1.** Determine which of the following sets of vectors are subspaces of $\mathbb{R}^3$.

(a) $\left\{ \begin{bmatrix} x \\ y \\ y-4x \end{bmatrix} : x, y \in \mathbb{R} \right\}$

(b) $\left\{ \begin{bmatrix} y+z+1 \\ y \\ z \end{bmatrix} : y, z \in \mathbb{R} \right\}$

(c) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x, y \in \mathbb{R} \text{ and } z = x+y \right\}$

**Problem 2.** Is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ in $\operatorname{Span}\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$?

**Problem 3.** Let $V$ be a vector space and let $v_1, \ldots, v_n \in V$. Verify that $\operatorname{Span}(v_1, \ldots, v_n)$ is a linear subspace. *(Hint: there are three conditions that you need to check.)*

**Problem 4.** Is $\begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}$ in $\operatorname{Span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$?

**Problem 5.** Determine if the vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ span $\mathbb{R}^4$.

**Problem 6.** Determine whether the given vectors are linearly independent or linearly dependent.

(a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} -9 \\ 6 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix}$

**Problem 7.** Show that $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ form a basis for $\mathbb{R}^2$

**Problem 8.** Show that $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ form a basis for $\mathbb{R}^3$.

**Problem 9.** Let $A$ be an $n \times n$ matrix. Prove (or at least explain convingingly) why $\det(A^\top) = \det(A)$ *(Hint: think about Theorem 31 from lecture 9.)*

**Problem 10.** Show that

$$\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

do not form a basis for $\mathbb{R}^3$.

**Problem 11** (Extra credit)**.** Let $A = (a_{ij})$ be a lower triangular $n \times n$ matrix. Using induction, prove that determinant of $A$ is the product of its diagonal entries (i.e., that $\det(A) = a_{11}a_{22} \cdots a_{nn}$).