

## Math 307: Homework 06

Due Wednesday, October 15 (at the beginning of class). This homework is mostly based on sections 2.3 and 2.4 in the textbook.

**Problem 1.** Is  $\begin{bmatrix} 3 & 5 \\ 3 & 4 \end{bmatrix}$  in  $\text{Span} \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ ?

**Problem 2.** Find the determinant of the matrix  $\begin{bmatrix} 4 & 3 & 2 & 1 \\ -2 & 5 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & -2 \end{bmatrix}$ .

**Problem 3.**

(a) Find the inverse of the matrix  $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 4 & -1 \\ 2 & 1 & 2 \end{bmatrix}$  using row reduction.

(b) Use the inverse matrix from part (a) to solve the linear system

$$\begin{cases} -2y + z = 2 \\ 2x + 4y - z = -1 \\ 2x + y + 2z = 5 \end{cases}$$

**Problem 4.** Show that  $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$  form a basis for  $\mathbb{R}^3$ .

**Problem 5.** For each part, determine whether the vectors form a basis for  $\mathbb{R}^3$ .

(a)  $\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ -1 \\ 8 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$ ,  $\begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$

(c)  $\begin{bmatrix} -7 \\ -9 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

**Instructions for problems 6-9:**

(a) Find a basis for the nullspace of the matrix

(b) Find a basis for the row space of the matrix

(c) Find a basis for the column space of the matrix

(d) Determine the rank of the matrix

(e) Determine whether the matrix is invertible

Note that parts (a)-(c) do not have unique answers.

**Problem 6.**  $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$

**Problem 7.**  $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

**Problem 8.**  $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

**Problem 9.**  $\begin{bmatrix} 1 & 1 & 0 & 3 \\ 1 & 1 & 1 & -2 \\ 3 & 3 & 2 & -1 \end{bmatrix}$

**Problem 10.** Determine whether the given vectors are linearly dependent or linearly independent.

(a)  $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b)  $x^3 - 1, x^2 - 1, x - 1, 1.$