20   2025-02-28 | Week 7 | Lecture 19

An example of normal random variable and standardization:

Example 66. Due to variation in the manufacturing process, the peak power consumption X of the latest Nvidia GPU varies slightly from chip to chip. Assume the power consumption is normally distributed $X \sim \mathcal{N}(\mu, \sigma^2)$. What is the probability that X is within one standard deviation of its mean?

$$\mathbb{P}\left[\mu - \sigma \leq X \leq \mu + \sigma\right] = \mathbb{P}\left[\frac{(\mu - \sigma) - \mu}{\sigma} \leq Z \leq \frac{(\mu + \sigma) - \mu}{\sigma}\right]$$
$$= \mathbb{P}\left[-1 \leq Z \leq 1\right]$$
$$\Phi(1) - \Phi(-1)$$
$$\approx .68$$

So if $\mu = 700$ watts and $\sigma = 35$ watts (realistic values for the Nvidia's new \$30,000 H100 GPU), we have

$$\mathbb{P}\left[665 \leq X \leq 735\right] \approx 0.68.$$

In other words, about 68% of chips will have peak power consumption between 665 and 735 watts.

End of Example 66. □

20.1   Joint distributions

Please read chapter 5.1

We are interested in understanding the properties of *random vectors*, which are vectors whose entries are random variables.

Let X and Y be two discrete r.v.s defined on the sample space $\mathcal{S}$. The *joint pmf* $p(x, y)$ is defined as

$$p(x, y) = \mathbb{P}\left[X = x, Y = y\right]$$

for all $x, y \in \mathcal{S}$. It must be the case that $p(x, y) \geq 0$ and $\sum_{x, y \in \mathcal{S}} p(x, y) = 1$.

Let A be any set consisting of pairs of (x, y) values. For example,

$$A = \{(x, y) : x + y = 5\}$$

or

$$A = \{(1, 2), (1, 4), (1, 6), (2, 5)\}$$

Then the probability

$$\mathbb{P}\left[(X, Y) \in A\right] = \sum_{\substack{x, y \in S \\ (x, y) \in A}} p(x, y)$$

Example 67. Cars made in a factory experience two kinds of defects: defective joint welds, and and improperly tightened bolts. Let

- X = number of defective welds in a new car

- Y = number of improperly tightened bolts

Past data suggests that the joint pdf of (X, Y) is given by the following table:

		Y			
		0	1	2	3
	0	.840	.030	.020	.010
X	1	.060	.010	.008	.002
	2	.010	.005	.004	.001

The probability that there are no defects in the car is

$$p(0,0) = \mathbb{P}\left[X = 0, Y = 0\right] = .84$$

The probability that there will be exactly one defect is

$$p(0,1) + p(1,0) = \mathbb{P}\left[X = 0, Y = 1\right] + \mathbb{P}\left[X = 1, Y = 0\right] = .06 + .03 = .09.$$

What is the probability that there will be no improperly tightened bolts? This concerns only the variable Y. This can be obtained by summing over all values of x:

$$\mathbb{P}\left[Y = 0\right] = \sum_{x=0}^{2} \mathbb{P}\left[X = x, Y = 0\right]$$
$$= p(0,0) + p(1,0) + p(2,0)$$
$$= .84 + .06 + .01$$
$$= .91$$

End of Example 67. □

Given the joint pmf for a two-dimensional discrete random vector (X, Y), it is easy to derive the individual pmfs for X and Y. The manner in which this is done is suggested by the previous example:
If p is the pdf of (X, Y), then the pdf of X alone is obtained by the formula

$$p_X(x) = \sum_{\text{all } y} p(x, y)$$

and the pdf of Y alone is given by

$$p_Y(y) = \sum_{\text{all } x} p(x, y)$$

In this setting, the pdfs p_X and p_Y are called the *marginal pmfs* of X and Y (respectively).

Similarly, if X, Y are continuous r.v.s, then the *joint pdf* $f(x, y)$ is a function satisfying $f(x, y) \geq 0$ and $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$ such that for any 2-dimensional set A,

$$\mathbb{P}\left[(X, Y) \in A\right] = \int_A \int f(x, y) dx dy$$

In addition, we can compute the *marginal density* of X and Y as

$$f_X(x) = \int_{\text{all } y} f(x, y) dy$$

and

$$f_Y(y) = \int_{\text{all } x} f(x, y) dx$$