7 2025-01-27 | Week 3 | Lecture 6

7.1 Conditional probability

This lecture is based on section 2.4 in the text.

First, play craps.

Tallying the results, we saw that shooters won about 50% of the time. Is this a fair game? We will set out to answer this question. To do so, we'll need to introduce some new ideas. A key idea is that of conditional probability:

Definition 20 (Conditional Probability)**.** Let A, B be events, and assume that $\mathbb{P}[A] > 0$. Then the *conditional probability of B, given A*, denoted $\mathbb{P}[B \mid A]$, is given by the formula

$$\mathbb{P}[B \mid A] := \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[A]}.$$

[draw picture]

- Intuitively, $\mathbb{P}[B \mid A]$ is the probability of B when we know that the event H as occurred.
- The idea is that if we know that event A has occured, then the sample space becomes A, and the new event is $A \cap B$.

For example,

Example 21 (Conditional probability)**.** I roll a dice behind a screen. I tell you that I rolled an even number. What's the probability that the dice roll is a 4 or a 6?

Solution: We have $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$, $B = \{4, 6\}$. We want to compute $\mathbb{P}[B \mid A]$. By Definition 20,

$$\mathbb{P}[B \mid A] = \frac{\mathbb{P}[AB]}{\mathbb{P}[A]} = \frac{2/6}{1/2} = \frac{2}{3}.$$

Put differently, we have three outcomes in A, which are all equally likely. Two of them (4 and 6) mean that event B occurs. So by the enumeration principle, the probability is $2/3$.

End of Example 21. □

Then we worked on `worksheet-02.pdf`.