Math 372: Homework 02

Due Wednesday, Jan 28

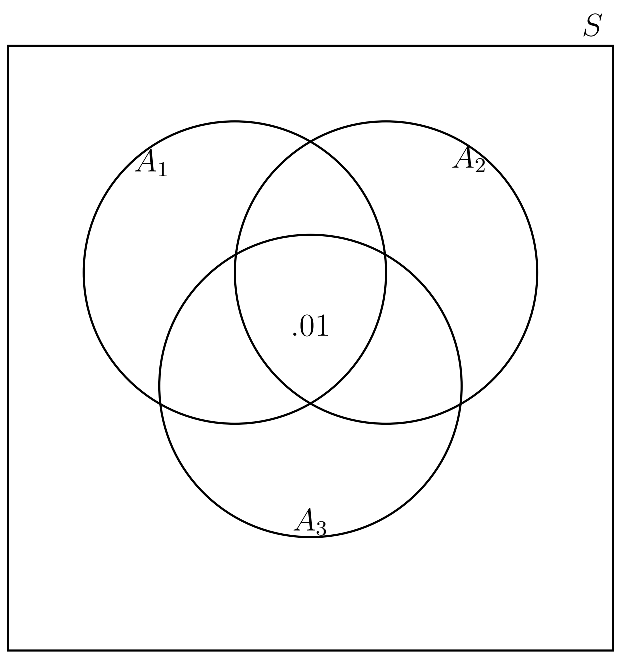
Problem 1 (Sets). A consulting firm has bids on 3 projects. Let

$$A_i = [\text{awarded project } i]$$

for $i \in \{1, 2, 3\}$. Suppose that

$$\mathbb{P}[A_1] = .22$$
$$\mathbb{P}[A_2] = .25$$
$$\mathbb{P}[A_3] = .28$$
$$\mathbb{P}[A_1 \cap A_2] = .11$$
$$\mathbb{P}[A_1 \cap A_3] = .05$$
$$\mathbb{P}[A_2 \cap A_3] = .07$$
$$\mathbb{P}[A_1 \cap A_2 \cap A_3] = .01$$

(a) In the following Venn diagram, the three overlapping circles represent events A_1, A_2, and A_3, all contained within the sample space box S. Each enclosed region represents an event, all of which have some probability of occurring. I've already filled in the probability of one of the events (namely, the event $A_1 \cap A_2 \cap A_3$). Using the probabilities above, fill in the probabilities of other 7 regions:



(b) Express in words each of the following events, and compute the probability of each event:

(i.) $A_1 \cup A_2$

(ii.) $A_1^c \cap A_2^c$ *[Hint: use De Morgan's laws. Note: the textbook uses the notation A_i' in place of A_i^c.]*

(iii.) $A_1 \cup A_2 \cup A_3$

(iv.) $A_1^c \cap A_2^c \cap A_3^c$

(v.) $A_1^c \cap A_2^c \cap A_3$

(vi.) $(A_1^c \cap A_2^c) \cup A_3$

Problem 2 (Sets)**.** Suppose that 55% of all people regularly consume coffee, 45% regularly consume soda, and 70% regularly consume at least one of these two drinks. *[Hint: draw a Venn diagram.]*

(a.) What is the probability that a randomly-selected person consumes both coffee and soda?

(b.) What is the probability that a randomly-selected person doesn't regularly consume at least one of these drinks?

Problem 3 (The geometric distribution)**.** Suppose we have an urn containing exactly 3 balls: two white balls and one gold ball. Consider the experiment specified by the following algorithm:

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Algorithm:

Step 1. Draw a ball at random from the urn.

Step 2. If you drew the gold ball, stop. Otherwise, put the
        ball back into the urn and return to step 1.

Output: the number of times X you drew a ball from the urn.
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In other words, you repeatedly draw a ball from the urn at random until you get the gold ball, where, if you draw a white ball, you have to put it back and draw again. The output is the (random) quantity X, which is the number of times you drew a ball in this process.

(a.) Find a way to actually run this experiment, and repeat it 10 times. Record (i) how you did it, (ii) the ten outputs you got, and (iii) their average.

(b.) What is the sample space S of this experiment? What possible values can X take?

(c.) Compute following probabilities: $\mathbb{P}[X = 1]$, $\mathbb{P}[X = 2]$, $\mathbb{P}[X = 3]$, $\mathbb{P}[X = 4]$, $\mathbb{P}[X = 5]$,

(d.) Give a formula for $\mathbb{P}[X = n]$, where n is a positive integer.

(e.) Write the event $[X \leq 5]$ as a subset of the sample space S . Compute $\mathbb{P}[X \leq 5]$.

(f.) Write the event $[X \text{ is even}]$ as a subset of the sample space. Use the *geometric series formula*

$$1 + x + x^2 + x^3 + x^4 + \ldots = \frac{1}{1-x} \quad \text{(valid whenever } |x| < 1)$$

to compute $\mathbb{P}[X \text{ is odd}]$.

(g.) Compute $\mathbb{E}[X]$, the expectation of X. Use the formula

$$\mathbb{E}[X] = \sum_{n=1}^{\infty} n \mathbb{P}[X = n] \,.$$

Compare with your answer to part (iii) of (a).

Problem 4 (Counting)**.** How many 7-character license plates are possible if the first 3 are to be occupied by letters and the final 4 by numbers?

Problem 5 (Counting)**.** How many ways can you get dealt 5 cards containing exactly 2 jacks?

Problem 6 (Counting)**.** How many ways are there of flipping 10 coins and getting 5 heads? What is the probability of this occurring?

Problem 7 (Disjoint events)**.** Suppose I have an urn with 6 balls, labeled $1, \ldots, 6$. If I draw two balls, what is the probability that they are either (1) both even or (2) both odd?

Problem 8 (Counting)**.** A dessert company has 31 distinct desserts on its menu: 5 types of pie, 9 types of cake, and 17 flavors of ice cream. *(You don't need to simplify your answers for this problem)*

(a) After purchasing 4 different desserts, a customer decides he will eat them all in a row, and he cares about the order in which he eats them. How many different ways can he eat his 4 desserts?

(b) Suppose 7 different desserts are selected from the menu. How many ways are there to do this if (i) we care about the order in which the 7 deserts are selected, and (ii) if we don't care about the order?

(c) If 9 desserts are randomly selected, how many ways are there to obtain 3 deserts of each type (pie, cake, and ice cream)?

(d) If 5 desserts are randomly selected, what's the probability that they are all of the same type (i.e., all pie, all cake, or all ice cream)?

Problem 9. Consider randomly selecting a student at a large university. Let A be the event that the selected student has a Visa card and B be the event that the student has a MasterCard. Suppose that $\mathbb{P}[A] = .6$ and $\mathbb{P}[B] = .4$

(a) Could it be the case that $\mathbb{P}[A \cap B] = .5$? Why or why not?

(b) From now on, suppose that $\mathbb{P}[A \cap B] = .3$. What is the probability that the selected student has at least one of these two types of cards?

(c) What is the probability that the selected student has neither type of card?

(d) Describe, in terms of A and B, the event that the student has a Visa card but not a MasterCard. Then calculate the probability of this event.

(e) Calculate the probability that the selected student has exactly one of the two types of cards.