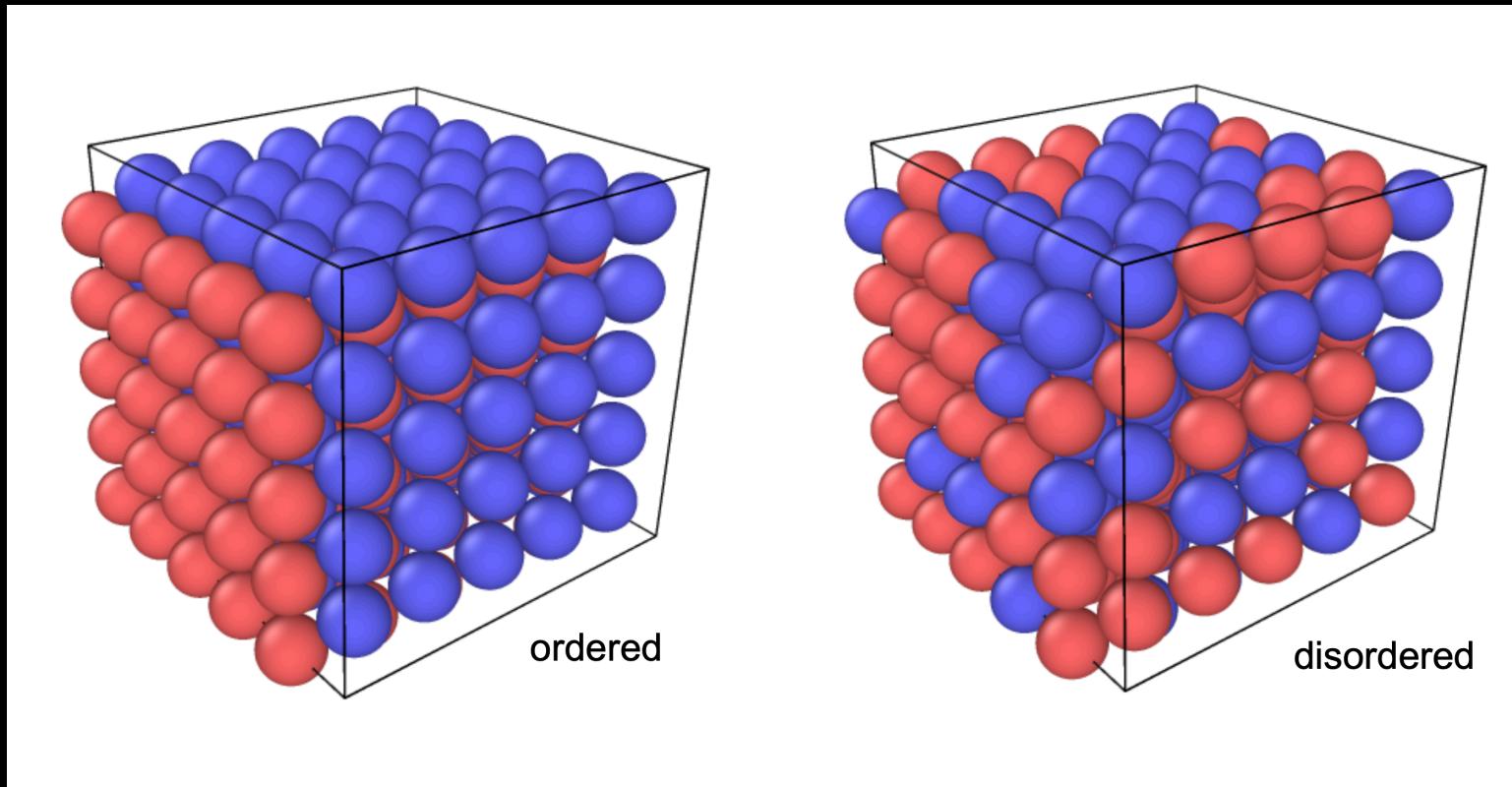


Symmetry Implementation for Pair Distribution Function

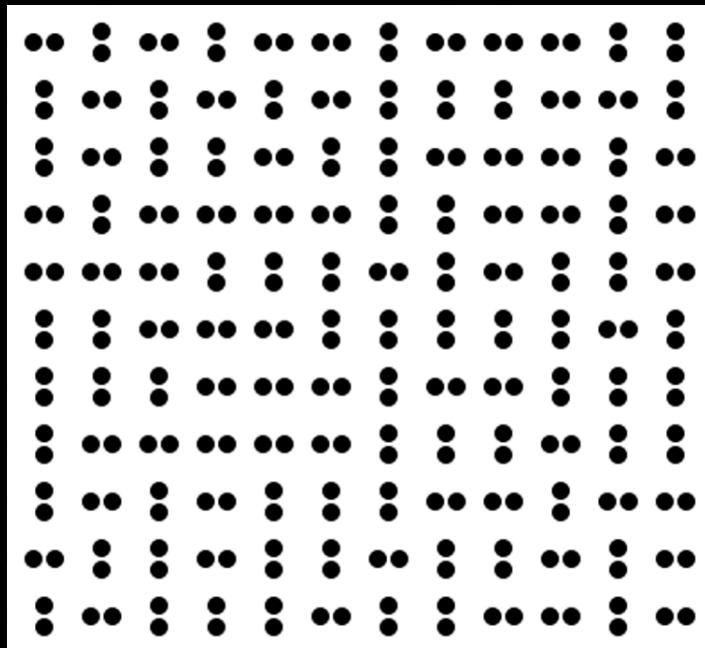
Max Krummenacher

July 2024

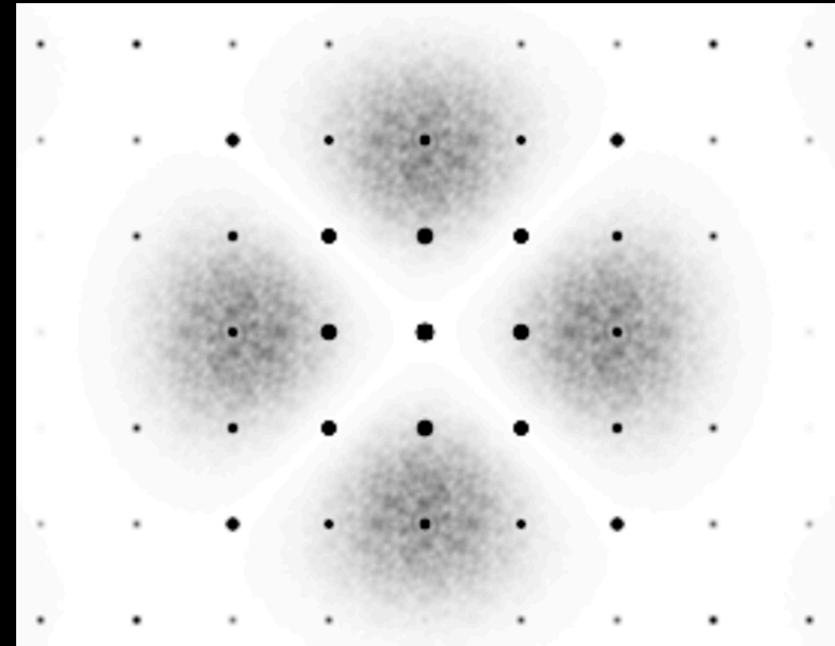
Disordered Crystals



Diffraction



Structure



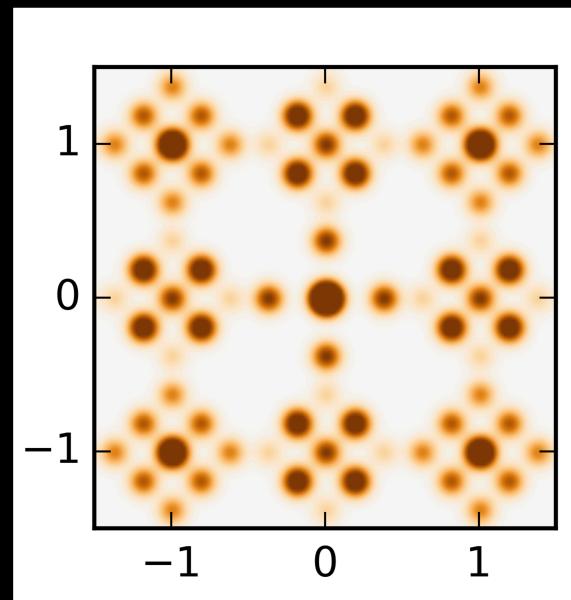
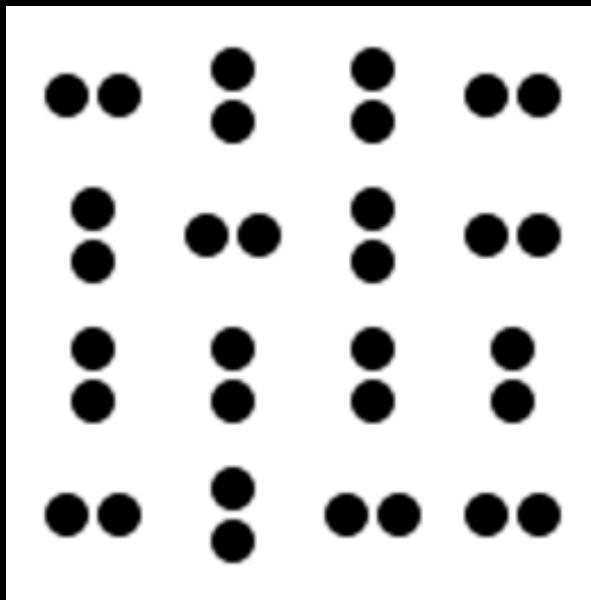
Diffraction Pattern

Pair Distribution Function

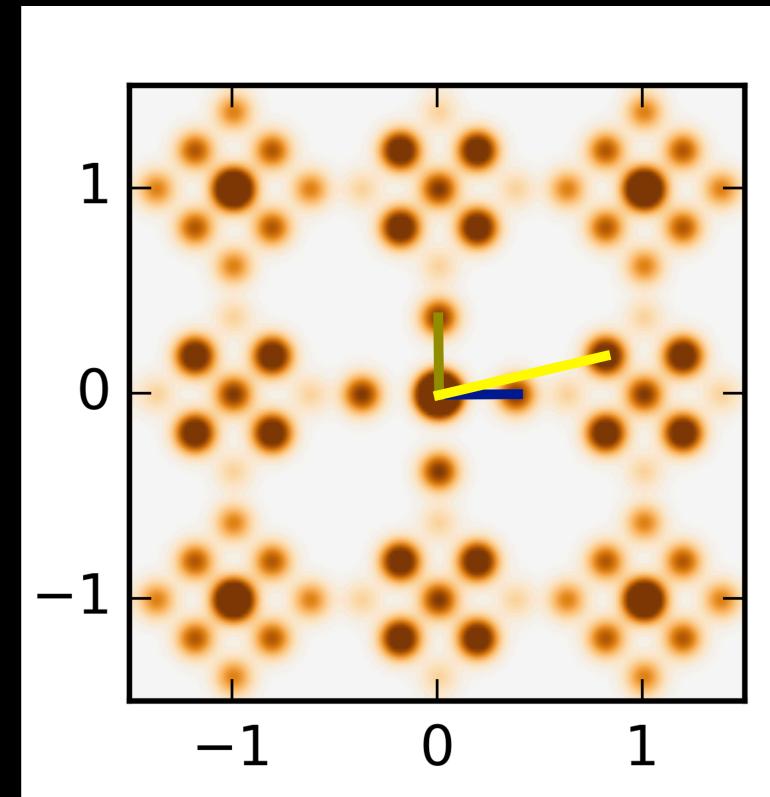
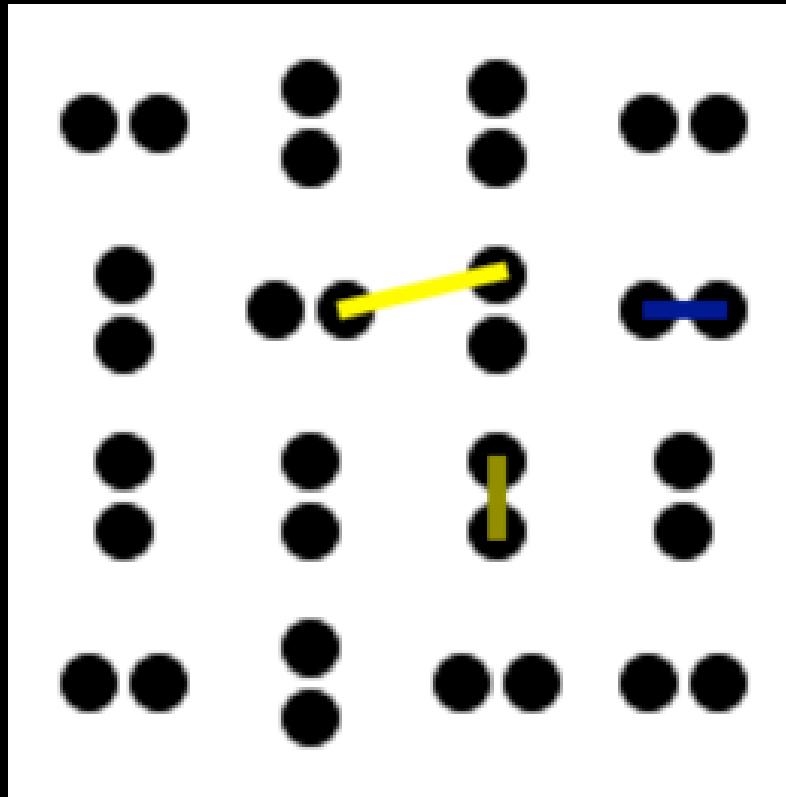
$$\text{PDF}(\vec{x}) = \int_{\mathbb{R}^3} \rho(\vec{\xi}) \rho(\vec{\xi} - \vec{x}) d\vec{\xi}$$

Pair Distribution Function

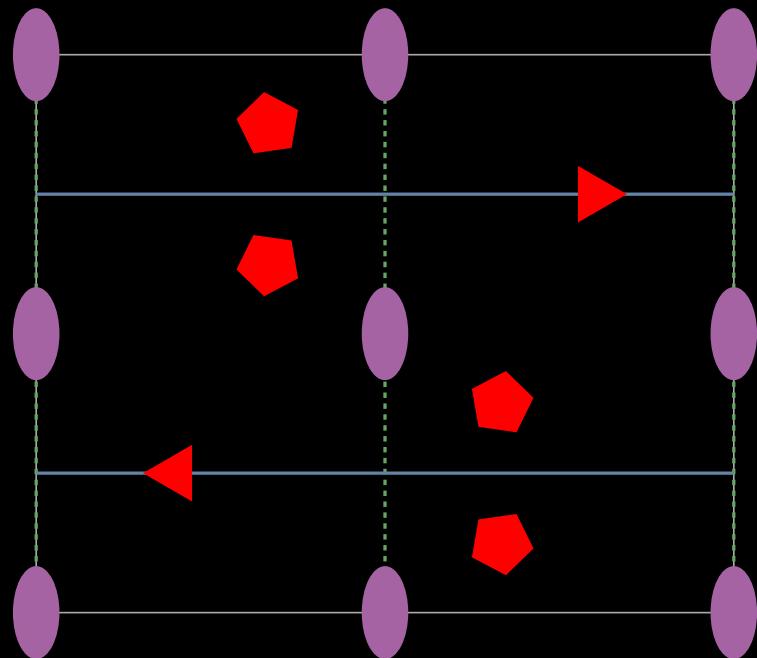
$$\text{PDF}(\vec{x}) = \int_{\mathbb{R}^3} \rho(\vec{\xi}) \rho(\vec{\xi} - \vec{x}) d\vec{\xi}$$



Pair Distribution Function



Wyckoff Positions

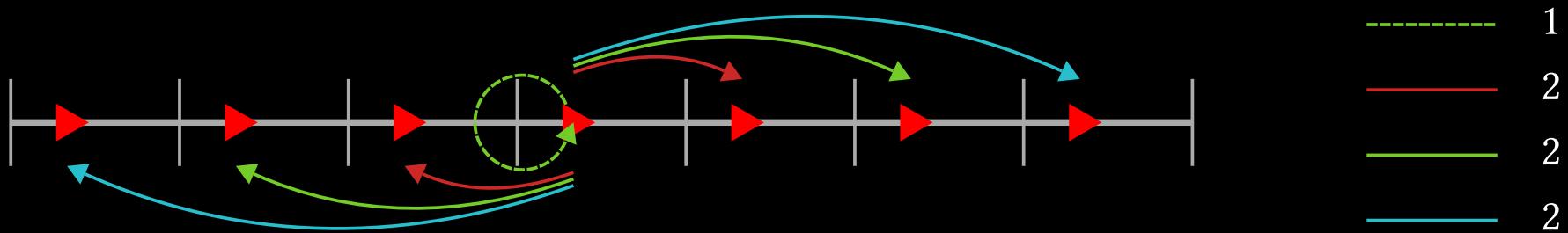


Wallpaper Group p2mg

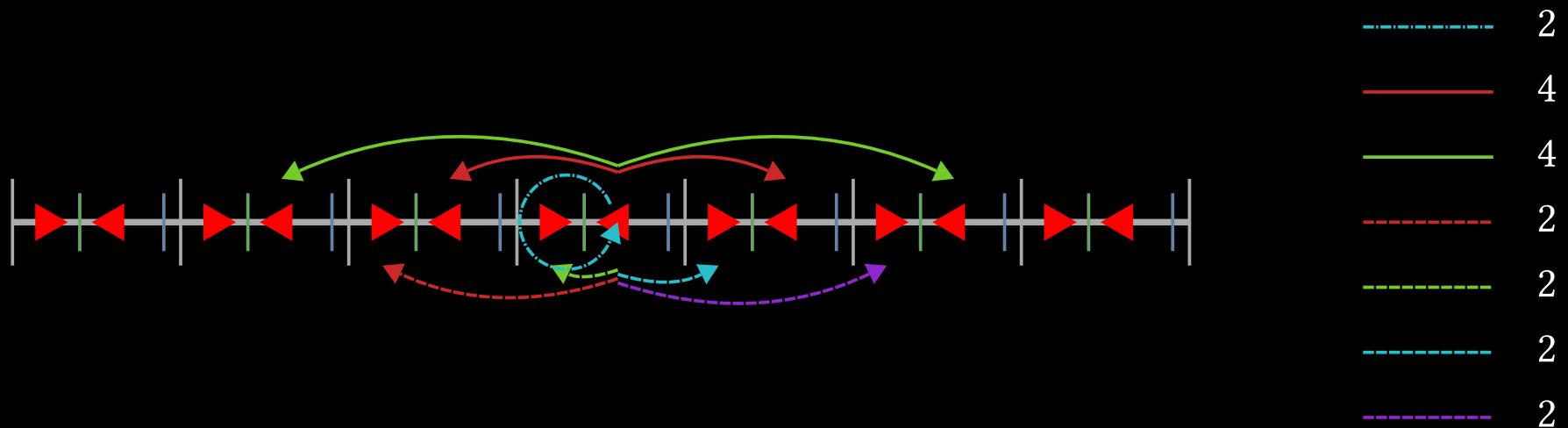
Examples

- Line Group p1
- Line Group p1m
- Wallpaper Group p2mg
- Space Group Pm $\bar{3}$ m

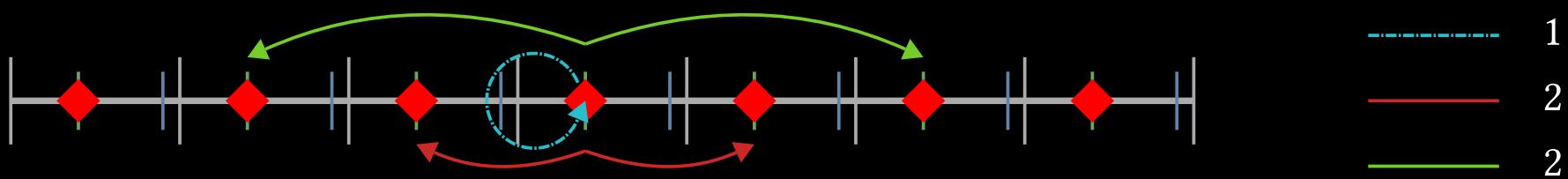
Line Group p1



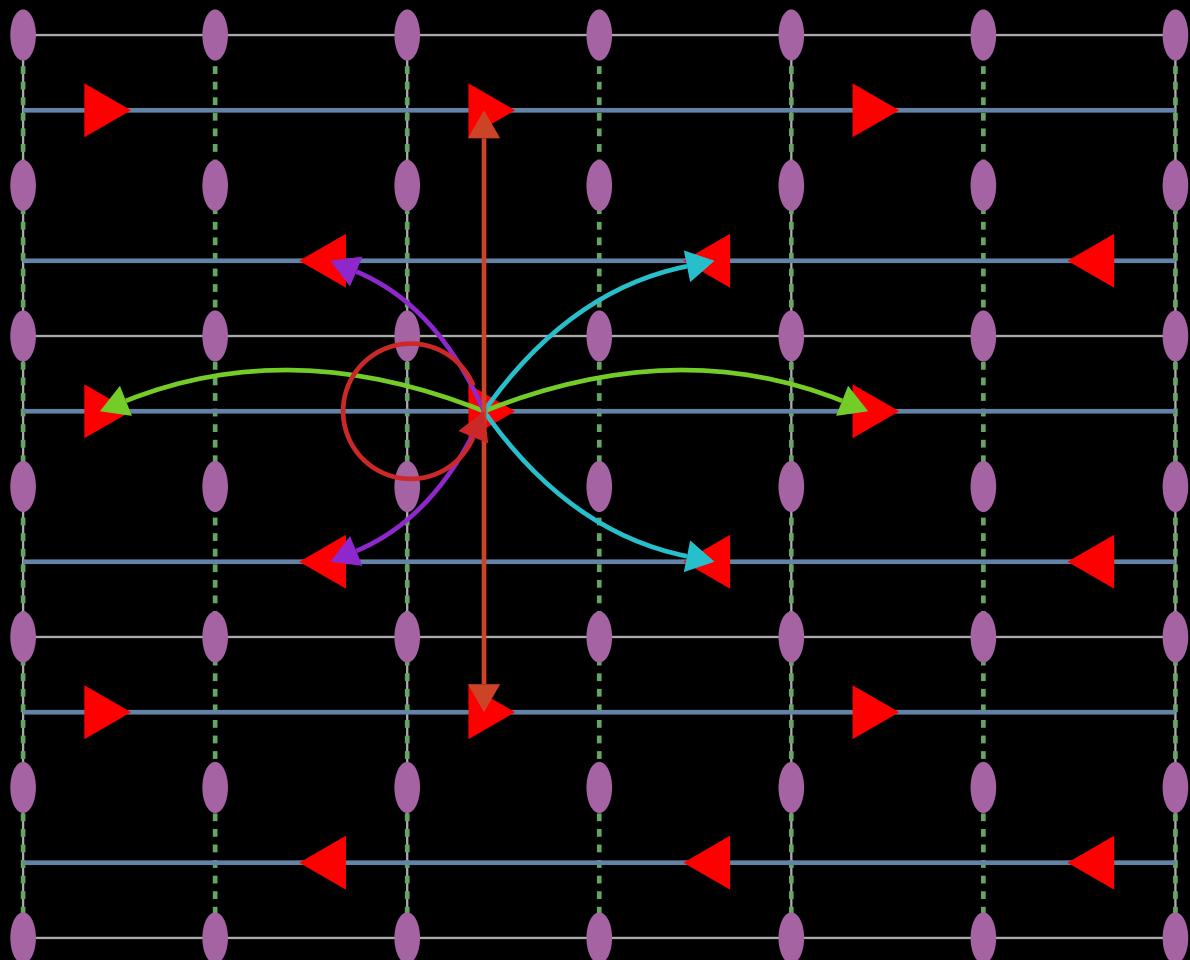
Line Group p1m General Position



Line Group p1m Special Position

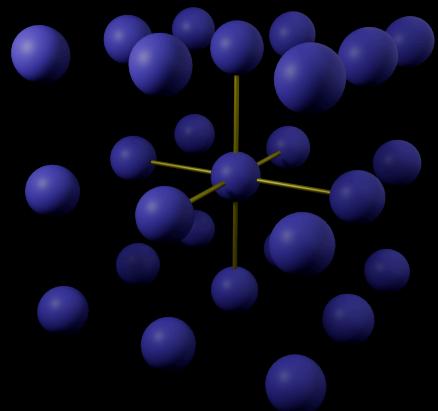


Wallpaper Group p2mg

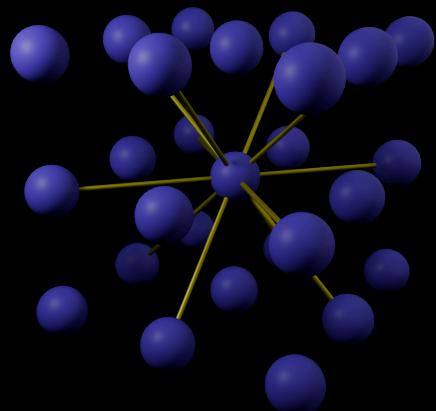


2
4
4
4
4

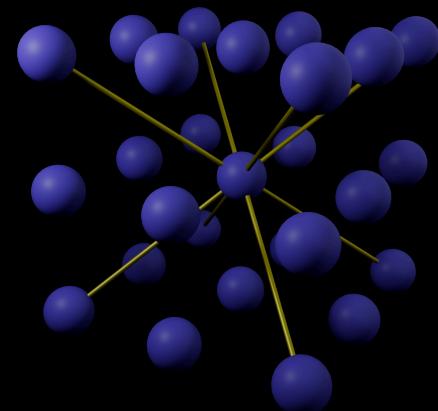
Space Group $\text{Pm}\bar{3}\text{m}$



$\langle 100 \rangle : 6$



$\langle 110 \rangle : 12$



$\langle 111 \rangle : 8$

Example Yell Input

The Algorithms and their Implementation

- Isometries
- Space Groups
- Bounds
- Wyckoff Positions
- Pair Expansions
- Main Algorithm

Isometry Implementation

Isometry Implementation

- Rational Numbers $\frac{p}{q}$

Isometry Implementation

- Rational Numbers $\frac{p}{q}$
- Vectors a

Isometry Implementation

- Rational Numbers $\frac{p}{q}$
- Vectors a
- Matrices A

Isometry Implementation

- Rational Numbers $\frac{p}{q}$
- Vectors a
- Matrices A
- Positions X

Isometry Implementation

- Rational Numbers $\frac{p}{q}$
- Vectors a
- Matrices A
- Positions X
- Affine Transformations $\mathcal{A} = (A, a)$

Isometry Implementation

- Rational Numbers $\frac{p}{q}$
- Vectors a
- Matrices A
- Positions X
- Affine Transformations $\mathcal{A} = (A, a)$

$$\tilde{X} = \mathcal{A}X = AX + a$$

Isometry Implementation

- Rational Numbers $\frac{p}{q}$
- Vectors a
- Matrices A
- Positions X
- Affine Transformations $\mathcal{A} = (A, a)$

$$\tilde{X} = \mathcal{A}X = AX + a$$

- Isometries

Isometry Implementation

- Rational Numbers $\frac{p}{q}$
- Vectors a
- Matrices A
- Positions X
- Affine Transformations $\mathcal{A} = (A, a)$

$$\tilde{X} = \mathcal{A}X = AX + a$$

- Isometries

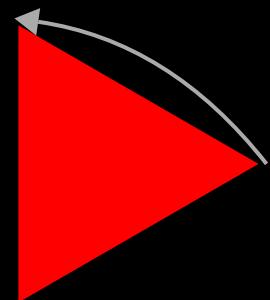
special affine transformation which preserve distances and angles

Space Groups

Generators

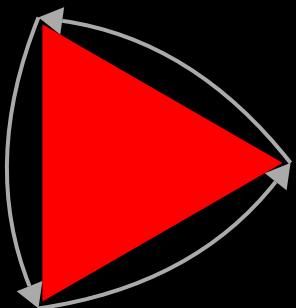
Space Groups

Generators



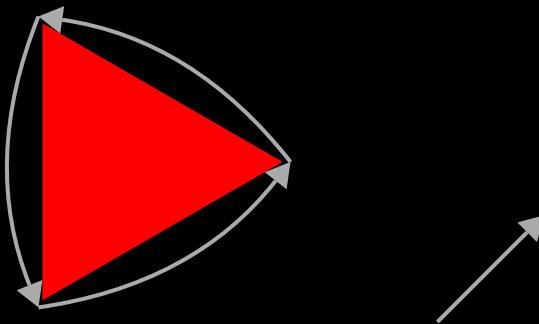
Space Groups

Generators



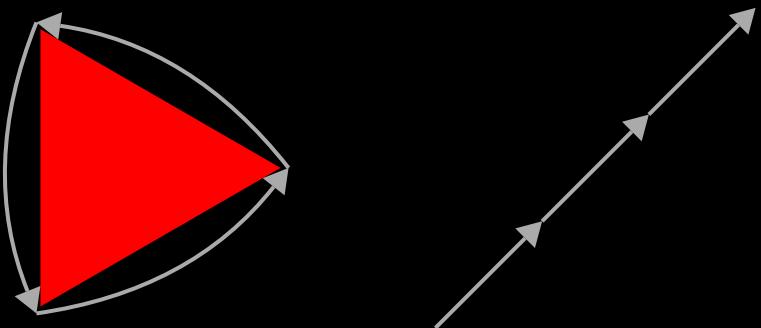
Space Groups

Generators



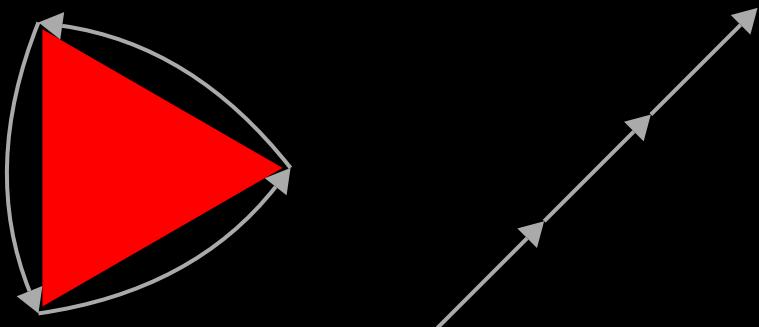
Space Groups

Generators



Space Groups

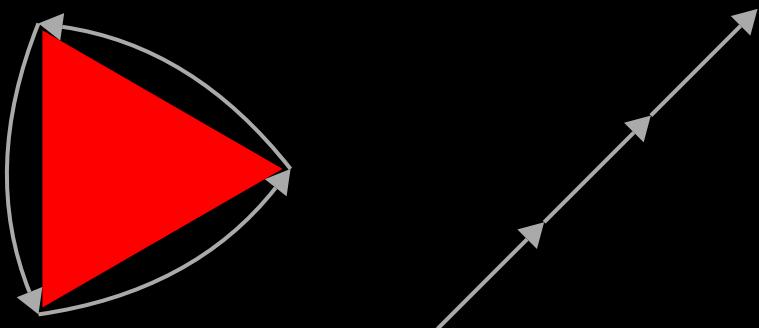
Generators



Space groups are infinite!
What should we do?

Space Groups

Generators



Bounds

- act on positions, vectors, and affine transformations
- allow finite representation of space group
- for unitcell and space group bound to [1, 1, 1]

Space groups are infinite!
What should we do?

Wyckoff Positions

```
fn site_new(position, group) -> Site {
    bounds = Bounds3(1, 1, 1);
    position = position % bounds;
    orbit = [position];
    stabilizer = [];

    for sym in group.get_symmetries() {
        new_pos = sym * position;
        if new_pos % bounds == position {
            stabilizer.push(symmetry_from_translation(position - new_pos) * sym);
        } else if new_pos % bounds not in orbit {
            orbit.push(new_pos % bounds);
        }
    }

    return Site { position, orbit };
}
```

Pair Expansions

Pair Expansions

- represents all equivalent pairs starting at a certain position

Pair Expansions

- represents all equivalent pairs starting at a certain position
- generated in a similar fashion to the orbit wyckoff positions

Pair Expansions

- represents all equivalent pairs starting at a certain position
- generated in a similar fashion to the orbit wyckoff positions
- can be compared against other pairs

Pair Expansions

- represents all equivalent pairs starting at a certain position
- generated in a similar fashion to the orbit wyckoff positions
- can be compared against other pairs

Main Algorithm

Pair Expansions

- represents all equivalent pairs starting at a certain position
- generated in a similar fashion to the orbit wyckoff positions
- can be compared against other pairs

Main Algorithm

- generates the space group

Pair Expansions

- represents all equivalent pairs starting at a certain position
- generated in a similar fashion to the orbit wyckoff positions
- can be compared against other pairs

Main Algorithm

- generates the space group
- determines the Wyckoff position of the sites

Pair Expansions

- represents all equivalent pairs starting at a certain position
- generated in a similar fashion to the orbit wyckoff positions
- can be compared against other pairs

Main Algorithm

- generates the space group
- determines the Wyckoff position of the sites
- determines all possible pairs between these position

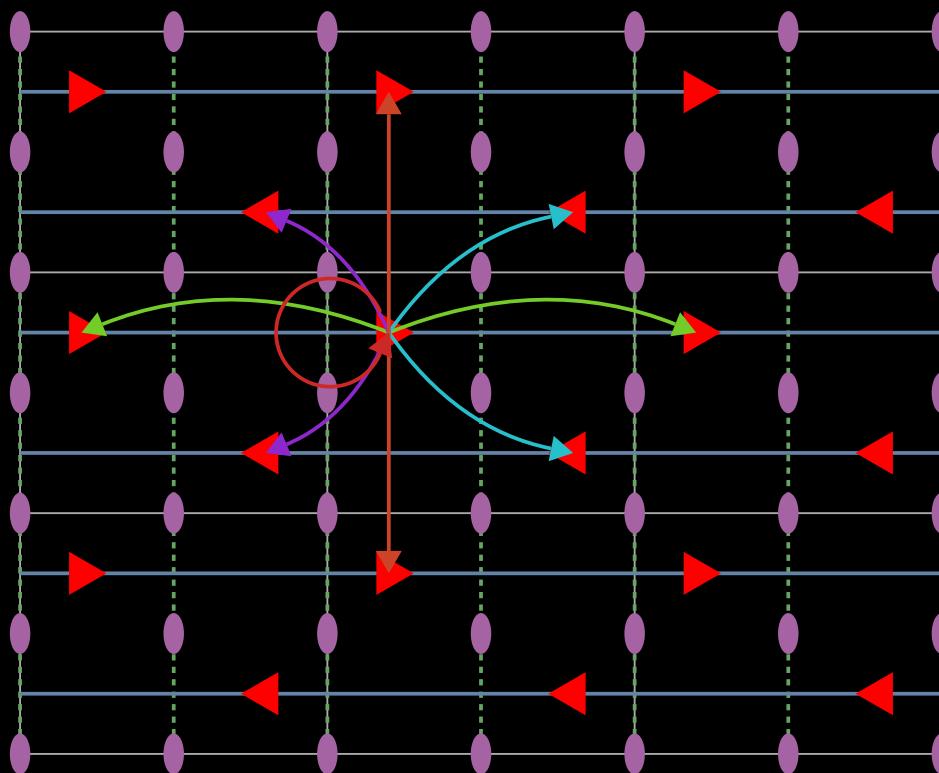
Pair Expansions

- represents all equivalent pairs starting at a certain position
- generated in a similar fashion to the orbit wyckoff positions
- can be compared against other pairs

Main Algorithm

- generates the space group
- determines the Wyckoff position of the sites
- determines all possible pairs between these position
- calculates the pair multiplicity

Website



Thank you for your Attention!

