

PROBLEM 1.1.

$$\frac{x^{k+2}}{x^{k-2}} = x^{k+2-k+2} = x^4$$

PROBLEM 1.2

$$x^{-1} \cdot 8 = 2$$

$$\frac{8}{x} = 2$$

$$2x = 8$$

$$x = 4$$

PROBLEM 1.3

$$a=5 \quad b=10$$

$$(5^{10})^0 = 5^{10 \cdot 0} = 5^0 = 1$$

PROBLEM 1.4

$$\frac{\sqrt{4x}}{\sqrt{x}} = \sqrt{\frac{4x}{x}} = \sqrt{\frac{4}{1}} = \sqrt{4} = 2$$

PROBLEM 1.5

$$x^2 + (x+1)^2 = (x+2)^2$$

~~$$x^2 + (x+1)^2 = (x+2)^2$$~~

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$$

$$x^2 - 2x - 3 = 0 = (x-3)(x+1)$$

$$x = 3 \quad x = -1$$

PROBLEM 1.6.

$$2^x > 1024$$

$$x > \log_2 1024$$

$$x > 10$$

PROBLEM 2.1.

$$ax + b = 32 \quad x = 0 \quad b = 32$$

$$ax + b = 212 \quad x = 100$$

$$100a + 32 = 212$$

$$a = 1.8$$

$$ay + b = 0 \quad y = 32$$

$$ay + b = 100 \quad y = 212$$

$$32a + b = 0$$

$$180a = 100$$

$$212a + b = 100$$

$$a = \frac{100}{180} = \frac{10}{18} = \frac{5}{9}$$

$$b = -(32 \cdot \frac{5}{9}) = -(\frac{160}{9}) = -17\frac{7}{9}$$

$$1.8x + 32 = \frac{5}{9}x - 17\frac{7}{9}$$

$$\frac{18}{10}x - \frac{5}{9}x + 32 + 17\frac{7}{9} = 0$$

$$(\frac{162}{90} - \frac{50}{90})x + 49\frac{7}{9} = 0$$

$$\frac{112}{90}x = -49\frac{7}{9}$$

$$x = 40 \checkmark$$

PROBLEM 2.2

$$f(x) = 5x + 4$$

$$x = 3$$

$$5 \cdot 3 + 4 = 15 + 4 = 19$$

$$y = 19$$

PROBLEM 2.3

$$x^2 - 4x + 3 = 0 =$$

$$= (x-3)(x-1)$$

$$x = 3$$

$$x = 1$$

PROBLEM 2.4

$$10(1+0.02)^{90} =$$

$$= 59.431$$

$$A = P(1 + \frac{r}{n})^{nt}$$

PROBLEM 2.5

$$e^{\ln 5} = 5$$

By definition of \ln

PROBLEM 3.1

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = 3$$

$$\sum_{i=1}^{\infty} \frac{a}{1-r} = \frac{a}{1-r} \quad a = \frac{12}{6} \quad r = \frac{12}{36} \quad |r| < 1$$

$$= \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = \frac{2 \cdot 3}{2} = 3$$

PROBLEM 3.5

$$\frac{d^2}{dx^2} 4x^4 + 4x^2 =$$

$$= \frac{d}{dx} 16x^3 + 8x =$$

$$= 48x^2 + 8$$

PROBLEM 3.6

$$\frac{d}{dx} \frac{\ln x}{e^x} = \frac{e^{-x}(1-x \log(x))}{x} = \frac{d}{dx} \ln x e^{-x}$$

PROBLEM 3.2

$$\lim_{x \rightarrow \infty} \frac{6^{1-x}}{x} = \frac{0}{\infty} = 0$$

$$\lim_{x \rightarrow \infty} 6^{1-x} = 0$$

PRB 3.2

$$\lim_{x \rightarrow 1} \frac{6^{1-x}}{x} =$$

$$x \rightarrow 1$$

$$\lim_{x \rightarrow 1} \frac{6^{1-x}}{x} = 1$$

PROBLEM 3.7

$$3x^2 - 5x + 2$$

$$(3x-2)(x-1) = 0$$

$$x = \frac{2}{3} \quad x = 1$$

$$P'(x) = 6x - 5 \quad P''(x) = 6$$

$$6x - 5 = 0$$

$$6x = 5$$

$$x = \frac{5}{6}$$

PROBLEM 3.3

$$f(x) = x^5 - 8$$

slope at $x = 3$

$$f'(x) = 5x^4$$

$$x = -3$$

$$f'(-3) = 5(-3)^4 = 5 \cdot 81 = 405$$

PROBLEM 3.4

$$\frac{d}{dx} \frac{x^3 + 2x - 1}{x - 2} = \frac{(3x^2 + 2)(x - 2) - (x^3 + 2x - 1) \cdot 1}{(x - 2)^2}$$

$$= \frac{2x^3 - 6x^2 - 3}{(x - 2)^2}$$

x	$-\infty$	$x < \frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3} < x < \frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6} < x < 1$	1	∞
$P(x)$	$\infty +$	$+$	0	$-$	$-$	$-$	0	$+$
$P'(x)$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$
SHAPE	\searrow	\searrow	\searrow	\searrow	MIN	\nearrow	\nearrow	\nearrow
$P''(x)$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
SHAPE	\cup	\cup	\cup	\cup	\cup	\cup	\cup	\cup

PROBLEM 3.8

$$f(x, y) = x^2 + y^3 \quad x = 2$$

$$y = 3$$

$$f(2, 3) = 2^2 + 3^3 = 4 + 27 = 31$$

PROBLEM 3.9

$$f(x, y) = \ln(x - y)$$

$$(x, y) \in \mathbb{R}^2; x > y$$

PROBLEM 3.10

$$\frac{\partial}{\partial x} x^5 + xy^3 = 5x^4 + y^3$$

PROBLEM 3.11

$$f(x, y) = x^2 y^2 + 10$$

$$\frac{\partial}{\partial x} = 2xy^2 \quad 2xy^2 = 0 \quad x = 0$$

$$\frac{\partial}{\partial y} = 2x^2 y \quad 2x^2 y = 0 \quad y = 0$$

$$f(x, 0) = 10 - \text{local minimum}$$

$$f(0, y) = 10 - \text{local minimum}$$

$$\text{Point } (0, 0) - \text{global minimum}$$

PROBLEM 3.12

$$\text{MAX } x^2 y^2$$

$$x + y = 10$$

$$x^2 y^2 - \lambda(x + y - 10)$$

$$\frac{\partial}{\partial x} = 2xy^2 - \lambda = 0$$

$$\frac{\partial}{\partial y} = 2x^2 y - \lambda = 0$$

$$\frac{\partial}{\partial \lambda} = -x - y + 10 = x + y - 10 = 0$$

$$x + y = 10$$

$$2xy^2 - \lambda = 0$$

$$2x^2 y - \lambda = 0$$

$$x + y = 10$$

$$x = 10 - y$$

$$\cancel{2x(10-y)^2 - \lambda = 0}$$

$$2xy^2 - \lambda = 2x^2 y - \lambda$$

$$2xy^2 = 2x^2 y \quad / : 2$$

$$xy^2 = x^2 y$$

$$y^2 = \frac{x^2 y}{x} \quad y^2 = xy$$

$$x = y$$

$$x = 10 - x$$

$$2x = 10$$

$$x = 5$$

$$y = 5$$

PROBLEM 4.1.

$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$$

~~$$A \cdot B = \begin{bmatrix} 14 \\ 13 \\ 25 \end{bmatrix}$$~~

$$A \cdot B = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

PROBLEM 4.2

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

PROBLEM 4.3

$$\begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix} = M$$

$$M^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

PROBLEM 4.4

$$M = \begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix}$$

$$\text{Det} = 8 - 18 = -10$$

PROBLEM 5.1

$$\Omega = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$$

$$SIZE = 36$$

PROBLEM 5.2

$$USES DRUGS = 1\% = 0.01$$

$$DRUG FREE = 99\% = 0.99$$

$$TRUE POSITIVE = 99\% = 0.99$$

$$FALSE NEGATIVE = 0.01\% = 0.01$$

$$TRUE NEGATIVE = 99.5\% = 0.995$$

$$FALSE POSITIVE = 0.05\% = 0.005$$

$$POSITIVE RESULT = (T.P.) \cdot (U.D.) + (F.P.) \cdot (D.F) =$$

$$= 0.99 \cdot 0.01 + 0.005 \cdot 0.99 = 0.01485$$

$$P.R. = 0.01485 = 1.485\%$$

$$P(U.D. | P.R.) = \frac{U.D. \cdot T.P.}{P.R.} = \frac{0.99 \cdot 0.01}{0.01485} = \frac{2}{3} = \frac{66\%}{100\%} = 66\%$$

PROBLEM 5.3