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Data and Signal Analysis

4th Exercise Sheet
Winter Semester 2024/2025

Submission unit 20/12/2024 1 p.m. CET

1. Exercise: Windowing (8 Points)

In the folder `Exercise Sheets`, you will find the file `windowing.txt`, which contains a synthetic time series representing a superposition of three sinusoidal oscillations with different frequencies and amplitudes. These three frequencies should be determined using the discrete Fourier transform. The sampling frequency is 1 mHz.

a) Plot the given time series.

b) Calculate the one-sided power spectral density spectrum and display it in a graph. Which window function have you implicitly applied?

c) At which frequencies does the power spectral density spectrum maximize? Can you identify the three superposed oscillations in the spectrum?

d) Multiply the time series by the Hanning window (e.g. `numpy.hanning`), recalculate the power spectral density spectrum, and plot it graphically. At which frequencies does this second power spectral density spectrum maximize?

e) Explain the differences between the two power spectral density spectra; discuss the effect of the so-called "windowing"!

2. Exercise: Empirical Mode Decomposition (7 Points)

a) In the folder `Exercise Sheets`, you will find the time series `signa_emd.txt`, where the first column is the time vector and the second column is the signal. Apply the Empirical Mode Decomposition (EMD) to the given signal to determine the intrinsic mode functions (IMF). Plot the signal and its IMFs. Interpret the results.

b) Determine the instantaneous amplitude, phase, and frequency of the individual IMFs using the method of the analytic signal. What frequencies are present in the signal? *Optional:* Display the instantaneous frequencies and amplitudes in a Hilbert-Huang spectrum.

To perform the EMD calculation, please install the Python library `emd` (`https://pypi.org/project/emd/`). Read the documentation of the library (`https://emd.readthedocs.org`) thoroughly to be able to complete the required steps of the task.

3. Exercise: Method of Least Squares (5 Points)

The height of a projectile in a homogeneous gravitational field, neglecting air resistance, is described by

$$y(t) = m_1 + m_2 \cdot t - \frac{1}{2} m_3 \cdot t^2 \,, \tag{1}$$

In a ballistic experiment, the following values were measured:

t [s]	1	2	3	4	5	6	7	8	9	10
y [m]	109.4	187.5	267.5	331.9	386.1	428.4	452.2	498.1	512.3	513.0

The parameters m_1, m_2, and m_3 are to be determined.

a) Set up the data kernel matrix $\underline{\underline{A}}$ of the linear inversion problem.

b) Solve the inversion problem using an unweighted least-squares method and determine the coefficients m_1, m_2, and m_3. Plot your result. Use only the vector algebra library `numpy.linalg` for matrix inversion, and do not use a least-squares library from `scipy.optimize`.

c) Calculate the residual variance.