Technische Universität Braunschweig

IGEP TU Braunschweig
Institut für Geophysik und extraterrestrische Physik

Prof. Dr. Ferdinand Plaschke
Patrick Kolhey M. Sc.

Data and Signal Analysis

6th Exercise Sheet
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1. Exercise: Signal Analysis (8 Points)

The file `signal_b5.txt` contains a signal composed of various wave packets. Find all the contained frequencies. Proceed as follows:

a) Plot the signal and the corresponding spectral power density.

b) Calculate the dynamic spectrum. What time resolution provides the best representation of the spectrum?

c) Perform a wavelet analysis. Use the Mexican-Hat wavelet and the Morlet wavelet.

d) Interpret the results and explain the advantage of the different methods. What should be considered when selecting the mother wavelet?

Use the `cwt` function of the `PyWavelets` library (external package!) to perform the continous wavlet transform. See: `https://pywavelets.readthedocs.io/en/latest/index.html`

2. Exercise: Magnetopause (4 Points)

In the file `mag_data_themis.txt`, you will find the time information, as well as the x, y, and z components of the magnetic field (in the GSE coordinate system), measured by a THEMIS satellite during its passage through the magnetopause. The direction $\vec{n}$ of the minimum variance

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\left(\vec{B}_i - \left\langle\vec{B}\right\rangle\right)\cdot\vec{n}\right]^2 \tag{1}$$

subject to the constraint $|\vec{n}|^2 = 1$ corresponds to the normal direction of the local magnetopause surface.

a) Physically justify the above statement.

b) Apply the minimum variance analysis to the THEMIS time series and thus determine the direction of the magnetopause normal in the GSE coordinate system.

3. Exercise: Filter (8 Points)

In the file `sinus13.txt`, you will find a signal composed of 3 superimposed sinusoidal oscillations; the sampling period is 1 s. The objective is to retrieve the 3 individual signals in the time domain. To do this, the signal must be filtered using a low-pass, a high-pass, and a band-pass filter.

a) Graphically represent the entire signal.

b) Perform a Fourier transform on the signal. Also calculate the amplitude spectrum and plot the result.

c) Multiply the Fourier-transformed signal by the system response function $H(f)$ for an ideal low-pass filter. This is constructed as follows: For frequencies $|f| < f_{\text{co}}$ (cut-off frequency), it is 1, otherwise 0. Note that you need a system response function of the same length as the signal, and that the Fourier transform of a real-valued function is mirrored in the middle and complex conjugated; the second half of the spectrum corresponds to the negative frequencies (not necessary with a real FFT). The cut-off frequency should be chosen so that only the sinusoidal oscillation of the lowest frequency is not filtered out.

d) Inversely transform the modified Fourier-transformed signal back to the time domain and plot the result. What has changed?

e) Now also inversely transform the system response function $H(f)$ and plot the result ($h(n)$). How can you tell that the ideal low-pass filter is not causal?

f) Repeat steps (c) and (d) using an ideal high-pass and band-pass filter to extract the other two signal components from the entire signal.

Contact information: Patrick Kolhey, Office 519, email: p.kolhey@tu-braunschweig.de