Technische Universität Braunschweig

IGEP TU Braunschweig Institut für Geophysik und extraterrestrische Physik

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Data and Signal Analysis

3rd Exercise Sheet
Winter Semester 2024/2025

Submission unit 06/12/2024 1 p.m. CET

1. Exercise: Analytical Signal (10 Points)

a) In the folder `Exercise Sheets`, you will find the time series `signal.txt`. Apply the method of the Analytic Signal to the first column of the file to determine the instantaneous amplitude, instantaneous phase, and instantaneous frequency. Explain your procedure. Plot all quantities and interpret the results. Use the `hilbert` function from the `scipy.signal` library for the Hilbert transform and thoroughly read the documentation for this function.

b) Now assume that the second and third columns each represent the y-component of a wave (with the first column as the x-component) and determine the ellipticity and azimuth of the wave. Plot the results and interpret them.

2. Exercise: Nyquist Frequency (5 Points)

Given is a signal that corresponds to the function

$$y(t) = 10 \cdot \sin(2\pi \cdot f_1 \cdot t) + 5 \cdot \sin(2\pi \cdot f_2 \cdot t) + 2 \cdot \sin(2\pi \cdot f_3 \cdot t), \tag{1}$$

where the frequencies are $f_1 = 2$ Hz, $f_2 = 32$ Hz, and $f_3 = 128$ Hz. Assume that the signal was recorded over a time interval of $t = [0, 1]$ s.

a) Initially, use two different sampling rates of $f_s = 50$ Hz and $f_s = 100$ Hz, and represent the one-sided auto-power spectral density spectrum of the time series using the Fast Fourier Transform. Describe your observations.

b) Provide an estimation for the sampling rate needed to reasonably resolve the time series. Again, represent the one-sided auto-power spectral density spectrum and describe your observations.

For this task, use the necessary functions from the `numpy.fft` library and thoroughly read their documentation.

3. Exercise: Correlation Function and Fourier Transformation (5 Points)

In this exercise, the correlations of two time series in the files `zeitreihe1.txt` and `zeitreihe2.txt`, which are located in the folder `Exercise Sheets`, will be examined. Write a code that uses the functions `correlate` and `correlation_lags` from the

`scipy.signal` library to calculate the auto- and crosscorrelation of the time series.

a) Plot both time series.

b) Determine the autocorrelation of time series 1 and the autocorrelation of time series 2. Interpret these results.

c) Determine the cross-correlation of time series 1 with time series 2. Interpret this.

d) Now determine the one-sided power spectral density function of the time series. Interpret this.

e) Determine the one-sided Fourier transform of the autocorrelation of the first time series. Show that the Wiener-Khinchine theorem (see lecture's manuscript) holds. Why does it not hold for the second time series?