Technische Universität Braunschweig

IGEP TU Braunschweig Institut für Geophysik und extraterrestrische Physik

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Data and Signal Analysis

1st Exercise Sheet
Winter Semester 2024/2025

Submission: Until 08/11/2024 1 p.m. CET

1. Exercise: Probability Density Function (5 Points)

For two independent random variables X and Y, the probability density function P_{X+Y} is equal to the convolution of $P_X(x)$ und $P_Y(x)$, such that

$$P_{X+Y}(t) = \int_{-\infty}^{\infty} P_X(x) \cdot P_Y(t-x)\,\mathrm{d}x\,. \tag{1}$$

Using equation (1), show that for independently normally distributed random variables $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$:

$$X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2) \tag{2}$$

2. Exercise: Analysing a time series (15 Points)

In the folder `Exercise Sheets` you will find the time series `timeseries.txt`, which will be analysed below.

a) Plot the time series using a Python script. What do you notice?

b) Detrend the time series and describe your procedure. To calculate the linear regression, either use the `least_squares` function from the `scipy.optimize` package or implement the formulae from the lecture.

c) Plot the distribution of the time series with and without trend (histogram). What differences are visible and where do they come from? What do you expect for $m \to \infty$, where m is the slope of the trend? What do you expect for $m \to 0$?

d) Write a Python script with which you can determine the first four (centred) moments of the distribution. Calculate the moments of the time series with and without trend and relate the values to the findings from task c) and to the values of the normal distribution. Do not use any existing Python libraries to calculate the moments.

The Python script for this exercise must be handed in and commented accordingly.

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