math_quiz

February 24, 2026

1 Math Quiz

Enter an answer for each question in the markdown box

1.1 Question 1:

Evaluate derivative with respect to x:

$$f(x) = x^4 e^{(2x+a)^2}$$

Product rule:

$$(fg)' = f'g + fg'$$

Chain rule:

$$f'(x) = h'(g(x)) \cdot g'(x)$$

Work:

$$f'(x) = 4x^3 e^{(2x+a)^2} + x^4 \cdot e^{(2x+a)^2} \cdot 2(2x+a)^{2-1} \cdot 2$$

$$f'(x) = 4x^3 e^{(2x+a)^2} + x^4 \cdot e^{(2x+a)^2} \cdot 4(2x+a)$$

$$= 4x^3 e^{(2x+a)^2} \left[1 + x(2x+a)\right]$$

1.2 Question 2:

Evaluate the following integral:

$$\int_{-1}^{2} 5t^4 \, dt$$

$$\int_{-1}^{2} 5t^4 \, dt = \left[t^5\right]_{-1}^{2} = 2^5 - (-1)^5 = 32 - (-1) = 33$$

1.3 Question 3:

If, $\hat{A} = \hat{A}^\dagger$, what kind of operator is $\hat{A}$?

$\hat{A}$ is a **Hermitian** operator.

1.4 Question 4:

If, $\hat{A}\hat{A}^\dagger = \hat{A}^\dagger\hat{A} = \hat{I}$, what kind of operator is $\hat{A}$?

$\hat{A}$ is a **Unitary** operator.

1.5 Question 5:

If, $\hat{A}\hat{A} = \hat{A}$, what kind of operator is $\hat{A}$?

$\hat{A}$ is an **Idempotent** operator.

1.6 Question 6:

If $\hat{A}$ is Idempotent and Involutory, what is $\hat{A}$?

Idempotent gives $\hat{A}\hat{A} = \hat{A}$, and Involutory gives $\hat{A}\hat{A} = \hat{I}$. Therefore $\hat{A} = \hat{I}$.

1.7 Question 7:

Given the following relationship, $\hat{A}|v_i\rangle = a_i|v_i\rangle$, define the following quantities:

- a_i :
- $|v_i\rangle$:

- a_i : **eigenvalue** of operator $\hat{A}$ corresponding to eigenstate $|v_i\rangle$
- $|v_i\rangle$: **eigenvector** of operator $\hat{A}$

1.8 Question 8:

Given the following : - $\mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix}$ - $\mathbf{y} = \begin{pmatrix} c \\ d \end{pmatrix}$ - $\mathbf{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ - $\mathbf{Y} = \begin{pmatrix} c & d \\ e & f \end{pmatrix}$

Compute: 1. $\mathbf{Yx} = ?$ 1. $\mathbf{x}^\dagger\mathbf{Y} = ?$ 1. $\mathbf{X}^\dagger\mathbf{Y} = ?$ 1. $Trace(\hat{X}) = ?$

1. $\mathbf{Yx} = \begin{pmatrix} c & d \\ e & f \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ca + db \\ ea + fb \end{pmatrix}$
2. $\mathbf{x}^\dagger\mathbf{Y} = \begin{pmatrix} a^* & b^* \end{pmatrix} \begin{pmatrix} c & d \\ e & f \end{pmatrix} = \begin{pmatrix} a^*c + b^*e & a^*d + b^*f \end{pmatrix}$

3. $\mathbf{X}^\dagger\mathbf{Y} = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}\begin{pmatrix} c & d \\ e & f \end{pmatrix} = \begin{pmatrix} a^*c + c^*e & a^*d + c^*f \\ b^*c + d^*e & b^*d + d^*f \end{pmatrix}$

4. $\text{Trace}(\hat{X}) = a + d$

1.9 Question 9:

Compute the following commutator: $\left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\right]$

where $[A, B] = AB - BA$, and the multiplication operation is the usual matrix multiplication.

$$AB = \begin{pmatrix} 1\cdot 5 + 2\cdot 7 & 1\cdot 6 + 2\cdot 8 \\ 3\cdot 5 + 4\cdot 7 & 3\cdot 6 + 4\cdot 8 \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

$$BA = \begin{pmatrix} 5\cdot 1 + 6\cdot 3 & 5\cdot 2 + 6\cdot 4 \\ 7\cdot 1 + 8\cdot 3 & 7\cdot 2 + 8\cdot 4 \end{pmatrix} = \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix}$$

$$[A, B] = AB - BA = \begin{pmatrix} 19 - 23 & 22 - 34 \\ 43 - 31 & 50 - 46 \end{pmatrix} = \begin{pmatrix} -4 & -12 \\ 12 & 4 \end{pmatrix}$$

1.10 Question 10:

Define the four Pauli matrices I, X, Y, Z: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Define $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Compute the expectation values: 1. $\langle 0|X|0\rangle$, 2. $\langle 1|X|1\rangle$, 3. $\langle 0|Y|0\rangle$, 4. $\langle 1|Y|1\rangle$, 5. $\langle 0|Z|0\rangle$, 6. $\langle 1|Z|1\rangle$.

1. $\langle 0|X|0\rangle = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$
2. $\langle 1|X|1\rangle = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$
3. $\langle 0|Y|0\rangle = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ i \end{pmatrix} = 0$
4. $\langle 1|Y|1\rangle = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} -i \\ 0 \end{pmatrix} = 0$
5. $\langle 0|Z|0\rangle = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$
6. $\langle 1|Z|1\rangle = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = -1$

1.11 Question 11:

Define the four Pauli matrices I, X, Y, Z as well as the basis states $|0\rangle, |1\rangle$ as in the previous question.

Define $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

Compute the expectation values: 1. $\langle\psi|X|\psi\rangle$, 2. $\langle\psi|Y|\psi\rangle$.

$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, so $\langle\psi| = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}$

1. $\langle\psi|X|\psi\rangle$: $X|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, so $\langle\psi|X|\psi\rangle = \frac{1}{2}(1+1) = \mathbf{1}$
2. $\langle\psi|Y|\psi\rangle$: $Y|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}-i\\i\end{pmatrix}$, so $\langle\psi|Y|\psi\rangle = \frac{1}{2}(-i+i) = \mathbf{0}$