Midterm Exam

● Graded

Student
Max Alan Niederman

Total Points
55 / 60 pts

Question 1
(no title) **9** / 10 pts

1.1 **(no title)** 6 / 7 pts

- **+ 7 pts** Correct
- ✔ **+ 6 pts** correct with calculation error
- **+ 5 pts** correct idea with some errors
- **+ 3 pts** solved without shifting by (2, -3, 1)
- **+ 1 pt** attempted projection
- **+ 0 pts** no points

1.2 **(no title)** **3** / 3 pts

- ✔ **+ 3 pts** Correct
- **+ 2.5 pts** correct with calculation error
- **+ 1 pt** reasonable progress
- **+ 0 pts** no points

Question 2
(no title) **10** / 10 pts

- ✔ **+ 10 pts** Correct
- **+ 5 pts** Correctly set up Lagrange system
- **+ 2 pts** Significant progress in solving system of equations
- **+ 3 pts** Solved Lagrange system with some errors
- **+ 3 pts** Attempted to use Lagrange multipliers
- **+ 0 pts** Incorrect
- **+ 1 pt** Some relevant work

Question 3

(no title) **10** / 10 pts

3.1 (no title) **2** / 2 pts

- ✔ **− 0 pts** Correct
- **− 2 pts** Incorrect/no answer

3.2 (no title) **4** / 4 pts

- ✔ **+ 2 pts** First column correct
- ✔ **+ 2 pts** Second column correct
- **+ 0 pts** None correct/no answer
- **− 1 pt** Computation mistake

3.3 (no title) **4** / 4 pts

- ✔ **+ 2 pts** Correct A[-1,1] = [0,4]
- ✔ **+ 2 pts** Correct A[1,-3] = [2,0]
- **+ 0 pts** None correct / no answer
- **− 0.5 pts** Right computation but incorrect drawing
- **− 1 pt** Work not shown

Question 4

(no title) **6** / 10 pts

4.1 **(no title)** **1** / 5 pts

+ 5 pts Correct M with correct and complete justification

+ 3 pts Significant progress towards correct answer

+ 1 pt Some correct ideas and progress

✓ **+ 1 pt** Correct M with no or almost no justification

+ 0 pts No credit

− 1 pt Minor slip or omission in explanation or answer

4.2 **(no title)** **5** / 5 pts

✓ **+ 5 pts** Correct answer with correct and complete justification

+ 3 pts Significant progress towards correct answer

+ 1 pt Some correct ideas and progress

+ 1 pt Correct M with no or almost no justification

+ 0 pts No credit

− 1 pt Minor slip or omission in explanation or answer

Question 5

(no title) **10** / 10 pts

5.1 **(no title)** **4** / 4 pts

- ✔ **+ 4 pts** Any correct solution arriving at point (2,-1,2)
- **+ 3.25 pts** Almost correct solution arriving at wrong point due to computation mistake.
- **+ 3 pts** Arrives at correct point but insufficient details of work
- **+ 1 pt** Partial credit: computing gradient of xz-y^2-3
- **+ 1 pt** Partial credit: computing normal vector to plane x+y+z=3
- **+ 1 pt** Partial credit: positing these vectors are scalar multiples
- **+ 0.5 pts** Reasonable attempt (if none of the above is marked)
- **+ 0 pts** No solution
- **− 0.25 pts** Additional computation mistake

5.2 **(no title)** **2** / 2 pts

- ✔ **+ 0.6 pts** Correct OB
- ✔ **+ 0.7 pts** Correct CA
- ✔ **+ 0.7 pts** Correct CB
- **+ 0 pts** None correct

5.3 **(no title)** **4** / 4 pts

- ✔ **+ 4 pts** Correct
- **− 1 pt** Correct, but explanation has oversights
- **+ 2 pts** Partial credit: Reasonable progress
- **+ 1 pt** Partial credit: Any attempt
- **+ 0 pts** No solution

Question 6

(no title) **10** / 10 pts

6.1 **(no title)** **Resolved** **5** / 5 pts

✔ **+ 1 pt** Correct answer (FALSE)

✔ **+ 4 pts** Correct explanation

+ 2 pts Correct idea with gaps in reasoning

+ 1 pt Some good ideas

− 0.5 pts calculation error

+ 0 pts no points

Regrade Request **Submitted on: Aug 02**

I believe that the idea behind my reasoning was solid, and deserves "Correct idea with gaps in reasoning" or at the very least "Some good ideas." Here's a full treatment of the idea I was trying to get across in my haste to finish the exam: http://web.stanford.edu/~maxnie/math51/midterm_6-a_justification.pdf. I admit that I made a few mistakes and inferences which could be considered gaps in reasoning, but I believe that my explanation deserves more than zero points.

I've adjusted your score to give you full points.

I remember your solution, and I took a long time deliberating the fair grade. Looking back, I see that I misunderstood a few parts of your solution that made me give a lower score than it deserved: for instance, I didn't notice the subscripts x_1 / y_1 in the second parts of your integral, which led me to think that you had calculated the wrong function values. Since I only gave points to students that accurate antiderivatives, I had decided to not give points to you.

Anyways, your solution is solid as you said, and good work for coming up with the gradient formula on your own! Your solution isn't perfect, since you did not consider the integral constant / had some typos in your solution. For instance, the integration in the first line should have been up to x_1 and y_1, not x and y. However, I felt that these were minor enough that they deserved full marks.

I apologize for making you go through the regrading process, and congratulations on your new score.

Reviewed on: Aug 04

6.2 **(no title)** **5** / 5 pts

✔ **+ 1 pt** Correct answer (FALSE)

✔ **+ 4 pts** correct explanation

+ 3 pts correct idea with gaps/minor flaws in reasoning

+ 1 pt mention of multiple z values

+ 2 pts incorrect answer, but found $z = \pm\sqrt{x^2 + y^2 + 1}$.

+ 1 pt incorrect answer, wrote $z = \sqrt{x^2 + y^2 + 1}$.

+ 0 pts no points

Math 51 Midterm Exam — July 28, 2023

Name: Max Niederman SUNet ID: maxnie ID #: 006776283

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result discussed in class or the text, but clearly state the result before using it, and verify that the hypotheses are satisfied.
- Please check that your copy of this exam contains 9 pages of exam questions, *numbered* in the upper-right, and that it is adequately stapled.

  DO NOT REMOVE ANY PAGES; if any page is missing, your exam will be considered incomplete. Incomplete exams will be assessed a 5-point penalty.
- You may use 1 piece of 8.5″ × 11″ paper (both sides) with formulas and other notes as a "reference sheet". No electronic devices, including phones, headphones, or calculation aids, are permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by course staff (apart from your own reference sheet) is prohibited. If you need extra room for your answers, use one of the blank pages provided (those pages except for the one at the end are labeled at the bottom by lower-case Roman numerals, starting with "ii"), and clearly indicate that that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to look over your graded exam in a timely manner. You have until **August 9, 5 p.m.**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

  Signature: Max Niederman

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2) $8\frac{y}{x} = \lambda = \frac{9}{2}\cdot\frac{x}{y}$

$16y^2 = 9x^2$

$y^2 = \frac{9}{16}x^2$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\frac{x^2}{16} + \frac{1}{9}\left(\frac{9}{16}x^2\right) = 1$

$\frac{x^2}{16} + \frac{x^2}{16} = 1$

$x^2 = 8$

$x = \sqrt{8}$

$y = \sqrt{\frac{9}{16}\sqrt{8}^2}$

$= \frac{3}{4}\sqrt{8}$

We double these coordinates to get the dimensions:

$2x = 2\sqrt{8} = 4\sqrt{2}$

$2y = 2\left(\frac{3}{4}\right)\sqrt{8} = \frac{3}{2}\sqrt{8} = 3\sqrt{2}$

So, the maximal rectangle is $4\sqrt{2}$ by $3\sqrt{2}$.

DO NOT DETACH THIS PAGE. If you use any of this space to continue your answer, please clearly indicate the problem number(s) and **indicate on the original page of the problem that your answer continues here.**

DO NOT DETACH THIS PAGE. If you use any of this space to continue your answer, please clearly indicate the problem number(s) and **indicate on the original page of the problem that your answer continues here.**

1. (10 points) Consider the plane $\mathcal{P}$ defined by

$$\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t' \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

for $t, t' \in \mathbf{R}$.

(a) (7 points) Find the point on $\mathcal{P}$ closest to $\mathbf{v} = (4, -7, 4)$.

Remark. Note that $\mathbf{0}$ is not on $\mathcal{P}$.

First we translate so that P is a subspace:

$$P' = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right) \qquad v' = \begin{bmatrix} 4 \\ -7 \\ 4 \end{bmatrix} - \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ are orthogonal, so

$$\text{Proj}_{P'} v' = \text{Proj}_{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}} v' + \text{Proj}_{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}} v'$$

$$= \frac{4-7}{1+1}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{4-(-7)+2(4)}{1+1+4}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$= -\frac{3}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{19}{6}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{9}{6} + \frac{19}{6} \\ -\frac{9}{6} - \frac{19}{6} \\ \frac{38}{6} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5 \\ -14 \\ 19 \end{bmatrix}$$

Then, we translate to find that the closest point is $\frac{1}{3}\begin{bmatrix} 5 \\ -14 \\ 19 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 11 \\ -23 \\ 22 \end{bmatrix}$

(b) (3 points) Give an equational form for $\mathcal{P}$.

We can find a normal vector to P by solving

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}\vec{n} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 2 \\ 1 & -1 & 2 \end{bmatrix}\vec{n} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}\vec{n} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}\vec{n} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{n} = \lambda\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

We can use any one of these vectors, so let us take $\lambda = 1$ to write the equational form:

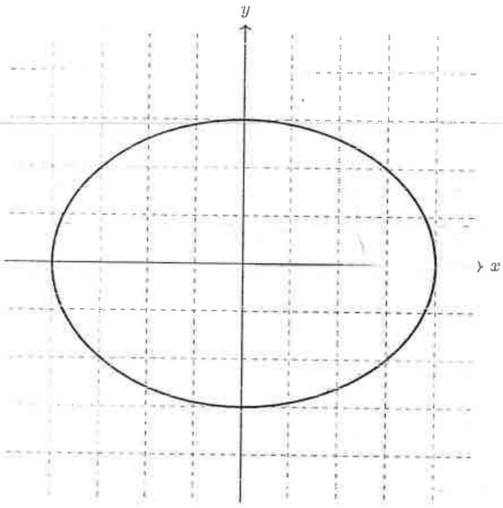
$$\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}\right) \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$-x + y + z = -2 - 3 + 1$$

$$-x + y + z = -4$$

2. (10 points) Find the dimensions of the rectangle of greatest area that can be inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ (shown below) with sides parallel to the coordinate axes. *[Note: We say that a rectangle is inscribed in an ellipse if the corners of the rectangle lie on the ellipse.]*



Let $g(x,y) = \frac{x^2}{16} + \frac{y^2}{9}$ so the set $\{g=1\}$ is the ellipse.

Let $f(x,y) = xy$ so that $f(\vec{x})$ is the area of $\frac{1}{4}$ a rectangle inscribed so that its corner is at $\vec{x}$.

We solve using Lagrange multipliers, considering only points in Quadrant I because of the mirror symmetry across the x- and y- axes:

$\nabla g = \vec{0}$

$\begin{bmatrix} \frac{2x}{16} \\ \frac{2y}{9} \end{bmatrix} = \vec{0}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \vec{0}$

Does not satisfy $g=1$.

$\nabla f = \lambda \nabla g$

$\begin{bmatrix} y \\ x \end{bmatrix} = \lambda \begin{bmatrix} \frac{x}{8} \\ \frac{2y}{9} \end{bmatrix}$

If $\lambda = 0$:

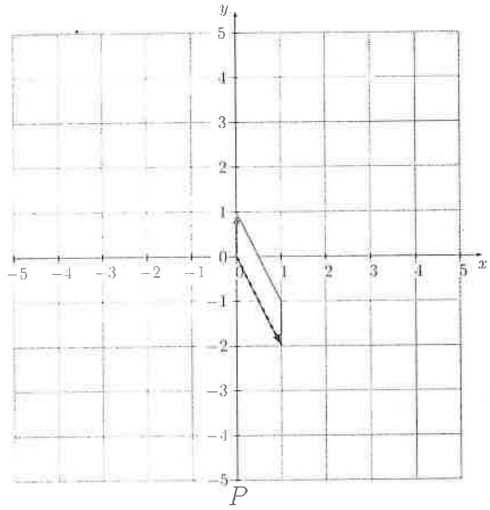
$\begin{bmatrix} x \\ y \end{bmatrix} = \vec{0}$

Does not satisfy $g=1$.

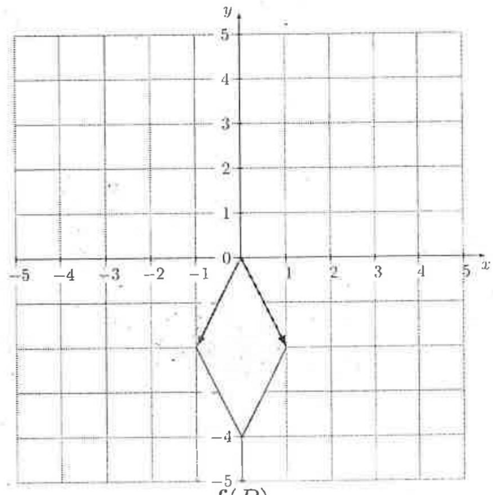
If $\lambda \neq 0$:

Continue on ii

3. (10 points) Suppose $\mathbf{f} : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear function that transforms P, the parallelogram on the left, to $\mathbf{f}(P)$, the parallelogram on the right.



P



$\mathbf{f}(P)$

Note that $\mathbf{f}$ transforms the light gray vector on the left into the light gray vector on the right, and that $\mathbf{f}$ transforms the dotted black vector on the left into the dotted black vector on the right.

(a) (2 points) Express $\begin{bmatrix}1\\0\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\-2\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$.

$$\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\-2\end{bmatrix} + 2\begin{bmatrix}0\\1\end{bmatrix}$$

(the first coefficient must be one to get the x-coord, then the second is trivial)

(b) (4 points) Determine A, the matrix associated to $\mathbf{f}$; i.e., for which $\mathbf{f}(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $\mathbf{R}^2$.

We know $f(e_2) = f\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\-2\end{bmatrix}$.

We can compute $f(e_1)$ using the coefficients from part (a):

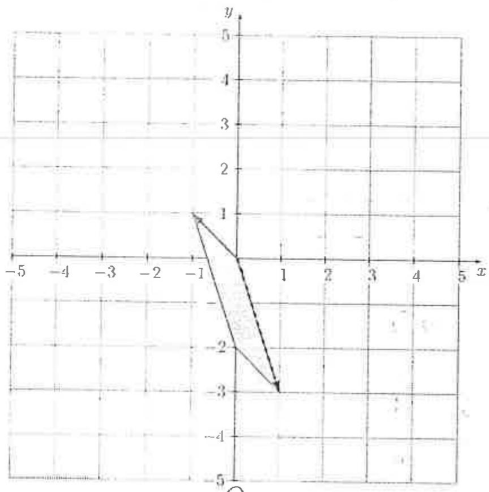
$$f(e_1) = f\left(\begin{bmatrix}1\\-2\end{bmatrix} + 2\begin{bmatrix}0\\1\end{bmatrix}\right)$$
$$= f\left(\begin{bmatrix}1\\-2\end{bmatrix}\right) + 2f\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$$
$$= \begin{bmatrix}1\\-2\end{bmatrix} + 2\begin{bmatrix}-1\\-2\end{bmatrix}$$
$$= \begin{bmatrix}-1\\-6\end{bmatrix}$$

So the matrix A is

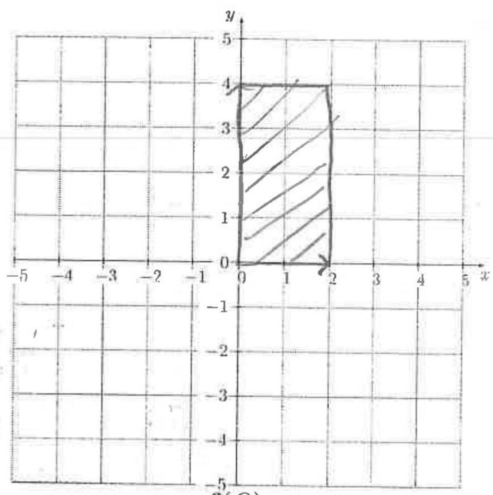
$$A = \begin{bmatrix} f(e_1) & f(e_2)\end{bmatrix} = \begin{bmatrix}-1 & -1\\-6 & -2\end{bmatrix}$$

(problem continued from facing page)

(c) (4 points) Given Q, the parallelogram on the left below, provide a sketch of $\mathbf{f}(Q)$ on the blank coordinate plane on the right below. (*Note:* you do *not* need to indicate a "light gray" and "dotted black" vector in your sketch.)



Q



$\mathbf{f}(Q)$

$f\left(\begin{bmatrix}-1\\1\end{bmatrix}\right)=\begin{bmatrix}0\\4\end{bmatrix}$

$f(1,-3)=\begin{bmatrix}2\\0\end{bmatrix}$

4. (10 points) Suppose that Derek and Gene each have \$2, and they play a game with a fair coin (i.e., it comes up heads 50% of the time and tails 50% of the time). If the coin comes up heads, Derek gives Gene \$1, and if the coin comes up tails, Gene gives Derek \$1.

   The game ends once someone has all \$4.

   (a) (5 points) Define the probability vector

$$\mathbf{p}_n = \begin{bmatrix} \text{probability that Derek has \$4 after } n \text{ turns} \\ \text{probability that Derek has \$3 after } n \text{ turns} \\ \text{probability that Derek has \$2 after } n \text{ turns} \\ \text{probability that Derek has \$1 after } n \text{ turns} \\ \text{probability that Derek has \$0 after } n \text{ turns} \end{bmatrix}.$$

   Compute the 5×5 matrix M for which $\mathbf{p}_{n+1} = M\mathbf{p}_n$. Justify your answer.

$$M = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 1 \end{bmatrix}.$$

(b) (5 points) It is a known result that if player A has \$$a$ and player B has \$$b$, where a and b are integers, the probability of player A eventually getting all $\$(a+b)$ is $\frac{a}{a+b}$ and the probability of player B eventually getting all $\$(a+b)$ is $\frac{b}{a+b}$.
For example, if Derek has \$20 and Gene has \$10, the probability of Derek winning all the money is $\frac{20}{20+10} = \frac{2}{3}$.
For the matrix M you found in part (a), compute

$$\lim_{n\to\infty} M^n,$$

and justify your answer.
Hint. It may be helpful to think about what each column of $\lim_{n\to\infty} M^n$ represents.

~~Each column of $\lim_{n\to\infty} M^n$ is the probability vector Derek after infinite time given he starts with \$(5-j), where j is the column index.~~

Better Explanation

We can use this to compute the matrix

$$\lim_{n\to\infty} M^n = \begin{bmatrix} P(\$4|\$4) & P(\$4|\$3) & \cdots \\ P(\$3|\$4) & P(\$3|\$3) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & \frac{3}{4} & \frac{2}{4} & \frac{1}{4} & \frac{0}{4} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{0}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} \end{bmatrix}$$

Each column is the probability vector

$$\begin{bmatrix} P(\text{Derek wins} \mid \text{Start w/ } \$a) \\ 0 \\ 0 \\ 0 \\ P(\text{Derek loses} \mid \text{Start w/ } \$a) \end{bmatrix}$$

where a is the number of dollars corresponding to that column. We can use the $\frac{a}{a+b}$ formula to compute these columns.

5. (10 points) (a) (4 points) Let

$$S = \{(x, y, z) \in \mathbf{R}^3 \mid xz - y^2 = 3\},$$

and $\mathcal{P}$ be the plane $x + y + z = 3$. Find the point Q on S where the plane $\mathcal{P}$ is tangent to S at Q.

Let $f(x,y,z) = x+y+z$ So $P = \{f=3\}$

Let $g(x,y,z) = xz - y^2$ So $S = \{g=3\}$.

Then S is tangent to P at $Q = (x,y,z)$ in $S \cap P$ if

the gradients of f and g are aligned, i.e.

if $\nabla g(x,y,z) = \vec{0}$ or $\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$

$\nabla g(x,y,z) = \vec{0}$

$$\begin{bmatrix} z \\ -2y \\ x \end{bmatrix} = \vec{0}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{0}$$

Not on $P \cap S$.

$\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} z \\ -2y \\ x \end{bmatrix}$$

$$\frac{1}{\lambda} = z = -2y = x \quad (\lambda = 0 \Rightarrow 1 = 0)$$

$$y = -\frac{x}{2}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ -\frac{x}{2} \\ x \end{bmatrix}$$

$$x + y + z = 3$$

$$x - \frac{x}{2} + x = 3$$

$$\frac{3}{2}x = 3$$

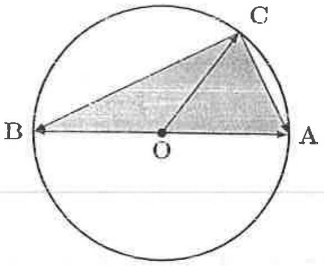
$$x = 2$$

$$Q = (2, -1, 2)$$

$$g(2,-1,2) = 2(2) - (-1)^2 = 3$$

$$Q = (2, -1, 2)$$

For parts (b) and (c), consider the circle below where O is the center of the circle, points A, B, C are on the cirlc, and AB is a diameter of the circle.



Suppose $\overrightarrow{OA} = \mathbf{u}$, and $\overrightarrow{OC} = \mathbf{v}$.

(b) (2 points) Express the vectors $\overrightarrow{OB}$, $\overrightarrow{CA}$, and $\overrightarrow{CB}$ as linear combinations of $\mathbf{u}$ and $\mathbf{v}$.

$$\overrightarrow{OB} = -\overrightarrow{OA} = -u$$

$$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = u - v$$

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = (-u) - v = -u - v$$

(c) (4 points) Show that $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are perpendicular to each other.

$$\overrightarrow{CA} \cdot \overrightarrow{CB} = (u - v) \cdot (-u - v)$$

$$= -u \cdot u - u \cdot v + v \cdot u + v \cdot v$$

$$= -u \cdot u + v \cdot v$$

$$= -\|u\|^2 + \|v\|^2$$

$$= 0$$

(because u and v are on the same circle)

Their dot product is zero, so they are perpindicular.

6. (10 points) For each of the following statements, circle either TRUE (meaning, "always true") or FALSE (meaning, "not always true"), and briefly and convincingly justify your answer. 1 point for the correct choice, and the rest for convincing justification.

(a) (5 points) There is a continuous function $f : \mathbf{R}^2 \to \mathbf{R}$ with

$$f_x = e^{x^2} \sin y \qquad \text{and} \qquad f_y = e^{x^2} \cos y.$$

Circle one, and justify below: TRUE FALSE

If there was, then $f(x_1,y_1) = \int_0^x f_x(x,0)\,dx + \int_0^y f_y(x_1,y)\,dy$

$= \int_0^y f_y(0,y)\,dy + \int_0^x f_x(x,y_1)\,dx$

x, then y

$\int_0^{x_1} e^{x^2} \sin 0\,dx + \int_0^{y_1} e^{x_1^2} \cos y\,dy$

$= 0 + e^{x_1^2} \sin y_1$

y, then x

$\int_0^{y_1} e^{(0)^2} \cos(y)\,dy + \int_0^{x_1} e^{x^2} \sin(y_1)\,dx$

$= \sin y_1 + \sin y_1 \int_0^{x_1} e^{x^2}\,dx$

$e^{x_1^2} \sin y_1 \neq \sin y_1 + \sin y_1 \int_0^{x_1} e^{x^2}\,dx$

(b) (5 points) The set $S = \{(x,y,z) \in \mathbf{R}^3 : x^2 + y^2 - z^2 = -1\}$ is the graph of some function $f : \mathbf{R}^2 \to \mathbf{R}$.

Circle one, and justify below: TRUE FALSE

A function $f: \mathbb{R}^2 \to \mathbb{R}$ can have only one output z for a given x and y, but the set S contains two points for each x and y: (x, y, z) and (x, y, -z).

DO NOT DETACH THIS PAGE. If you use any of this space to continue your answer, please clearly indicate the problem number(s) and **indicate on the original page of the problem that your answer continues here.**