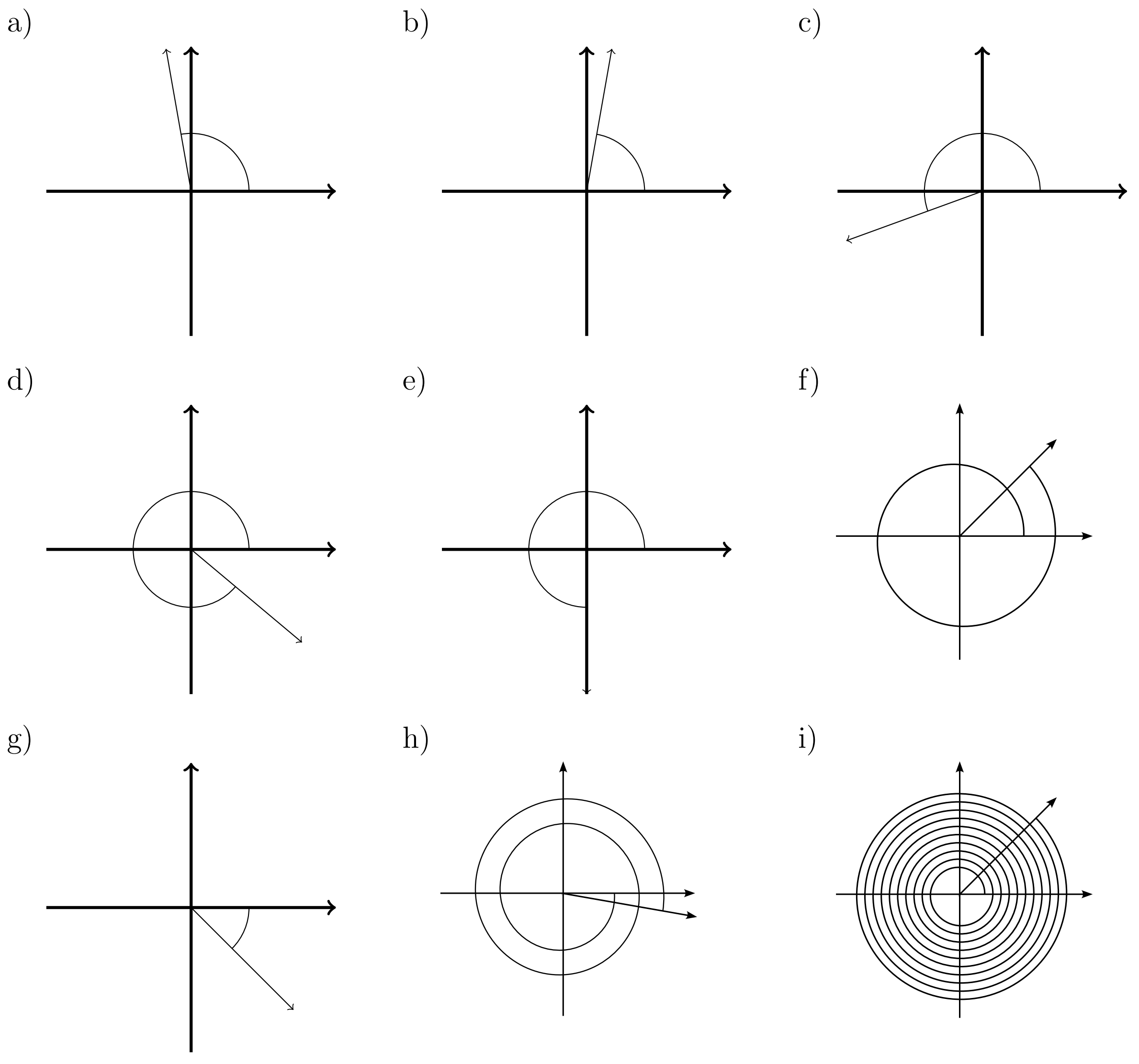
Selected Solutions.

Section 1 Solutions

1.1.



1.2.

a) 48° is is quadrant I. b) 300° is in quadrant IV. c) 185° is in quadrant III.
d) 75° is in quadrant I. e) 220° is in quadrant III. f) 160° is in quadrant II.

1.3.

a) 32° b) 32° c) 67° d) 67° e) 38°
f) 38° g) 56° h) 56° i) 68° j) 47°

1.4.

a) $135°, 225°$, and $315°$.

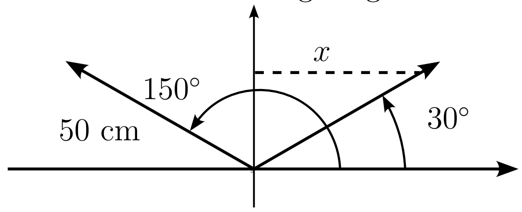
b) $120°, 240°$, and $300°$.

c) $150°, 210°$, and $330°$.

d) $105°, 255°$, and $285°$.

1.5. $146°$

1.6. Consider the following diagram.



$$\begin{aligned}\cos 30° &= \frac{x}{50}\\ \frac{\sqrt{3}}{2} &= \frac{x}{50}\\ x &= 25\sqrt{3}\end{aligned}$$

By symmetry, the horizontal distance that the tip of the wiper travels in one swipe will be $2x$, or $50\sqrt{3}$ cm.

1.7.

a) The coordinates of the other three trees are found using symmetries of the diagram: flowering dogwood $(-3.5, 2)$, river birch $(-3.5, -2)$, white pine $(3.5, -2)$.

b) For the red maple,

$$\begin{aligned}\tan\theta &= \frac{2}{3.5}\\ \theta &= \tan^{-1}\left(\frac{2}{3.5}\right)\\ \theta &= 29.744\ldots\end{aligned}$$

The angle in standard position for the red maple is $30°$, to the nearest degree. Then, the angle in standard position for the flowering dogwood is $180° - 30°$ or $150°$, to the nearest degree. The angle in standard position for the river birch is $180° + 30°$ or $210°$, to the nearest degree. The angle in standard position for the white pine is $360° - 30°$ or $330°$, to the nearest degree.

c) On the grid, there are 4 vertical units of distance between the red maple and the white pine. Since each grid mark represents 10 m, the distance between these two trees is 40 m.

Section 2 Solutions

2.1.

a) $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, \tan\theta = \frac{3}{4}$

b) $\sin\theta = \frac{5}{\sqrt{29}}, \cos\theta = -\frac{2}{\sqrt{29}}, \tan\theta = -\frac{5}{2}$

c) $\sin\theta = -\frac{\sqrt{3}}{2}, \cos\theta = -\frac{1}{2}, \tan\theta = \sqrt{3}$

d) $\sin\theta = -\frac{\sqrt{5}}{3}, \cos\theta = \frac{2}{3}, \tan\theta = -\frac{\sqrt{5}}{2}$

e) $\sin\theta = 0, \cos\theta = -1, \tan\theta = 0$

f) $\sin\theta = -1, \cos\theta = 0, \tan\theta$ is undefined

2.2.

a) $\sin\theta = -\frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = -\frac{4}{3}$

b) $\sin\theta = \frac{5}{13}, \cos\theta = -\frac{12}{13}, \tan\theta = -\frac{5}{12}$

c) $\sin\theta = -\frac{24}{25}, \cos\theta = -\frac{7}{25}, \tan\theta = \frac{24}{7}$

d) $\sin\theta = \frac{15}{17}, \cos\theta = \frac{8}{17}, \tan\theta = \frac{15}{8}$

e) $\sin\theta = \frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}, \tan\theta = -\frac{1}{\sqrt{3}}$

f) $\sin\theta = \frac{\sqrt{7}}{3}, \cos\theta = \frac{\sqrt{2}}{3}, \tan\theta = \frac{\sqrt{7}}{\sqrt{2}}$

g) $\sin\theta = -\frac{1}{\sqrt{2}}, \cos\theta = -\frac{1}{\sqrt{2}}, \tan\theta = 1$

h) $\sin\theta = 1, \cos\theta = 0, \tan\theta$ is undefined

i) $\sin\theta = 0, \cos\theta = -1, \tan\theta = 0$

j) $\sin\theta = \frac{2}{3}, \cos\theta = -\frac{\sqrt{5}}{3}, \tan\theta = -\frac{2}{\sqrt{5}}$

k) $\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = -\frac{1}{\sqrt{2}}, \tan\theta = -1$

l) $\sin\theta = \frac{1}{3}, \cos\theta = -\frac{2\sqrt{2}}{3}, \tan\theta = -\frac{1}{2\sqrt{2}}$

m) $\sin\theta = -\frac{9}{41}, \cos\theta = -\frac{40}{41}, \tan\theta = \frac{9}{40}$

n) $\sin\theta = -\frac{40}{41}, \cos\theta = \frac{9}{41}, \tan\theta = -\frac{40}{9}$

o) $\sin\theta = \frac{2\sqrt{2}}{3}, \cos\theta = \frac{1}{3}, \tan\theta = 2\sqrt{2}$

p) $\sin\theta = \frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}, \tan\theta = \frac{-\sqrt{3}}{3}$

q) $\sin\theta = -\frac{\sqrt{5}}{4}, \cos\theta = -\frac{\sqrt{11}}{4}, \tan\theta = \frac{\sqrt{55}}{11}$

r) $\sin\theta = \frac{2\sqrt{2}}{5}, \cos\theta = \frac{\sqrt{17}}{5}, \tan\theta = \frac{2\sqrt{34}}{17}$

2.3.

a) $\sin\theta = -\frac{2}{\sqrt{5}}, \cos\theta = -\frac{1}{\sqrt{5}}, \tan\theta = 2$

b) $\sin\theta = -\frac{2}{\sqrt{29}}, \cos\theta = \frac{5}{\sqrt{29}}, \tan\theta = -\frac{2}{5}$

c) $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = \frac{4}{3}$

d) $\sin\theta = \frac{3}{\sqrt{13}}, \cos\theta = -\frac{2}{\sqrt{13}}, \tan\theta = -\frac{3}{2}$

e) $\sin\theta = -\frac{5}{\sqrt{29}}, \cos\theta = -\frac{2}{\sqrt{29}}, \tan\theta = \frac{5}{2}$

f) $\sin\theta = -\frac{1}{\sqrt{17}}, \cos\theta = \frac{4}{\sqrt{17}}, \tan\theta = -\frac{1}{4}$

g) $\sin\theta = 1, \cos\theta = 0, \tan\theta = \infty$

h) $\sin\theta = 0, \cos\theta = -1, \tan\theta = 0$

2.4.

a) $\cos\theta = \frac{3}{5}, \tan\theta = \frac{4}{3}$

b) $\sin\theta = \frac{5}{13}, \tan\theta = -\frac{5}{12}$

c) $\sin\theta = -\frac{7}{25}, \cos\theta = -\frac{24}{25}$

d) $\cos\theta = \frac{1}{2}, \tan\theta = -\sqrt{3}$

e) $\sin\theta = \frac{\sqrt{3}}{2}, \tan\theta = \sqrt{3}$

f) $\sin\theta = \frac{2}{3}, \cos\theta = \frac{\sqrt{5}}{3}$

g) $\cos\theta = -\frac{1}{\sqrt{10}}, \tan\theta = 3$

h) $\sin\theta = -\frac{2}{\sqrt{13}}, \tan\theta = -\frac{2}{3}$

i) $\sin\theta = -\frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}$

j) $\cos\theta = \pm\frac{1}{4}, \tan\theta = \pm\sqrt{15}$

k) $\sin\theta = \pm\frac{\sqrt{13}}{4}, \tan\theta = \pm\frac{\sqrt{39}}{3}$

l) $\sin\theta = \pm\frac{\sqrt{3}}{2}, \cos\theta = \pm\frac{1}{2}$

2.5.

a) $\cos\theta = 0.767, \tan\theta = 0.837$

b) $\sin\theta = -0.844, \tan\theta = -1.571$

c) $\sin\theta = -0.894, \cos\theta = -0.447$

d) $\cos\theta = -0.972, \tan\theta = -0.244$

e) $\sin\theta = 0.926, \tan\theta = -2.450$

f) $\sin\theta = -0.816, \cos\theta = 0.578$

g) $\cos\theta = -0.658, \tan\theta = 1.144$

h) $\sin\theta = 0.871, \tan\theta = -1.770$

i) $\sin\theta = -0.655, \cos\theta = 0.756$

j) $\cos\theta = \pm 0.936, \tan\theta = \pm 0.375$

k) $\sin\theta = \pm 0.585, \tan\theta = \pm 0.721$

l) $\sin\theta = \pm 0.420, \cos\theta = \pm 0.907$

2.7. $\sin(-\theta) = -\sin\theta$

2.8. $\cos(-\theta) = \cos\theta$

2.9.

a) The measure for $\angle A$ is $45°$, for $\angle B$ is $135°$, for $\angle C$ is $225°$, and for $\angle D$ is $315°$.

b) Point A is on the terminal arm of $45° : A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. B is on the terminal arm of $135° : B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. C is on the terminal arm of $225° : C\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$. D is on the terminal arm of $315° : D\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

2.10. Since they are both radii, $OA = OB = 1$. Therefore, $\triangle OAB$ is isosceles with $\angle OAB = \angle ABO = 60°$. Hence $\triangle OAB$ is equilateral and $AB = 1$, and $CB = 2$.

Since OA is the terminal arm of $60°$, the coordinates of A are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The coordinates of C are $(-1, 0)$. Then, use the Pythagorean Theorem in the right triangle with hypotenuse AC.

$$
\begin{aligned}
AC^2 &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2 \\
AC^2 &= \frac{3}{4} + \frac{9}{4} \\
AC^2 &= 3 \\
AC &= \sqrt{3}
\end{aligned}
$$

Then, in $\angle ABC$, $BC^2 = 4$, and

$$AC^2 + AB^2 = 3 + 1 = 4$$

So the sides of $\triangle ABC$ satisfy the Pythagorean Theorem, with $\angle CAB = 90°$ because BC is the hypotenuse.

Section 3 Solutions

3.1. No. The two shorter sides combined must be longer than the longest side.

3.2. Side c is the shortest and a is the longest side. The shortest side is opposite the smallest angle ($\angle C = 53°$), and the longest side is opposite the largest angle ($\angle A = 65°$).

3.3.

a) The largest angle ($\angle C = 73°$) must have the largest side opposite it. This is not the case.

b) The three angles do not add up to $180°$.

c) The two smaller sides added are less than the largest side.

d) Two angles of different degrees, $\angle A$ and $\angle C$, cannot have sides of the same value.

3.4.

a) $170°$ b) $150°$ c) $138°$ d) $109°$ e) $59°$ f) $43°$

3.5.

a) This is an ambiguous case. First find the acute measure of $\angle$C :

$$\begin{aligned}\frac{\sin \mathrm{C}}{c} &= \frac{\sin \mathrm{B}}{b}\\ \frac{\sin \mathrm{C}}{13} &= \frac{\sin 67^\circ}{12}\\ \sin \mathrm{C} &= \frac{13 \sin 67^\circ}{12}\\ \angle \mathrm{C} &= \sin^{-1}\left(\frac{13\sin 67^\circ}{12}\right)\\ \angle \mathrm{C} &= 85.721\ldots\end{aligned}$$

The acute measure of $\angle$C is 86°, to the nearest degree. Then, the obtuse measure of $\angle$C is $180^\circ - 85.721^\circ = 94.279\ldots\circ$. The obtuse measure of $\angle$C is 94°, to the nearest degree.
Then, find the measure of $\angle$A.

$$\begin{aligned}\angle \mathrm{A} &= 180^\circ - (85.721\ldots^\circ + 67^\circ) \quad \text{or} \quad \angle \mathrm{A} = 180^\circ - (94.279\ldots^\circ + 67^\circ)\\ \angle \mathrm{A} &= 27.279\ldots^\circ \quad\quad \angle \mathrm{A} = 18.721\ldots^\circ\end{aligned}$$

The measure of $\angle$A is 27° or 19°, to the nearest degree. Now find the measure of side a.

$$\begin{aligned}\frac{a}{\sin \mathrm{A}} &= \frac{b}{\sin \mathrm{B}} \quad \text{or} \quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}}\\ \frac{a}{\sin 27.279...0^\circ} &= \frac{12}{\sin 67^\circ}\\ a &= \frac{12\sin 27.279...^\circ}{\sin 67^\circ} \quad \frac{a}{\sin 18.721...^\circ} = \frac{12}{\sin 67^\circ}\\ a &= 5.974\ldots \quad\quad a = \frac{12\sin 18.721...^\circ}{\sin 67^\circ}\\ a &= 4.184\ldots\end{aligned}$$

Summary:
Acute case: $\angle \mathrm{C} = 86^\circ, \angle \mathrm{A} = 27^\circ$, side a is 6.0 m, to the nearest tenth of a metre.
Obtuse case: $\angle \mathrm{C} = 94^\circ, \angle \mathrm{A} = 19^\circ$, side a is 4.2 m, to the nearest tenth of a metre.
$<$ Note: If you use the rounded degree values, then a is 5.9 m for the acute case. For the obtuse case, using rounded degree values results in the same answer.

b) $\angle \mathrm{C} = 54^\circ$. The length of side a is 33.6 m, to the nearest tenth of a metre. The length of side c is 40.7 m, to the nearest tenth of a metre.

c) $\angle \mathrm{B} = 119^\circ$. The length of side a is 12.4 mm, to the nearest tenth of a millimetre. The length of side c is 20.9 mm, to the nearest millimetre.

d) $\angle B = 71^\circ$. The length of side a is 16.5 cm, to the nearest tenth of a centimetre. The length of side c is 19.4 cm, to the nearest centimetre.

3.6. Let C be the position of the coast guard ship and H the foot of the perpendicular from C to the shoreline.

$$\frac{\sin \mathrm{D}}{500} = \frac{\sin 20^\circ}{250}$$
$$\sin \mathrm{D} = \frac{500 \sin 20^\circ}{250}$$
$$\angle \mathrm{D} = \sin^{-1}\left(\frac{500 \sin 20^\circ}{250}\right)$$
$$\angle \mathrm{D} = 43.160\ldots$$

Then, in $\triangle$CDH :

$$\cos 43.160\ldots^\circ = \frac{\mathrm{DH}}{250}$$
$$\mathrm{DH} = 250 \cos 43.160\ldots^\circ$$
$$\mathrm{DH} = 182.361\ldots$$

Since $\mathrm{CA} = \mathrm{CD}, \Delta\mathrm{ACH} \approx \triangle\mathrm{DCH}$ and $\mathrm{AH} + \mathrm{HD}$. So $\mathrm{AD} = 2(182.361\ldots)$ or $364.722\ldots$ The length of shoreline that is illuminated by the spotlight is 364.7 m, to the nearest tenth of a metre.

3.7.

a) $\angle B = 15^\circ, b = 8.1, c = 13.1$

b) $\angle A = 27.5^\circ, \angle C = 114.5^\circ, c = 11.8$

c) $\angle A = 107^\circ, b = 90.3, c = 57.0$

d) Impossible triangle.

e) $\angle A = 90^\circ, \angle C = 30^\circ, c = 4$

f) $\angle A = 41^\circ, \angle B = 98^\circ, b = 13.6$

g) Impossible triangle.

h) $\angle C = 74^\circ, a = 34.8, c = 39.5$

i) $\angle C = 99^\circ, a = 11.9, b = 10.7$

j) $\angle B = 14.4^\circ, \angle C = 132.6^\circ, c = 36.7$

k) $\angle B = 38.7^\circ, 141.3^\circ, \angle C = 111.3^\circ, 8.7^\circ, c = 14.9.2.4$

l) $\angle B = 70.4^\circ, 109.6^\circ, \angle C = 51.6^\circ, 12.4^\circ, c = 8.3, 2.3$

m) $\angle C = 110^\circ, b = 22.4, c = 24.4$

n) $\angle A = 35^\circ, a = 48.7, b = 14.7$

o) Impossible triangle.

p) Impossible triangle.

3.8. 13.48 km

3.9. 1.42 km

3.10. 921 km

3.11. $AB' = 133.6$ million miles $AB = 26.4$ million miles

3.12. 12.65 cm