question_01_02

November 15, 2017

```
In [1]: import numpy as np
        from matplotlib import pyplot as plt
        from sklearn import cross_validation
        from scipy.stats import multivariate_normal
        from sklearn.datasets import load_digits
        from sklearn.model_selection import KFold
```

```
/Users/maxsimon/anaconda/lib/python3.6/site-packages/sklearn/cross_validation.py:44: Deprecati
  "This module will be removed in 0.20.", DeprecationWarning)
```

```
In [2]: digits = load_digits()
        print(digits.keys())
```

```
dict_keys(['data', 'target', 'target_names', 'images', 'DESCR'])
```

```
In [3]: data = digits["data"]
        images = digits["images"]
        target = digits["target"]
        target_names = digits["target_names"]
```

```
In [4]: # use only 1 and 7 for this exercise
        mask_all = np.logical_or(target == 1, target == 7)
        X_all = data[mask_all]
        y_all = target[mask_all]
```

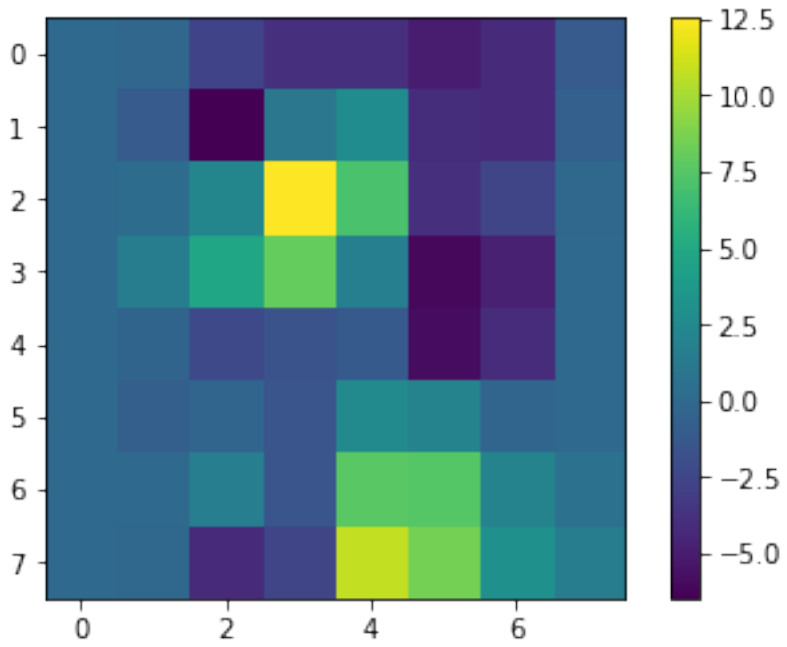
```
In [5]: # create a kfold instance (for the performance measurements)
        kf = KFold(n_splits=10)
```

1 Question 1: Dimension reduction

```
In [6]: def get_class(x, y, desired):
            """
            Returns a subarray of x where y = desired
            """
            return x[y == desired]
```

First of all we are looking onto the difference of a mean image of all 1's and a mean image of all 7's.

```
In [7]: fig, ax = plt.subplots(1, 1)
        im = ax.imshow(np.mean(get_class(images, target, 1), axis=0) - np.mean(get_class(images
        plt.colorbar(im)
        plt.show()
```



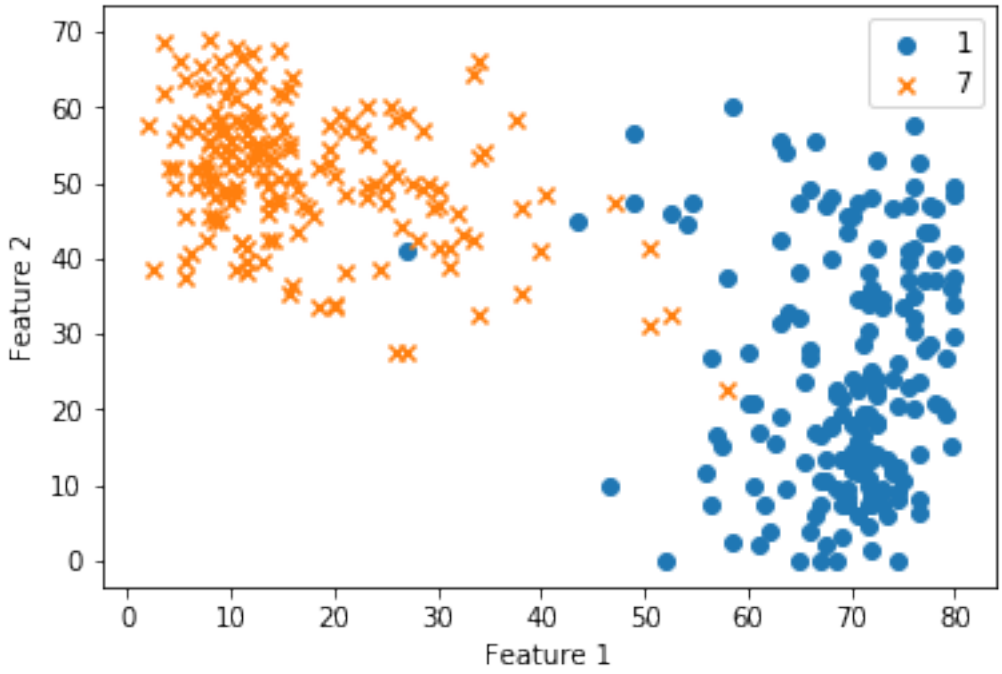
One can recognize, that pixels (2,3), (2,4), (3,3) as well as pixels (6,4), (6,5), (7,4), (7,5) are often populated in a 1 whereas pixels (1,2), (3,5), (4,5) and (7,2) are often populated in a 7.

```
In [8]: pixels_1 = [(2,3,3), (2,4,.5), (3,3,.5), (6,4,0.5), (6,5,0.5)]
        pixels_7 = [(1,2,1.5), (3,5,1.5), (4,5,1), (0,5,0.5)]

        def flat_ind(index):
            return np.ravel_multi_index(index, (8, 8))

        def reduce_dim(x):
            reduced = np.empty((x.shape[0], 2))
            # first feature is large for a digit 1
            reduced[:,0] = np.sum([pixel[2]*x[:,flat_ind(pixel[:2])] for pixel in pixels_1], ax
            # second feature is large for a digit 7
            reduced[:,1] = np.sum([pixel[2]*x[:,flat_ind(pixel[:2])] for pixel in pixels_7], ax
            return reduced
```

1.1 Validation of feature choice

```
In [9]: X_all_r = reduce_dim(X_all)
```

```
In [10]: fig, ax = plt.subplots(1, 1)
         # plot the 1's
         ax.scatter(get_class(X_all_r, y_all, 1)[:,0], get_class(X_all_r, y_all, 1)[:,1], marke
         # plot the 7's
         ax.scatter(get_class(X_all_r, y_all, 7)[:,0], get_class(X_all_r, y_all, 7)[:,1], marke
         ax.set_xlabel('Feature 1')
         ax.set_ylabel('Feature 2')
         ax.legend()
         plt.show()
```



Not to good, as required :D

2 Question 2: Nearest Mean Classifier

```
In [11]: def nearest_mean(training_features, training_labels, test_features):
             """
             This function implements the nearest mean classifier
             """
             # calculating the mean of feature 1 and feature 2 for the training set of digit 1
             mean_1 = np.mean(get_class(training_features, training_labels, 1), axis = 0)
             # calculating the mean of feature 1 and feature 2 for the training set of digit 7
             mean_7 = np.mean(get_class(training_features, training_labels, 7), axis = 0)

             # create distance matrix
             distances = np.empty_like(test_features)
```

```
    distances[:,0] = np.sqrt(np.sum([(test_features[:,i] - mean_1[i])**2 for i in rang
    distances[:,1] = np.sqrt(np.sum([(test_features[:,i] - mean_7[i])**2 for i in rang

    # create prediction
    prediction = np.empty((test_features.shape[0]), dtype=float)
    prediction[:] = 1 # set all to label 1
    # if distance to 7 is smaller: set to label 7
    prediction[distances[:,0] > distances[:,1]] = 7

    return prediction
```

2.1 Visualisation of decision regions

For this exercise we need a training and a test set. However, the performance measurement is done with cross validation.

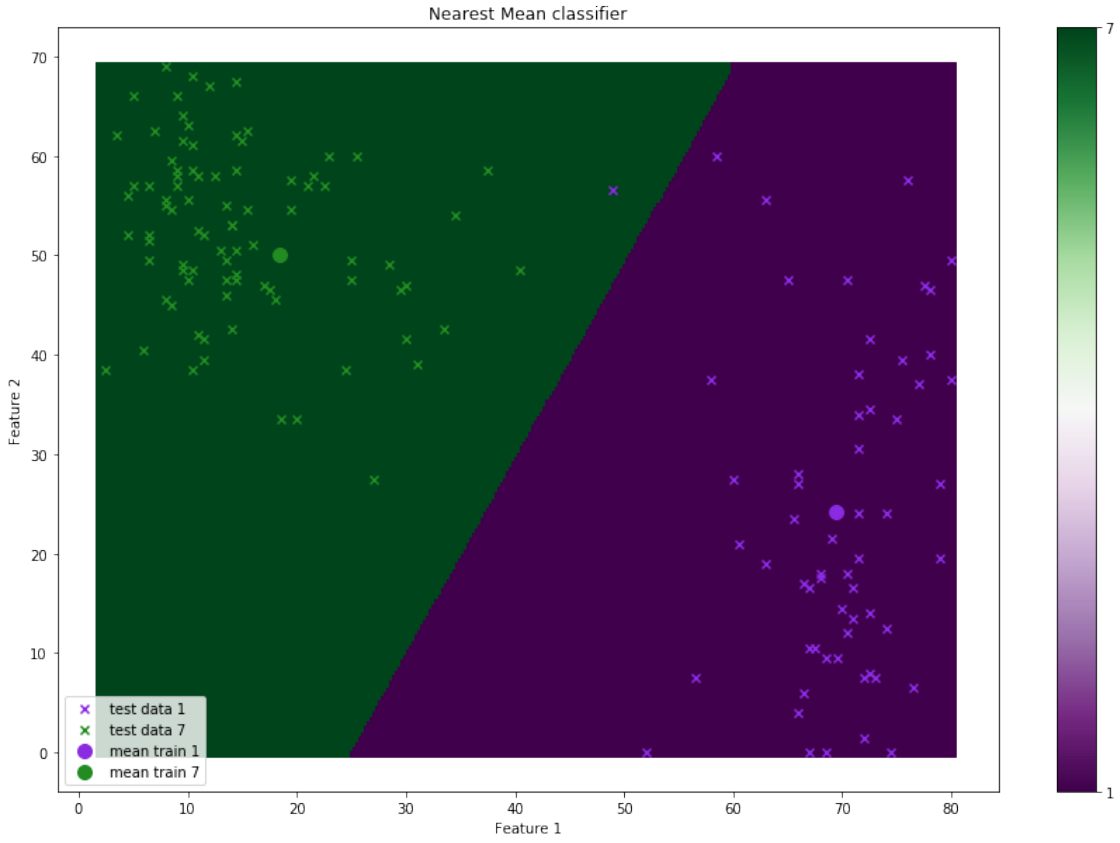
```
In [12]: X_train_r , X_test_r , y_train , y_test = cross_validation.train_test_split(X_all_r, y
```

```
In [13]: # create a grid for decision regions
         grid_feat_1 = np.linspace(np.min(X_all_r[:,0]), np.max(X_all_r[:,0]), 300)
         grid_feat_2 = np.linspace(np.min(X_all_r[:,1]), np.max(X_all_r[:,1]), 300)

         # thanks to SO, this has suitable dimensions for the algos
         grid = np.transpose([np.tile(grid_feat_1, len(grid_feat_2)), np.repeat(grid_feat_2, le
         # this has suitable dimensions for plotting contours
         mesh_feat1, mesh_feat2 = np.meshgrid(grid_feat_1, grid_feat_2)
```

```
In [14]: # apply classifier
         grid_pred_nm = nearest_mean(X_train_r, y_train, grid)
         test_pred_nm = nearest_mean(X_train_r, y_train, X_test_r)
```

```
In [33]: # plot that stuff

         # Create the plot
         fig, ax = plt.subplots(1, 1, figsize=(15, 10))
         # plot the decision region
         scat = ax.scatter(grid[:,0], grid[:,1], c=grid_pred_nm, marker='s', cmap='PRGn')
         cbar = plt.colorbar(scat, ticks=[1, 7])
         # plot the test data
         ax.scatter(get_class(X_test_r, y_test, 1)[:,0], get_class(X_test_r, y_test, 1)[:,1], m
         ax.scatter(get_class(X_test_r, y_test, 7)[:,0], get_class(X_test_r, y_test, 7)[:,1], m
         # plot the means for 1 and 7, yeah well nice implementation of the mean :D
         ax.scatter(*np.mean(get_class(X_train_r, y_train, 1), axis = 0), marker='o', color='bl
         ax.scatter(*np.mean(get_class(X_train_r, y_train, 7), axis = 0), marker='o', color='fo
         # labelling
         ax.set_xlabel('Feature 1')
         ax.set_ylabel('Feature 2')
         ax.set_title('Nearest Mean classifier')
```

```
ax.legend(loc=3)
plt.show()
```



In [16]:
```
# Performance measurement
error_rates_nm = []
for train, test in kf.split(X_all_r):
    # prediction
    pred_fold_nm = nearest_mean(X_all_r[train], y_all[train], X_all_r[test])
    # calculation of error
    error_rates_nm.append(np.count_nonzero(pred_fold_nm - y_all[test])/pred_fold_nm.sh

mean_error_nm = np.mean(error_rates_nm)
std_error_nm = np.std(error_rates_nm)

print('Mean error rate on 10 folds: {:f} (std: {:f})'.format(mean_error_nm, std_error_
print('This corresponds to {:f} wrong classifications'.format(mean_error_nm*len(test))
```

```
Mean error rate on 10 folds: 0.019444 (std: 0.032984)
This corresponds to 0.700000 wrong classifications
```

3 Question 3: QDA

3.1 Implementation of fitting

```
In [34]: def fit_qda(training_features, training_labels, possible_features=[1, 7]):
             D = training_features.shape[1] # dimension of features
             F = len(possible_features) # number of features we are dealing with

             # create an array of testsetdata
             ts = [get_class(training_features, training_labels, feature) for feature in possi
             # caculate the total amount of testdata (including all possible features)
             N_tot = sum([tsf.shape[0] for tsf in ts])

             # mu has the shape FxD
             mu = np.empty((F, D), dtype=float)
             # cov should have the shape FxDxD
             cov = np.empty((F, D, D))
             # the priors are scalars and have therefore the shape F
             p = np.empty(F)

             for i in range(F):
                 N = ts[i].shape[0] # number of training instances for the feature possible_fe
                 # calculating mu
                 mu[i] = np.mean(ts[i], axis=0)

                 # calculating the covariance matrix
                 ts_centralised = ts[i] - mu[i]
                 # some numpy magic
                 cov[i] = np.add.reduce(ts_centralised[:,:,np.newaxis] * ts_centralised[:,np.ne

                 # calculating the priors
                 p[i] = N/N_tot

             # done
             return mu, cov, p
```

3.2 Prediction

```
In [35]: def predict_qda(mu, covmat, p, test_features, possible_features = [1, 7]):
             # numpy array to store the results for each k
             results = np.empty((len(possible_features), test_features.shape[0]))

             # for each class
             for i in range(len(possible_features)):
                 # calculating b_k
                 bk = np.log(np.linalg.det(covmat[i])) - 2*np.log(p[i])

                 # calculating the inverse of the cov-matrix
```

```
            sig_inv = np.linalg.inv(covmat[i])
            # centralise the coordinates
            centralised = test_features - mu[i]
            # calculate the Mohalunsbis-distance with some numpy magic
            moha_dist = np.sum(centralised * np.tensordot(sig_inv, centralised, (1, 1)).T
            # store the result
            results[i] = moha_dist + bk

        # minimize the results
        minimized = np.argmin(results, axis = 0)

        # this will hold the predicted label instead of its index in possible_features
        test_labels = np.empty(test_features.shape[0])
        # populate this stuff
        for i, k in enumerate(possible_features):
            test_labels[minimized == i] = k

        return test_labels
```

3.2.1 Fitting with training data

I guess we are supposed to train with the complete data set since we are applying crossvalidation later.

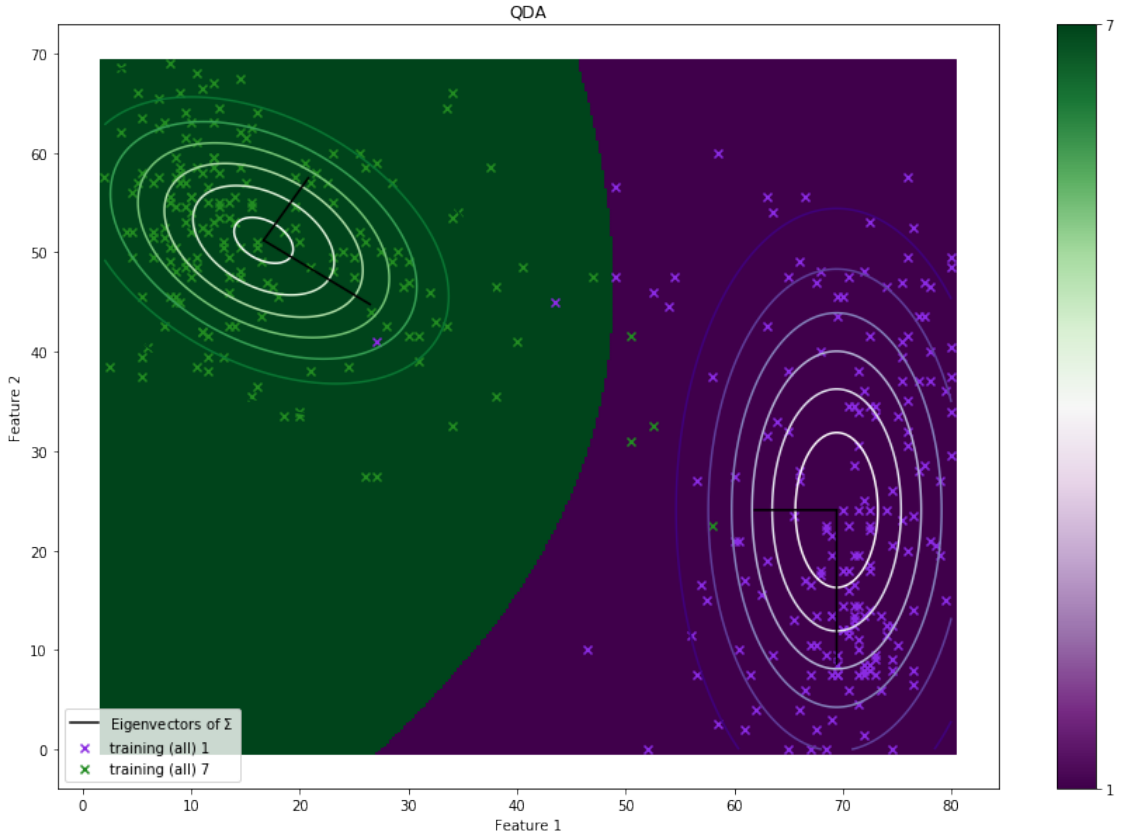
```
In [19]: mu_qda, cov_qda, p_qda = fit_qda(X_all_r, y_all)
```

```
In [20]: # apply to test set, training set and grid
         grid_pred_qda = predict_qda(mu_qda, cov_qda, p_qda, grid)
         train_pred_qda = predict_qda(mu_qda, cov_qda, p_qda, X_all_r)
```

3.3 Visualisation

```
In [21]: # Create the plot
         fig, ax = plt.subplots(1, 1, figsize=(15, 10))

         # plot the decision region
         scat = ax.scatter(grid[:,0], grid[:,1], c=grid_pred_qda, marker='s', cmap='PRGn')
         cbar = plt.colorbar(scat, ticks=[1, 7])
         # plot the training
         ax.scatter(get_class(X_all_r, y_all, 1)[:,0], get_class(X_all_r, y_all, 1)[:,1], marke
         ax.scatter(get_class(X_all_r, y_all, 7)[:,0], get_class(X_all_r, y_all, 7)[:,1], marke

         #plot the gaussians
         gauss_1_qda = multivariate_normal.pdf(grid, mean=mu_qda[0], cov=cov_qda[0])
         gauss_2_qda = multivariate_normal.pdf(grid, mean=mu_qda[1], cov=cov_qda[1])
         cg1 = ax.contour(mesh_feat1, mesh_feat2, gauss_1_qda.reshape(300, 300), cmap='Purples_
         cg2 = ax.contour(mesh_feat1, mesh_feat2, gauss_2_qda.reshape(300, 300), cmap='Greens_r

         # eigenvalue decomposition
```

```
e_val_1, e_vec_1 = np.linalg.eig(cov_qda[0])
e_val_2, e_vec_2 = np.linalg.eig(cov_qda[1])
# for label 1
ax.plot([mu_qda[0, 0], mu_qda[0, 0] + np.sqrt(e_val_1[0])*e_vec_1[0, 0]], [mu_qda[0,
ax.plot([mu_qda[0, 0], mu_qda[0, 0] + np.sqrt(e_val_1[1])*e_vec_1[0, 1]], [mu_qda[0,
# for label 7
ax.plot([mu_qda[1, 0], mu_qda[1, 0] + np.sqrt(e_val_2[0])*e_vec_2[0, 0]], [mu_qda[1,
ax.plot([mu_qda[1, 0], mu_qda[1, 0] + np.sqrt(e_val_2[1])*e_vec_2[0, 1]], [mu_qda[1,

# labelling
ax.set_xlabel('Feature 1')
ax.set_ylabel('Feature 2')
ax.set_title('QDA')
ax.legend(loc=3)
plt.show()
```



There are some missclassifications. We can find 4 instances of 7 inside the decision area of 1 and 2 instances of 1 inside the decision area for 7. There are 182 training instances for label 1 and 179 training instances for label 7. Therefore the training error is $\approx \frac{6}{179+182} = 1.7\%$. In comparison to the Nearest Neighbour classifier, QDA does not use the training instances directly for classification, but the fitted Gaussian. Therefore outlayers of the cluster can be misclassified.

3.4 Performance measure

```
In [22]: error_rates_qda = []
         for train, test in kf.split(X_all_r):
             # fitting
             mu_fold_qda, cov_fold_qda, p_fold_qda = fit_qda(X_all_r[train], y_all[train])
             # prediction
             pred_fold_qda = predict_qda(mu_fold_qda, cov_fold_qda, p_fold_qda, X_all_r[test])
             # calculation of error
             error_rates_qda.append(np.count_nonzero(pred_fold_qda - y_all[test])/pred_fold_qda

         mean_error_qda = np.mean(error_rates_qda)
         std_error_qda = np.std(error_rates_qda)

         print('Mean error rate on 10 folds: {:f} (std: {:f})'.format(mean_error_qda, std_erro
         print('This corresponds to {:f} wrong classifications'.format(mean_error_qda*len(test
```

```
Mean error rate on 10 folds: 0.019444 (std: 0.030556)
This corresponds to 0.700000 wrong classifications
```

4 Question 4: LDA

4.1 Implementation of Fitting

```
In [36]: # mainly copy paste except for calculation of covariance matrix
         def fit_lda(training_features, training_labels, possible_features=[1, 7]):
             D = training_features.shape[1] # dimension of features
             F = len(possible_features) # number of features we are dealing with

             # create an array of testsetdata
             ts = [get_class(training_features, training_labels, feature) for feature in possi
             # caculate the total amount of testdata (including all possible features)
             N_tot = sum([tsf.shape[0] for tsf in ts])

             # mu has the shape FxD
             mu = np.empty((F, D), dtype=float)
             # cov should have the shape FxDxD
             cov = np.zeros((D, D))
             # the priors are scalars and have therefore the shape F
             p = np.empty(F)

             for i in range(F):
                 N = ts[i].shape[0] # number of training instances for the feature possible_fe
                 # calculating mu
                 mu[i] = np.mean(ts[i], axis=0)

                 # calculating the covariance matrix
```

```
            ts_centralised = ts[i] - mu[i]
            # some numpy magic
            cov += np.add.reduce(ts_centralised[:,:,np.newaxis] * ts_centralised[:,np.newa

            # calculating the priors
            p[i] = N/N_tot

        w = 2*np.dot(mu, cov)
        bk = -np.diag(np.tensordot(mu, np.dot(mu, cov), (1, 1))) - np.log(np.linalg.det(co

        # done
        return w, bk, mu, cov # mu and cov just need for plotting
```

4.2 Prediction

```
In [37]: def predict_lda(w, bk, test_features, possible_features = [1, 7]):
             # numpy array to store the results for each k
             results = np.empty((len(possible_features), test_features.shape[0]))

             # for each class
             for i in range(len(possible_features)):
                 # store the result
                 results[i] = np.dot(test_features, w[i]) + bk[i]

             # maximize the results
             maximize = np.argmax(results, axis = 0)

             # this will hold the predicted label instead of its index in possible_features
             test_labels = np.empty(test_features.shape[0])
             # populate this stuff
             for i, k in enumerate(possible_features):
                 test_labels[maximize == i] = k

             return test_labels
```

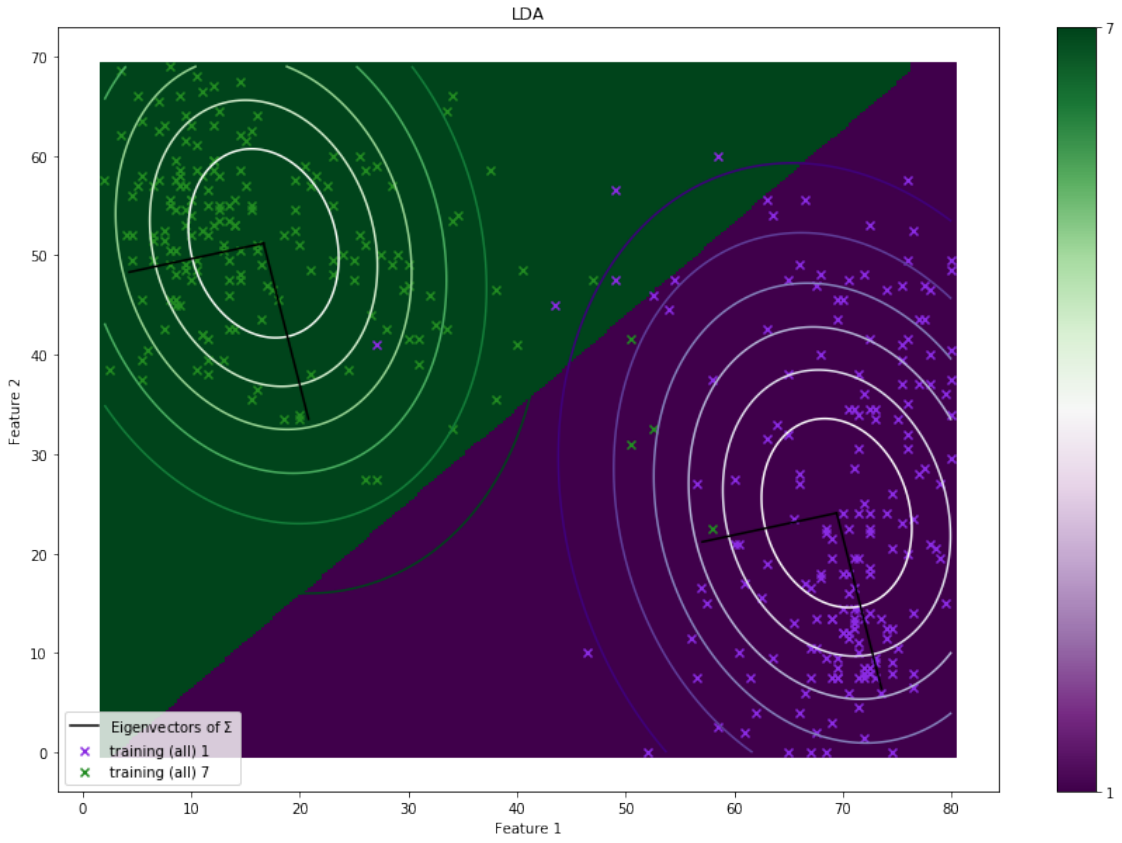
4.2.1 Fitting with training data

```
In [38]: w_lda, bk_lda, mu_lda, cov_lda = fit_lda(X_all_r, y_all)

In [39]: # apply to test set, training set and grid
         grid_pred_lda = predict_lda(w_lda, bk_lda, grid)
         train_pred_lda = predict_lda(w_lda, bk_lda, X_all_r)
```

4.3 Visualisation

```
In [40]: # Create the plot
         fig, ax = plt.subplots(1, 1, figsize=(15, 10))
```

```
# plot the decision region
scat = ax.scatter(grid[:,0], grid[:,1], c=grid_pred_lda, marker='s', cmap='PRGn')
cbar = plt.colorbar(scat, ticks=[1, 7])
# plot the training
ax.scatter(get_class(X_all_r, y_all, 1)[:,0], get_class(X_all_r, y_all, 1)[:,1], marke
ax.scatter(get_class(X_all_r, y_all, 7)[:,0], get_class(X_all_r, y_all, 7)[:,1], marke

#plot the gaussians
gauss_1_lda = multivariate_normal.pdf(grid, mean=mu_lda[0], cov=cov_lda)
gauss_2_lda = multivariate_normal.pdf(grid, mean=mu_lda[1], cov=cov_lda)
cg1 = ax.contour(mesh_feat1, mesh_feat2, gauss_1_lda.reshape(300, 300), cmap='Purples_
cg2 = ax.contour(mesh_feat1, mesh_feat2, gauss_2_lda.reshape(300, 300), cmap='Greens_r

# eigenvalue decomposition
e_val_lda, e_vec_lda = np.linalg.eig(cov_lda)
# for label 1
ax.plot([mu_lda[0, 0], mu_lda[0, 0] + np.sqrt(e_val_lda[0])*e_vec_lda[0, 0]], [mu_lda
ax.plot([mu_lda[0, 0], mu_lda[0, 0] + np.sqrt(e_val_lda[1])*e_vec_lda[0, 1]], [mu_lda
# for label 7
ax.plot([mu_lda[1, 0], mu_lda[1, 0] + np.sqrt(e_val_lda[0])*e_vec_lda[0, 0]], [mu_lda
ax.plot([mu_lda[1, 0], mu_lda[1, 0] + np.sqrt(e_val_lda[1])*e_vec_lda[0, 1]], [mu_lda

# labelling
ax.set_xlabel('Feature 1')
ax.set_ylabel('Feature 2')
ax.set_title('LDA')
ax.legend(loc=3)
plt.show()
```



4.4 Performance measure

```
In [41]: error_rates_lda = []
         for train, test in kf.split(X_all_r):
             # fitting
             w_fold_lda, bk_fold_lda, _, _ = fit_lda(X_all_r[train], y_all[train])
             # prediction
             pred_fold_lda = predict_lda(w_fold_lda, bk_fold_lda, X_all_r[test])
             # calculation of error
             error_rates_lda.append(np.count_nonzero(pred_fold_lda - y_all[test])/pred_fold_lda

         mean_error_lda = np.mean(error_rates_lda)
         std_error_lda = np.std(error_rates_lda)

         print('Mean error rate on 10 folds: {:f} (std: {:f})'.format(mean_error_lda, std_erro
         print('This corresponds to {:f} wrong classifications'.format(mean_error_lda*len(test

Mean error rate on 10 folds: 0.027778 (std: 0.044790)
This corresponds to 1.000000 wrong classifications
```

5 Comparison

In my case the Nearest Mean Classifier and the QDA perform equally well. The LDA has a slightly larger error rate. However, the digits 1 and 7 are well distinguishable because the selected features differ a lot for the two digits.