question_02

November 21, 2017

```
In [1]: %pylab inline
        import numpy as np
        from matplotlib import pyplot as plt
        from sklearn.datasets import load_digits
        from sklearn.model_selection import train_test_split, KFold
        np.seterr(divide='ignore', invalid='ignore');
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
In [2]: digits = load_digits()
        print(digits.keys())
```

```
dict_keys(['data', 'target', 'target_names', 'images', 'DESCR'])
```

```
In [3]: data = digits["data"]
        images = digits["images"]
        target = digits["target"]
        target_names = digits["target_names"]
```

```
In [4]: # create a kfold instance (for the performance measurements)
        kf = KFold(n_splits=10)
```

0.1 Dimension reduction

```
In [5]: # Dimension reduction

        # pixels representing 3
        pixels_3 = [(2,2,1), (3,2,1)]
        # pixels representing 9
        pixels_9 = [(4,4,1)]

        def flat_ind(index):
            return np.ravel_multi_index(index, (8, 8))

        def reduce_dim(x):
            reduced = np.empty((x.shape[0], 2))
```

```
    # first feature is large for a digit 3
    reduced[:,0] = np.sum([pixel[2]*x[:,flat_ind(pixel[:2])] for pixel in pixels_3], ax
    # second feature is large for a digit 9
    reduced[:,1] = np.sum([pixel[2]*x[:,flat_ind(pixel[:2])] for pixel in pixels_9], ax
    return reduced
```

0.2 Prepare training and testsets

```
In [6]: # use only 3 and 9 for this exercise
        mask_all = np.logical_or(target == 3, target == 9)
        X_all = data[mask_all]
        y_all = target[mask_all]

        X_train , X_test , y_train , y_test = train_test_split(X_all, y_all, test_size=0.4, ran

        X_all_r = reduce_dim(X_all)
        X_train_r , X_test_r , y_train , y_test = train_test_split(X_all_r, y_all, test_size=0
```

1 Implementation of Naive Bayes

```
In [7]: def fit_naive_bayes(features, labels, bincount, possible_labels=[3, 9]):

            np_bin_calc = bincount # this is the argument for numpy.histogram for determining

            if bincount == 0: # get our own binning
                all_l = np.array([])
                for i, label in enumerate(possible_labels):
                    # calculate for each label: iqr, n, d (full range of data) and out of thes
                    iqr = np.percentile(features[labels == label], 75, axis=0) - np.percentile
                    n = features[labels == label].shape[0]
                    d = (np.max(features[labels == label], axis=0) - np.min(features[labels ==

                    mask = iqr != 0
                    all_l = np.append(all_l, d[mask]/(2*iqr[mask]/n**(1/3)))

                # set the largest L as the bincount (see below)
                bincount = int(np.ceil(np.max(all_l)))
                # tell numpy later to use the Freedman Diaconis Estimator
                np_bin_calc = 'fd'

            # create arrays for later
            histo = np.zeros((len(possible_labels), features.shape[1], bincount))
            # the return variable binning will have the dimensions: CxDx3
            # this is because I decided to use for each histogram the optimal/predicted L, if 
            # in order to do so, the last dimension of the histogram must be the largest L in 
            # on the other hand, histogram overflow can only be handled by knowing, where the 
            binning = np.zeros((len(possible_labels), features.shape[1], 3)) # the last dimens
```

```
    for i, label in enumerate(possible_labels):
        for j in range(features.shape[1]): # loop over feature dimensions

            # create the histogram
            np_hist = np.histogram(features[labels == label, j], bins=np_bin_calc, dens
            # store histogram
            histo[i, j][:np_hist[0].shape[0]] = np_hist[0]
            # save first edge of histogram
            binning[i, j, 0] = np_hist[1][0]
            # save bin width of histogram
            binning[i, j, 1] = np.abs(np_hist[1][1] - np_hist[1][0])
            # store number of bins (L) of histogram
            binning[i, j, 2] = np_hist[0].shape[0]

    return histo, binning
```

```
In [8]: def predict_naive_bayes(test_features, histograms, binning, possible_labels=[3, 9]):
            scores = np.empty((len(possible_labels), test_features.shape[0]))
            for k in range(len(possible_labels)):
                reshaped_binning = np.array([binning[k] for _ in range(test_features.shape[0])]

                hist_ind = np.int_(np.floor((test_features - reshaped_binning[:, :, 0])/reshape
                # handle underflow
                hist_ind[hist_ind < 0] = 0
                # handle overflow
                hist_ind[hist_ind >= reshaped_binning[:, :, 2]] = reshaped_binning[hist_ind >=
                probs = np.array([histograms[k, j, hist_ind[:, j]] for j in range(test_feature

                scores[k] = np.sum(np.log(probs), axis=0)

            prediction = np.argmax(scores, axis=0)
            for i, k in enumerate(possible_labels):
                prediction[prediction == i] = k

            return prediction
```

```
In [9]: def get_confusion_matrix(predicted, truth, possible_labels=[3, 9]):
            conf = np.empty((len(possible_labels), len(possible_labels)))
            for i, k in enumerate(possible_labels):
                items, counts = np.unique(predicted[truth == k], return_counts=True)
                count_array = np.array([counts[np.where(items == tested_k)[0][0]] for tested_k
                conf[i] = count_array
            return conf
```

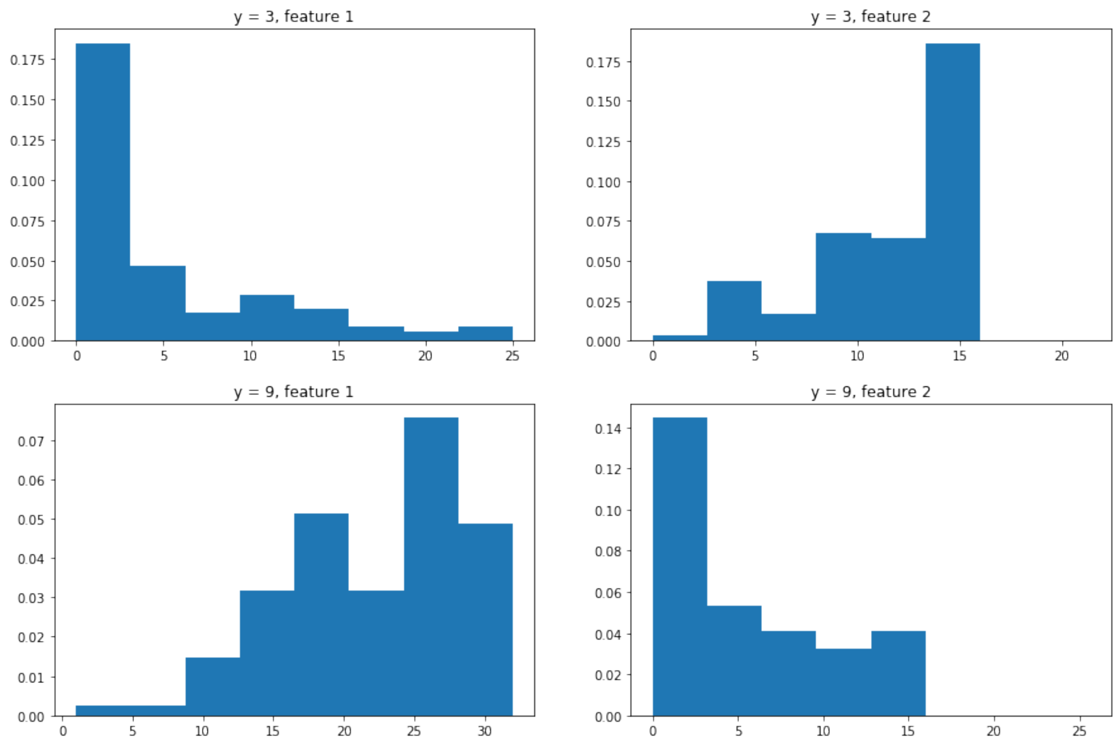
```
In [10]: # create a grid for decision regions
         grid_feat_1 = np.linspace(np.min(X_all_r[:,0]), np.max(X_all_r[:,0]), 300)
```

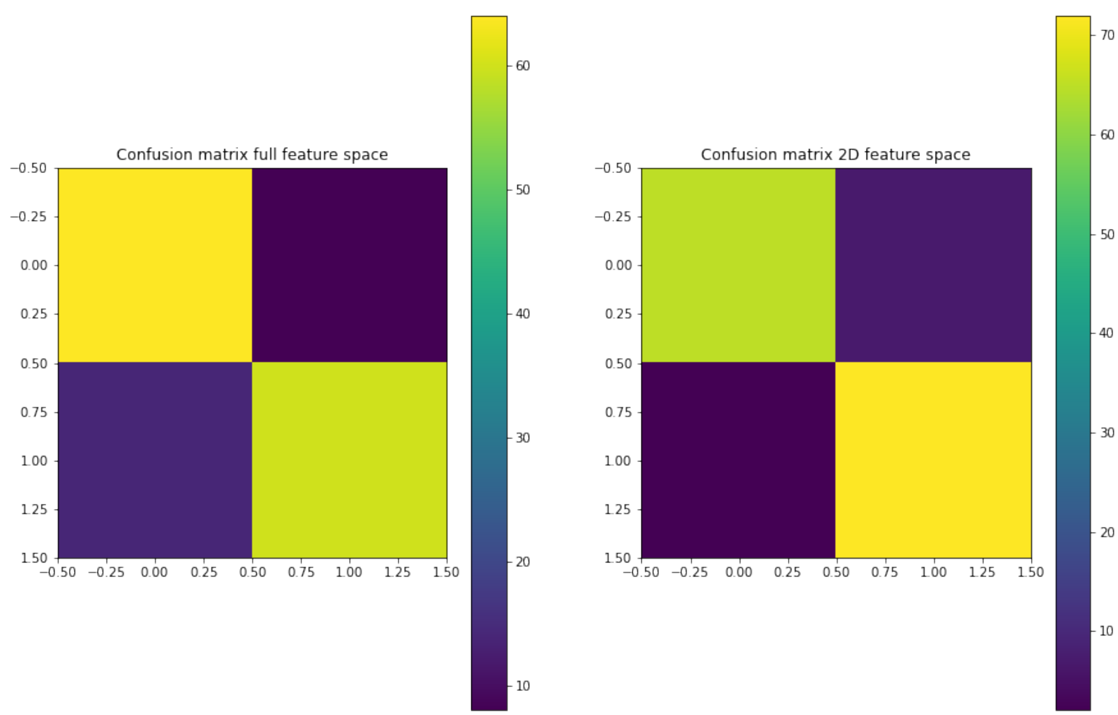
```
grid_feat_2 = np.linspace(np.min(X_all_r[:,1]), np.max(X_all_r[:,1]), 300)

# thanks to SO, this has suitable dimensions for the algos
grid = np.transpose([np.tile(grid_feat_1, len(grid_feat_2)), np.repeat(grid_feat_2, le
# this has suitable dimensions for plotting contours
mesh_feat1, mesh_feat2 = np.meshgrid(grid_feat_1, grid_feat_2)
```

In [11]:
```
# full feature space
histograms, binning = fit_naive_bayes(X_train, y_train, 8)
test_pred = predict_naive_bayes(X_test, histograms, binning)
conf_ff = get_confusion_matrix(test_pred, y_test)
# Decision region
histograms, binning = fit_naive_bayes(X_train_r, y_train, 0)
grid_pred = predict_naive_bayes(grid, histograms, binning)
# 2D-Feature space
histograms_2f, binning_2f = fit_naive_bayes(X_train_r, y_train, 0)
test_pred_r = predict_naive_bayes(X_test_r, histograms_2f, binning_2f)
conf_2f = get_confusion_matrix(test_pred_r, y_test)
```

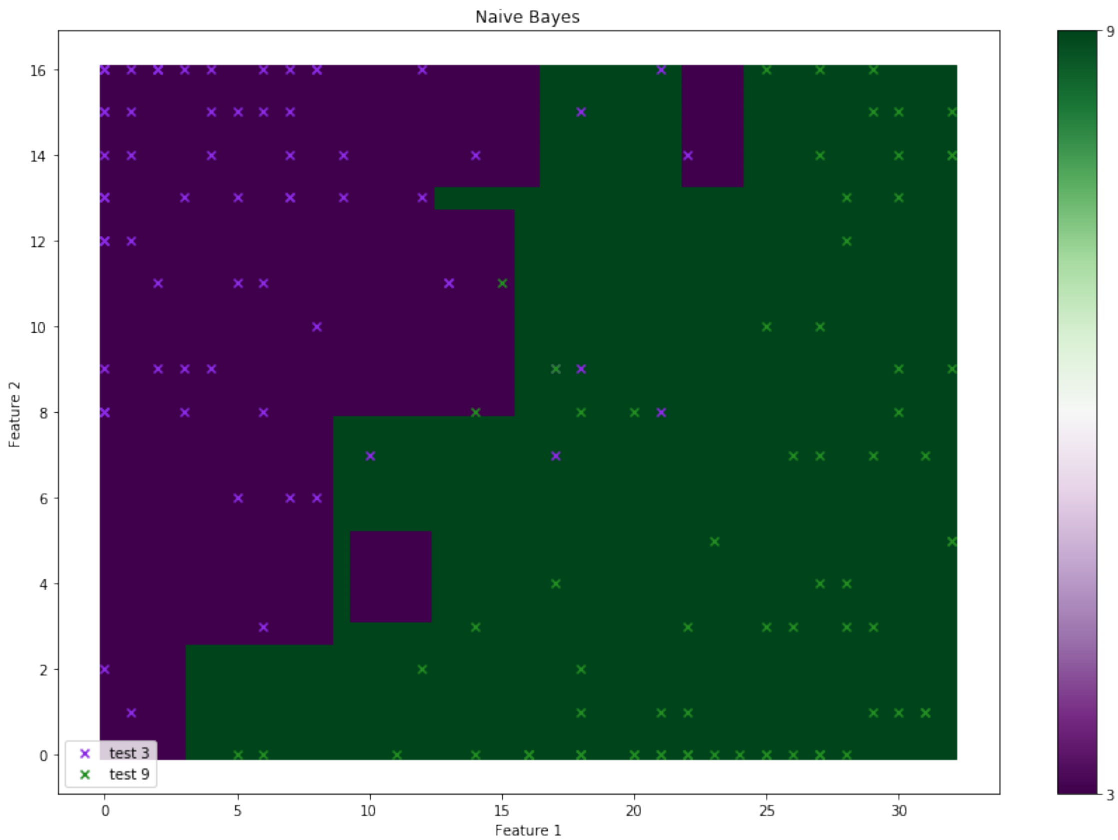
1.1 Visualisation

In [12]:
```
# Histogramms
fig, ax = plt.subplots(2, 2, figsize=(15, 10))
ax[0][0].set_title('y = 3, feature 1')
ax[0][0].bar([binning_2f[0][0][0] + (i + 0.5)*binning_2f[0][0][1] for i in range(histo
ax[0][1].set_title('y = 3, feature 2')
ax[0][1].bar([binning_2f[0][1][0] + (i + 0.5)*binning_2f[0][1][1] for i in range(histo
ax[1][0].set_title('y = 9, feature 1')
ax[1][0].bar([binning_2f[1][0][0] + (i + 0.5)*binning_2f[1][0][1] for i in range(histo
ax[1][1].set_title('y = 9, feature 2')
ax[1][1].bar([binning_2f[1][1][0] + (i + 0.5)*binning_2f[1][1][1] for i in range(histo
plt.show()

# Confusion matrices
fig, ax = plt.subplots(1, 2, figsize=(15, 10))
im = ax[0].imshow(conf_ff)
im2 = ax[1].imshow(conf_2f)
plt.colorbar(im, ax=ax[0])
plt.colorbar(im2, ax=ax[1])
ax[0].set_title('Confusion matrix full feature space')
ax[1].set_title('Confusion matrix 2D feature space')
plt.show()
```


y = 3, feature 1
y = 3, feature 2
y = 9, feature 1
y = 9, feature 2


Confusion matrix full feature space
Confusion matrix 2D feature space

In [13]:
```
# Decision region
fig, ax = plt.subplots(1, 1, figsize=(15, 10))

# plot the decision region
scat = ax.scatter(grid[:,0], grid[:,1], c=grid_pred, marker='s', cmap='PRGn')
cbar = plt.colorbar(scat, ticks=[3, 9])
# plot the test
ax.scatter(X_test_r[y_test == 3][:,0], X_test_r[y_test == 3][:,1], marker='x', color=
ax.scatter(X_test_r[y_test == 9][:,0], X_test_r[y_test == 9][:,1], marker='x', color=

# labelling
ax.set_xlabel('Feature 1')
ax.set_ylabel('Feature 2')
ax.set_title('Naive Bayes')
ax.legend(loc=3)
plt.show()
```



In [14]:
```
error_rates = []
for train, test in kf.split(X_all):
    # fitting
    histograms, binning = fit_naive_bayes(X_all_r[train], y_all[train], 8)
```

```
            # prediction
            prediction = predict_naive_bayes(X_all_r[test], histograms, binning)
            # calculation of error
            error_rates.append(np.count_nonzero(prediction - y_all[test])/prediction.shape[0])
        mean_error = np.mean(error_rates)
        std_error = np.std(error_rates)

        print('Mean error rate on 10 folds: {:f} (std: {:f})'.format(mean_error, std_error))
        print('This corresponds to {:f} wrong classifications'.format(mean_error*len(test)))
```

```
Mean error rate on 10 folds: 0.065916 (std: 0.070308)
This corresponds to 2.372973 wrong classifications
```

```
In [ ]:
```