Предел функции

4. c) $$\lim_{x\to\infty}\left(\frac{x+3}{x}\right)^{4x+1}=\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x+1}=$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{\frac{x}{3}\cdot\frac{3}{x}\cdot(4x+1)}=e^{\lim\limits_{x\to\infty}\frac{3}{x}(4x+1)}=\cancel{e^{(12+\frac{3}{x})}}$$

$$=e^{\lim\limits_{x\to\infty}(12+\frac{3}{x})}=e^{12}$$

Теорема о пределах

a) $$\lim_{x\to 0}\frac{\sin(2x)}{4x}=\lim_{x\to 0}\frac{1}{2}\frac{\sin(2x)}{2x}=\frac{1}{2}$$

b) $$\lim_{x\to 0}\frac{x}{\sin(x)}=\lim_{x\to 0}\frac{1}{\frac{\sin x}{x}}=1$$

c) $$\lim_{x\to 0}\frac{x}{\arcsin x}=\lim_{x\to 0}\frac{1}{\frac{\arcsin x}{x}}=1$$

d) $$\lim_{x\to\infty}\left(\frac{4x+3}{4x-3}\right)^{6x}=\lim_{x\to\infty}\left(1+\frac{6}{4x-3}\right)^{6x}=$$

$$\lim_{x\to\infty}\left(1+\frac{6}{4x-3}\right)^{\frac{6}{4x-3}\cdot\frac{4x-3}{6}\cdot 6x}=\lim_{x\to\infty}e^{\frac{36x}{4x-3}}=$$

$$=e^{\lim\limits_{x\to\infty}(9+\frac{27}{4x-3})}=e^{9}$$

e) $$\lim_{x\to\infty}\frac{\sin x+\ln x}{x}=\underbrace{\lim_{x\to\infty}\frac{\sin x}{x}}_{\to 0}+\lim_{x\to\infty}\frac{\ln x}{x}=$$

$$=\lim_{x\to\infty}\frac{\ln x}{x}\ \cancel{=\left|y=\frac{1}{x}\right|=\lim_{y\to 0}(-y\ln y)}$$

по правилу Лопиталя

$$=\lim_{x\to\infty}\frac{\ln' x}{x'}=\lim_{x\to\infty}\frac{1}{x}=0$$