MODÉLISATION ET RÉSOLUTION DE LA RÉPONSE À LA COLLISION

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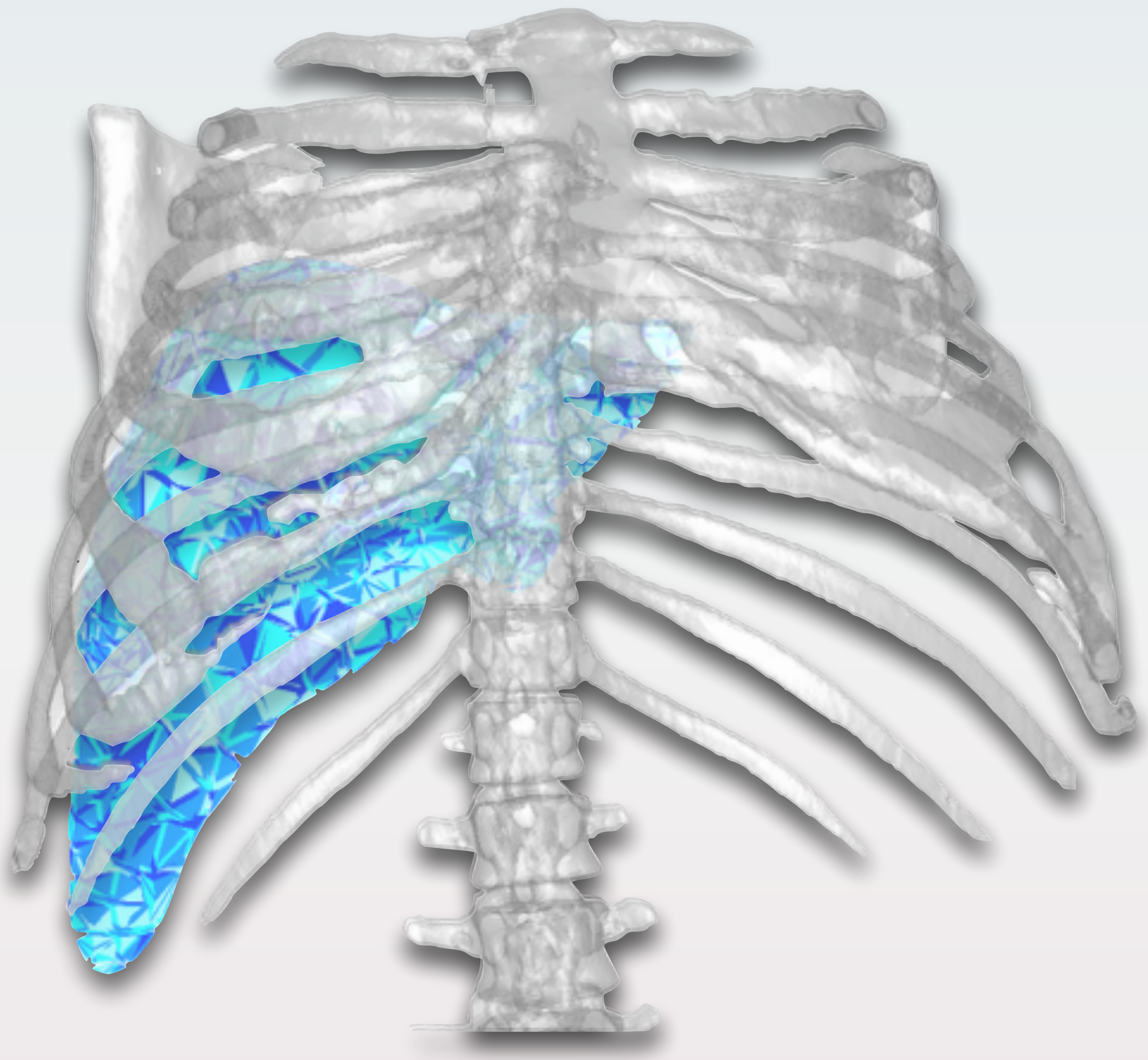
WHY IMPORTANT ?

- Tool-tissue interaction realism
  - Post interaction deformation
  - Behavior of the instruments
  - Haptic rendering
- Boundary conditions
  - For both tools and soft tissues
  - Interactions between anatomical structures
- Surgical simulator ≠ scripted game
  - Limited possible precomputation
  - Should allow for mistakes...
- NEED OF PHYSICS !

OUTCOME

- Mechanical models for real-time computation (rappel)
- Constraint-based modeling of biomechanical interactions



(RAPPELS) MECHANICAL MODELS FOR REAL-TIME COMPUTATION

MECHANICAL MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\boxed{\dot{\mathbf{v}}} = \mathbb{P}(\boxed{t}) - \mathbb{F}\left(\mathbf{q}, \boxed{\mathbf{v}}\right) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\mathbf{v} \in \mathbb{R}^n$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n\times n}$	Inertia Matrix
$\mathbb{F}\left(\mathbf{q}, \mathbf{v}\right)$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\mathbf{H}^T\lambda \in \mathbb{R}^n$	Constraint force contribution

TIME INTEGRATION SCHEMES

- Explicit Methods:

$$\mathbb{M}\,\underbrace{\dot{\mathbf{v}}}_{\text{unknown}} = \mathbb{P}(t) - \mathbb{F}\,\underbrace{(\mathbf{q},\mathbf{v})}_{\text{known from previous time steps}}$$

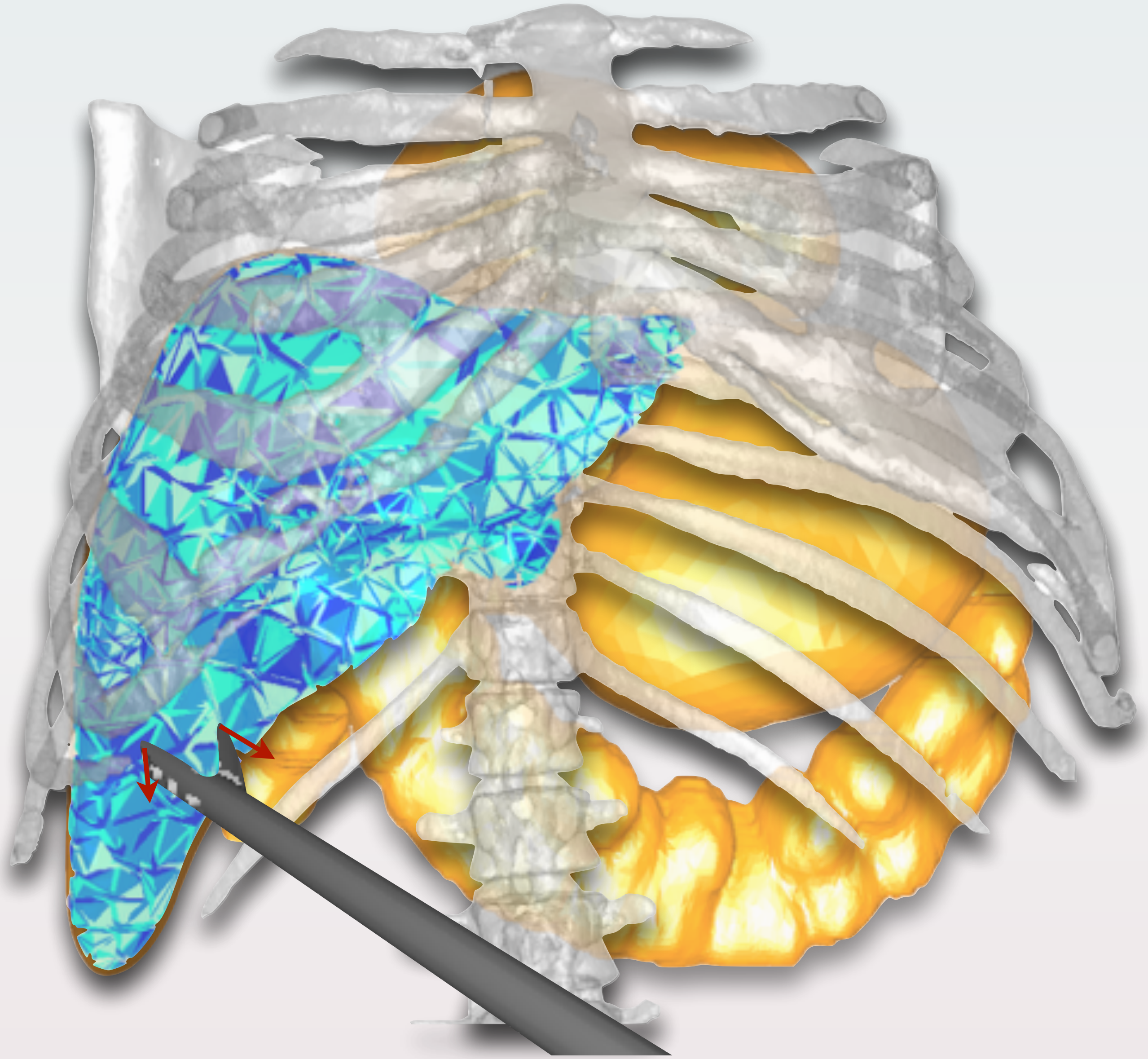
  - Conditionnally stable
  - High constraint on the time step used in the simulation
  - **h ≤ Le/c** (**h**: *time step,* **Le**: *Caracterstic lengh of smallest element,* **c**: *velocity of the deformation wave)*

TIME INTEGRATION SCHEMES

- Implicit Methods:

$$\mathbb{M}\,\underbrace{\dot{\mathbf{v}}}_{\text{unknown}} = \mathbb{P}(t) - \mathbb{F}\,\underbrace{(\mathbf{q},\mathbf{v})}_{\text{depends on } \dot{\mathbf{v}} \text{ and on previous time steps}}$$

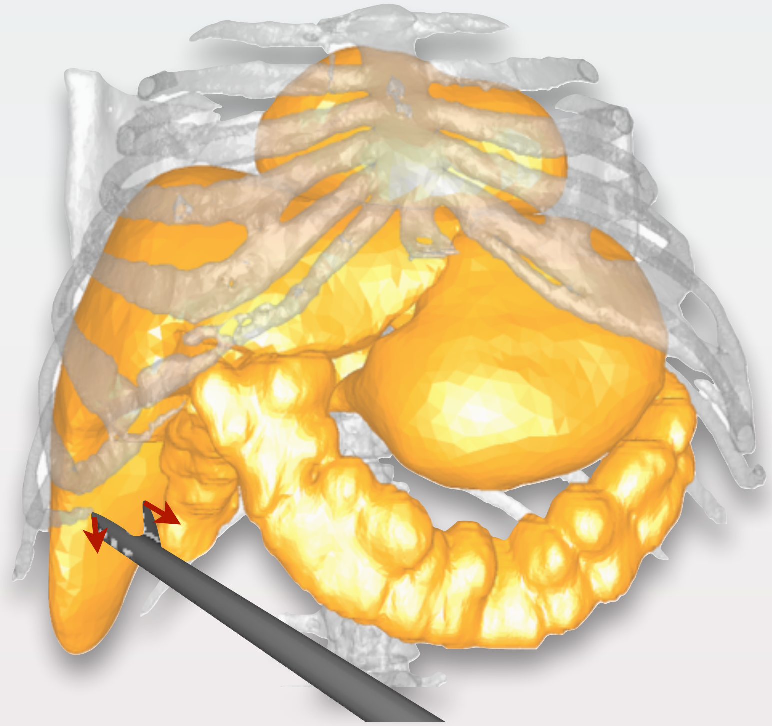
  - Unconditionnally stable
  - Possible use of «large» time step **h** in the simulation
  - Needs the resolution of a large non-linear problem

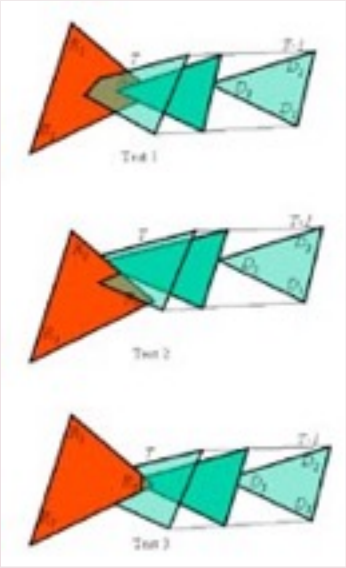


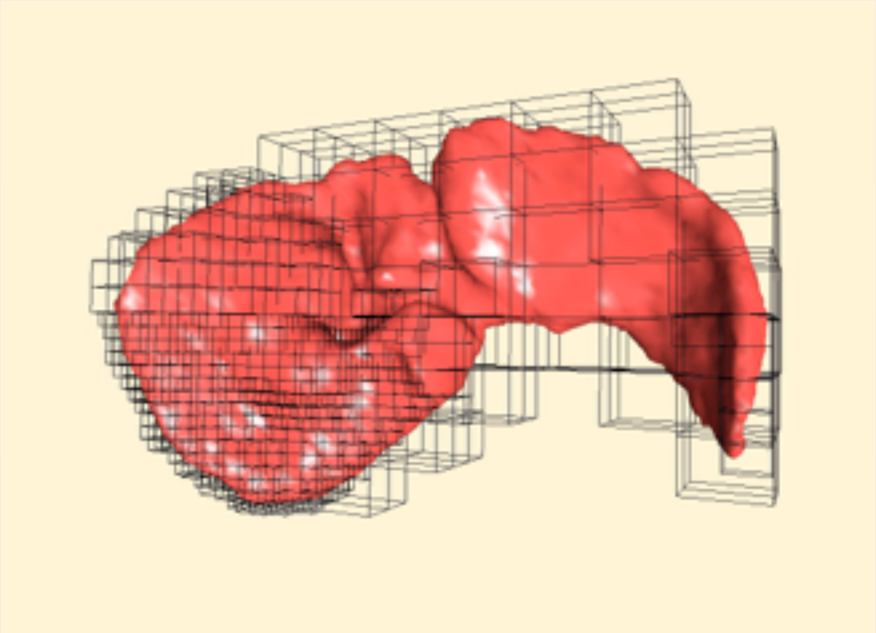
CONSTRAINT-BASED MODELING OF BIOMECHANICAL INTERACTIONS

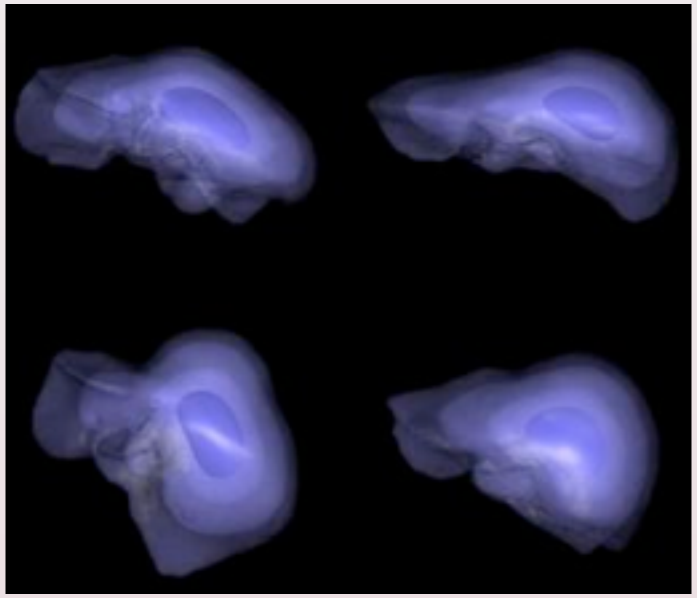
COLLISION DETECTION

- A simple problem: find the colliding primitives of two meshes...
  - ...that requires many computation !!!
  - Various optimization:
    - Bounding Volume hierarchy
    - Spatial coherency
    - Distance maps, implicit representation...





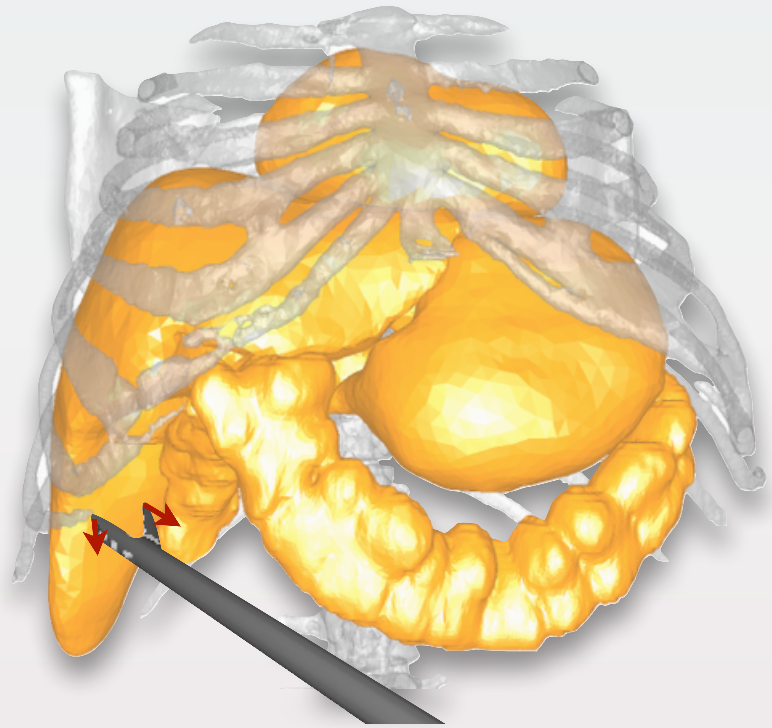




NON-SMOOTH (BIO)-MECHANICS IN REAL-TIME

- Why important ?
  - Boundary conditions between anatomical structures
  - Device-tissues interactions
- Why difficult ?
  - Non-smooth events
  - Multi-Contact response
    - Contact: Signorini's law (linear inequalities)
    - Friction: Coulomb's law (non-linear inequalities)
  - Many other interactions...
    - Complex anatomical and mechanical links between organs
    - Specific interactions for some devices

$$
\begin{aligned}
\mathbf{M}(\mathbf{v}_f - \mathbf{v}_i) &= h\left(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)\right) + \boxed{h\mathbf{H}^T\lambda_f} \\
\mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f
\end{aligned}
$$



Example : V- <0 before impact and V+ >0 after impact.. between them, an very small time step

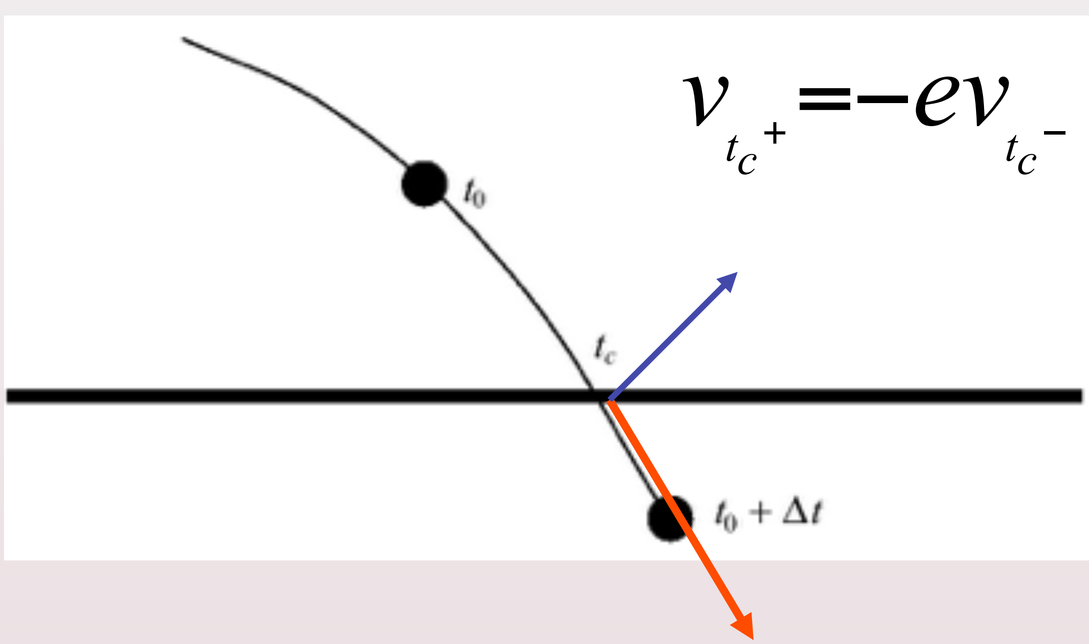
EVENT-DRIVEN

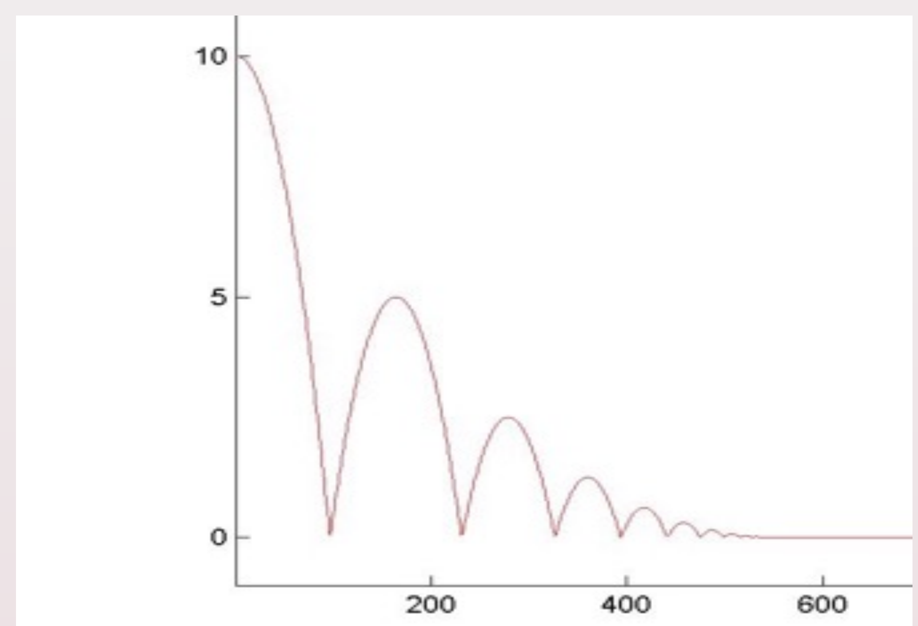
- A chaque nouvelle collision:
  - On détermine l'instant du premier impact,
  - On arrête l'intégration en temps,
  - On résout l'impact,
  - On redémarre l'intégration en temps

- Intérêt:
  - Entre deux évènements, on respecte la continuité nécessaire aux schémas d'intégration « élevés », donc précis.
- Défaut:
  - Coûteux en résolution si on a beaucoup de contacts,
  - Rebond « infini »… on est obligé de donner un critère d'arrêt

EVENT-DRIVEN

- Collision contre un mur
  - Temps de collision tc
  - Facteur de restitution e de Newton
  - Problème du rebond à l'infini





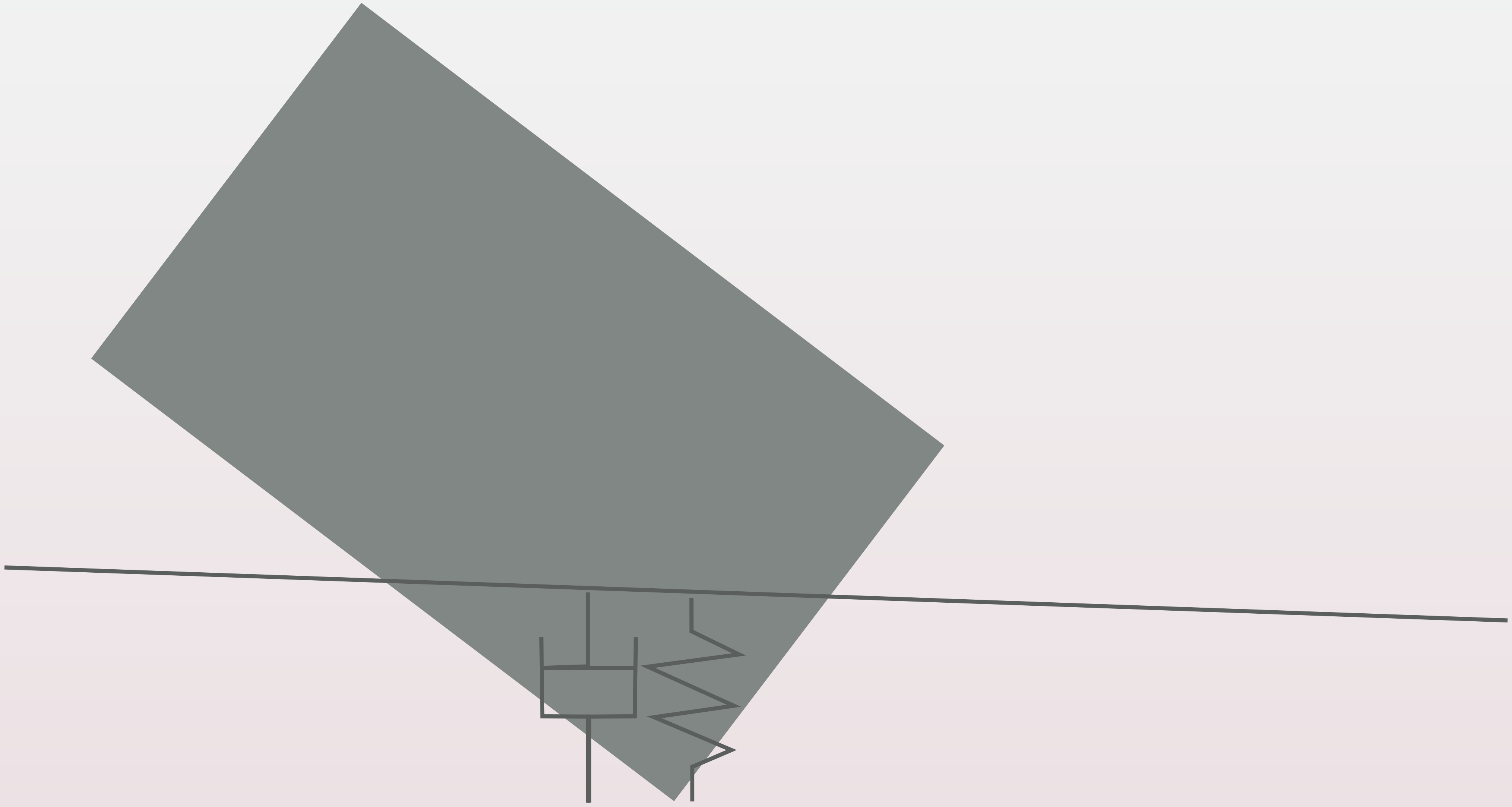
TIME-STEPPING

- Pas de temps FIXE
  - On détecte toutes les collisions apparues durant le pas de temps,
  - On résout toutes ces collisions « en même temps »,
  - Mouvement « contraint » avec toutes les forces

- Intérêt:
  - Plus rapide si il y a beaucoup de contacts,
- Défaut :
  - Utilisation de schéma d'intégration d'ordre faible
    (Forces de contact / impact deviennent des impulsions)
  - Pas de temps d'intégration petits

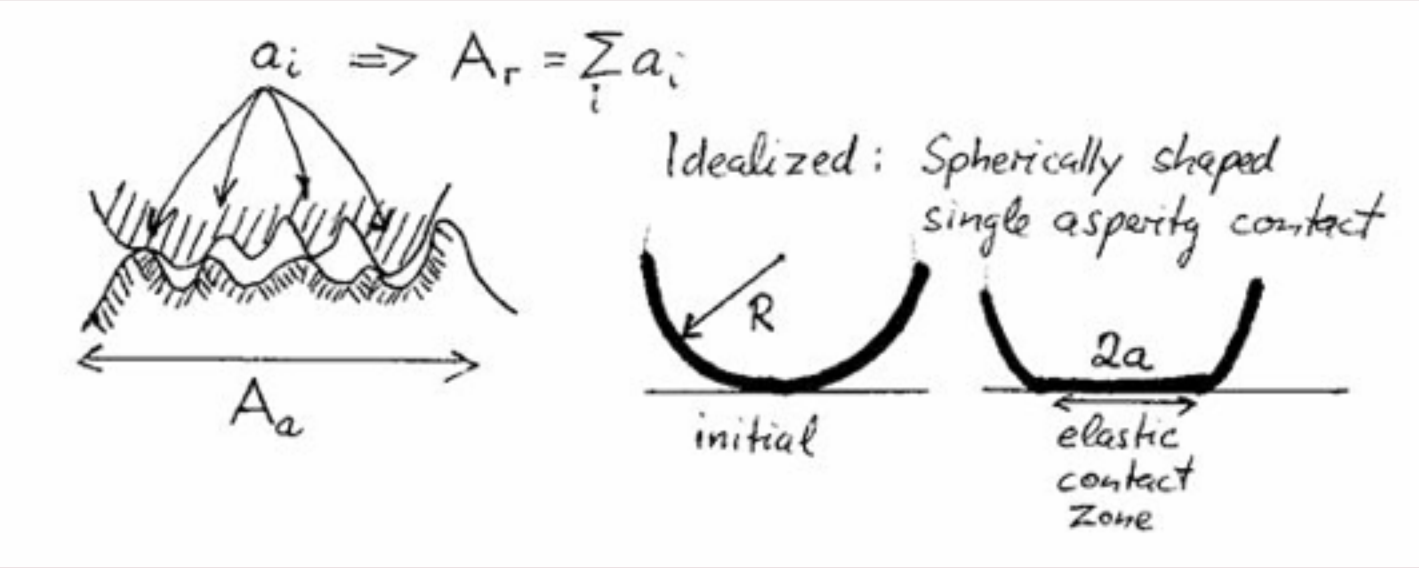
CONTACT'S LAW

- Penalty approach
  - Collision is modeled using (damped) spring



CONTACT'S LAW

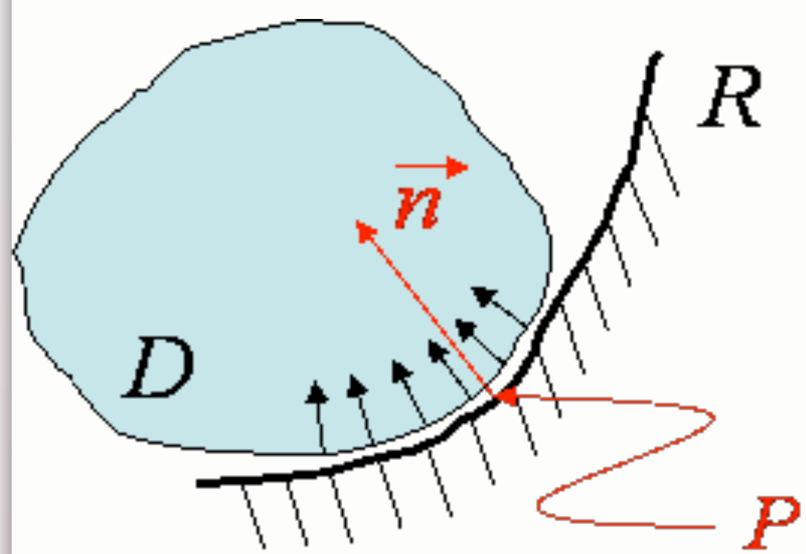
- Hertz model:
  - the strains are small and within the elastic limit,
  - each body can be considered an elastic half-space, i.e., the area of contact is much smaller than the characteristic radius of the body,
  - the surfaces are continuous and non-conforming
  - the surfaces are frictionless.
  - **analytical solutions**



$$a^3 = \frac{3PR}{4E^*}; \quad E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right); \quad a \ll R$$

CONTACT'S LAW

- Signorini's law
  - Complementarity
  - Contact betwen soft object and its environment



Non pénétration : $\delta_n(P) \geq 0$

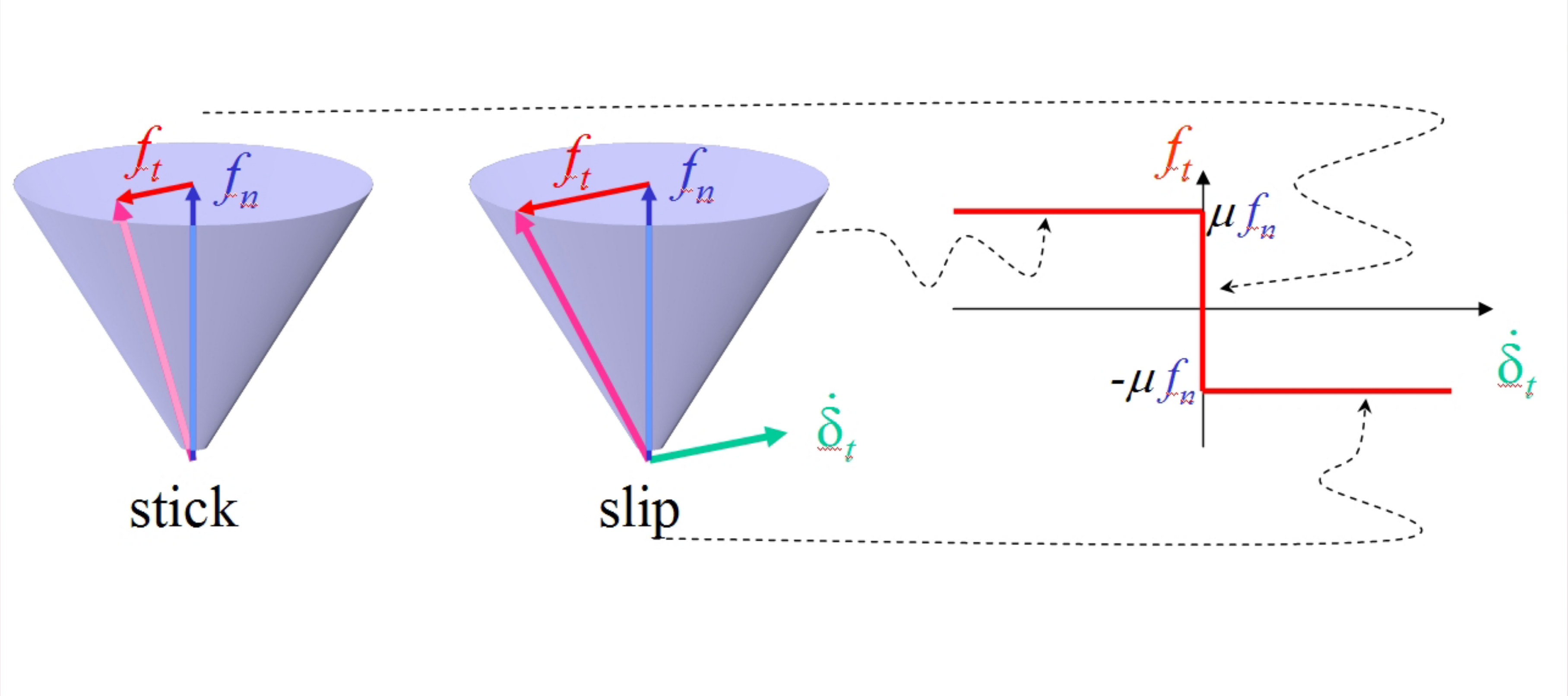
Pression à la surface : $\sigma_{nn}(P) \geq 0$

Complémentarité : $\delta_n(P) \perp \sigma_{nn}(P)$

FRICTION'S LAW

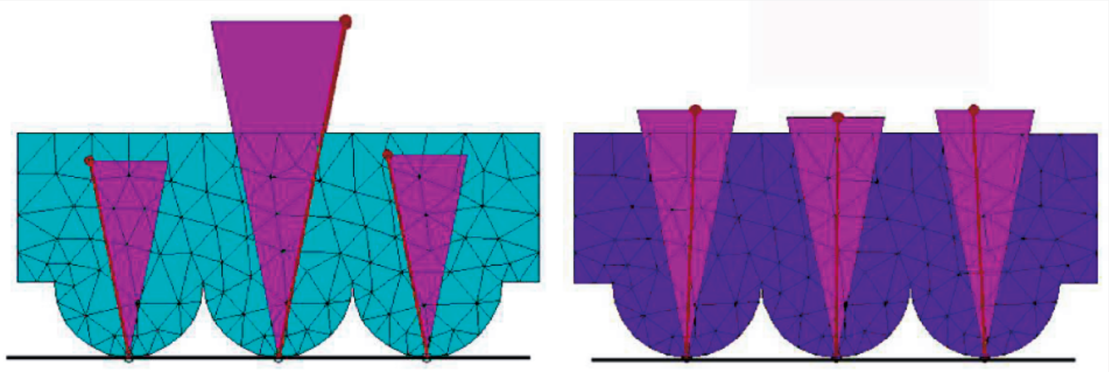
- Coulomb's law
  - Complementarity too...

$$\dot{\delta}_{\vec{T}} = \vec{0} \Rightarrow \|f_{\vec{T}}\| < \mu \|f_{\vec{n}}\| \quad \text{(stick)}$$
$$\dot{\delta}_{\vec{T}} \neq \vec{0} \Rightarrow f_{\vec{T}} = -\mu \|f_{\vec{n}}\| \frac{\dot{\delta}_{\vec{T}}}{\|\dot{\delta}_{\vec{T}}\|} = -\mu \|f_{\vec{n}}\| \vec{T} \quad \text{(slip)}$$



CHALLENGES...

- Non-unique solution (mainly with rigid objects)



- Non-linearity

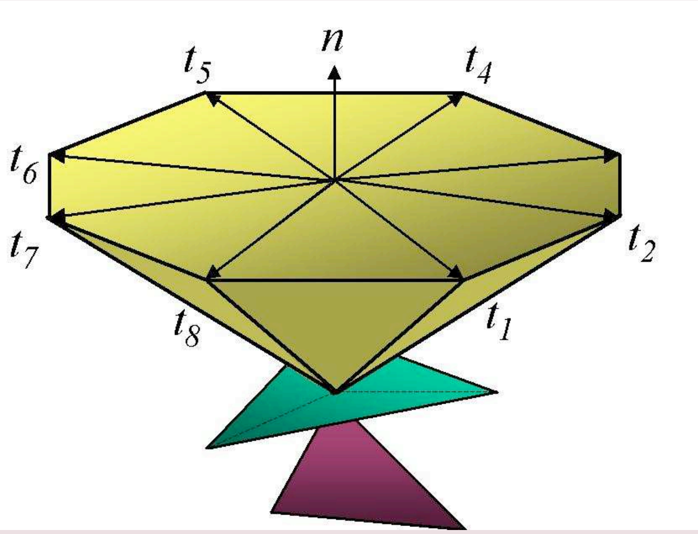
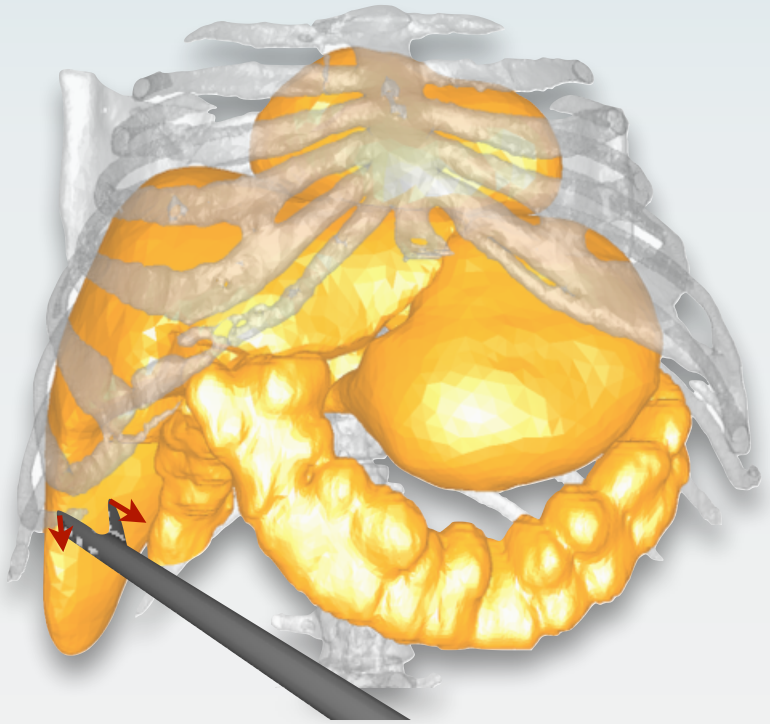


FIG. 2.27 – *Cône à $k = 8$ faces*

«CONTACT MAPPING»



- How to build matrix $\mathbf{H}^T$?
  - Direction of the contact $\mathfrak{F}_\alpha \doteq [\mathbf{n}_\alpha, \mathbf{t}_\alpha, \mathbf{s}_\alpha].$
  - Link between constraint motion and DOFs

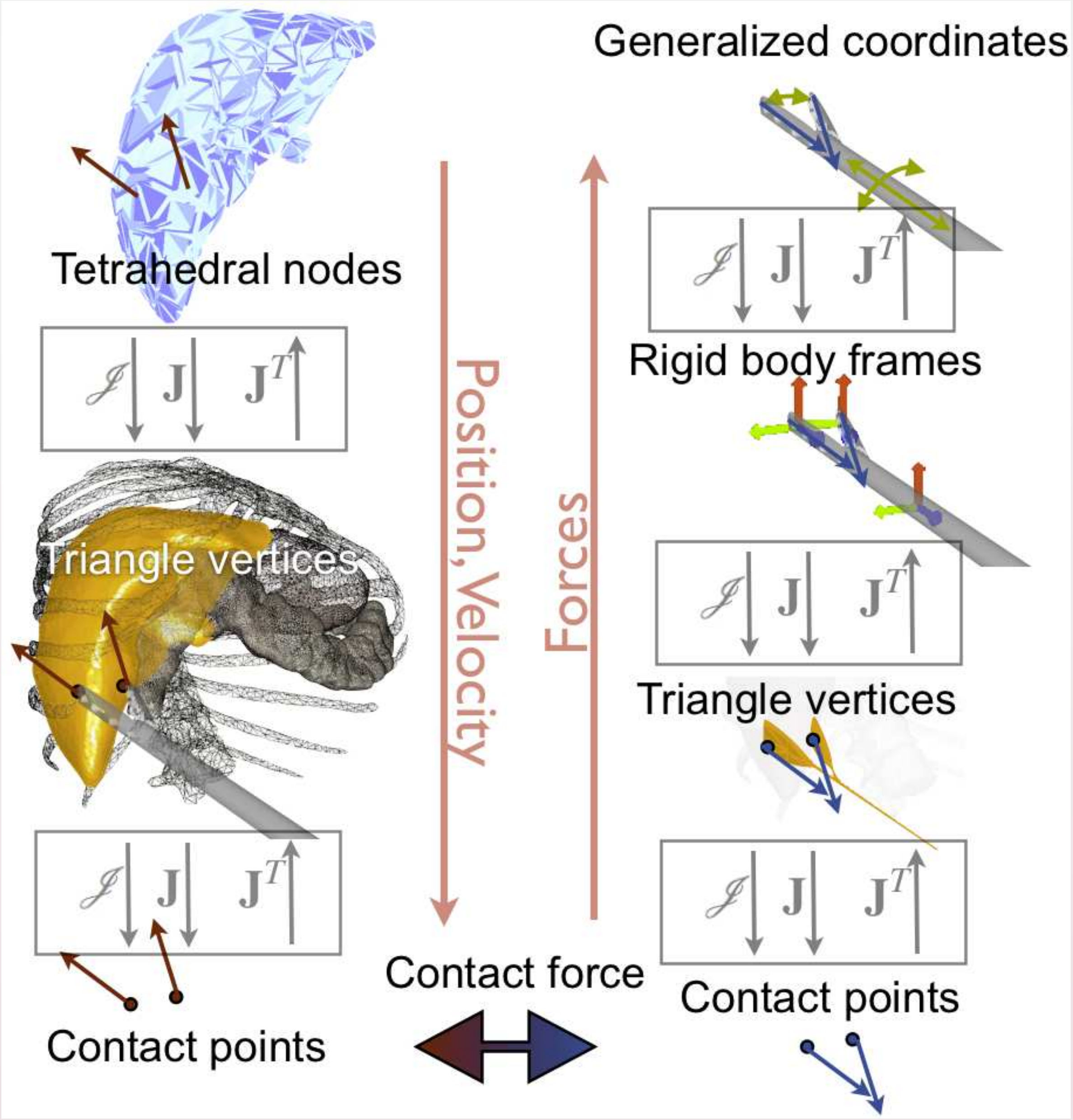
$$\boldsymbol{\delta}_\alpha = \mathbb{A}_\alpha(\mathbf{q_1}, t) - \mathbb{A}_\alpha(\mathbf{q_2}, t)$$

  - Derivation

$$\mathbb{H}_\alpha(\mathbf{q}) = \frac{\partial \mathbb{A}_\alpha}{\partial \mathbf{q}}$$

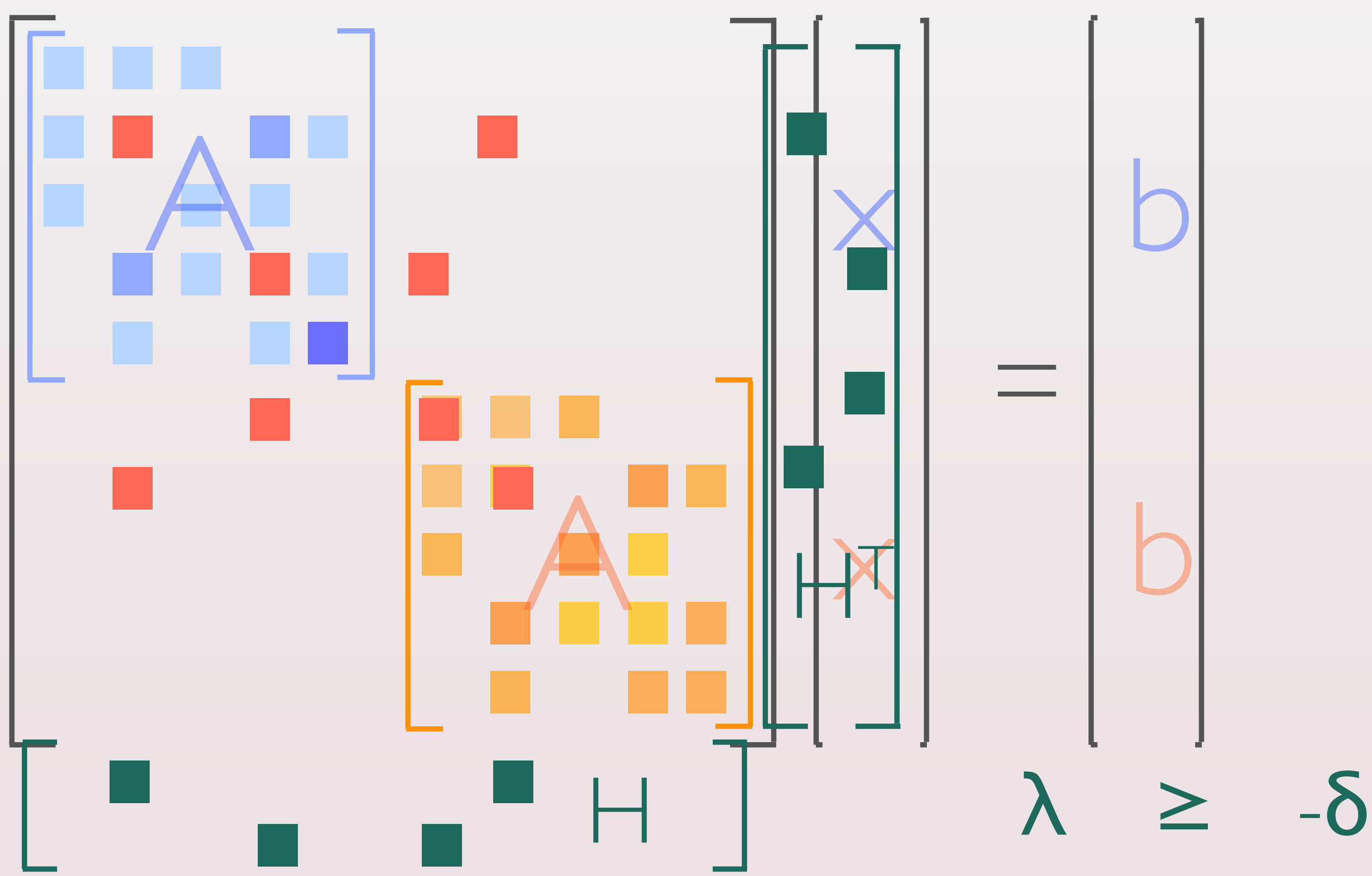
$$\dot{\boldsymbol{\delta}}_\alpha(t) = \mathbb{H}_\alpha(\mathbf{q_1})\mathbf{v}_1(t) - \mathbb{H}_\alpha(\mathbf{q_2})\mathbf{v}_2(t)$$

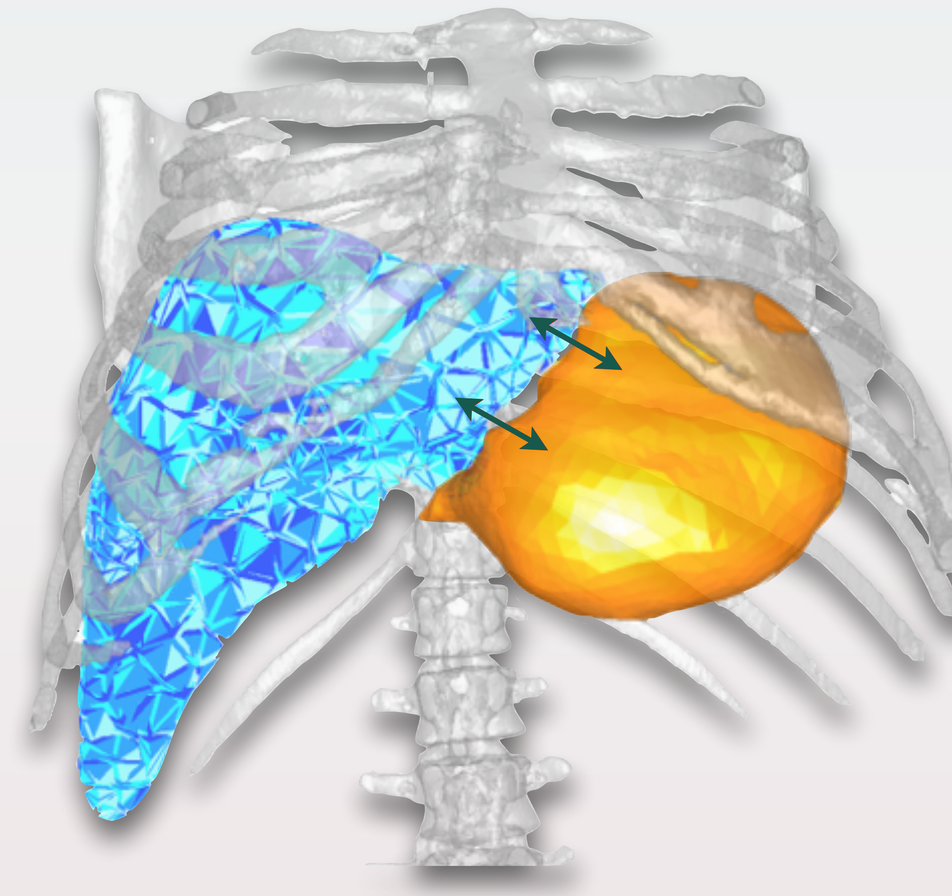
  - Virtual work principle
    - Force = $\mathbb{H}(\mathbf{q_i})^T \boldsymbol{\lambda}_f$



INTERACTION BETWEEN TWO DEFORMABLE MODELS

- 2 system of equations A x = b for each object
- «Direct approach»
  - Penalty forces: additional stiffness in the system
  - Lagrange Multipliers: very large system of (in)-equations





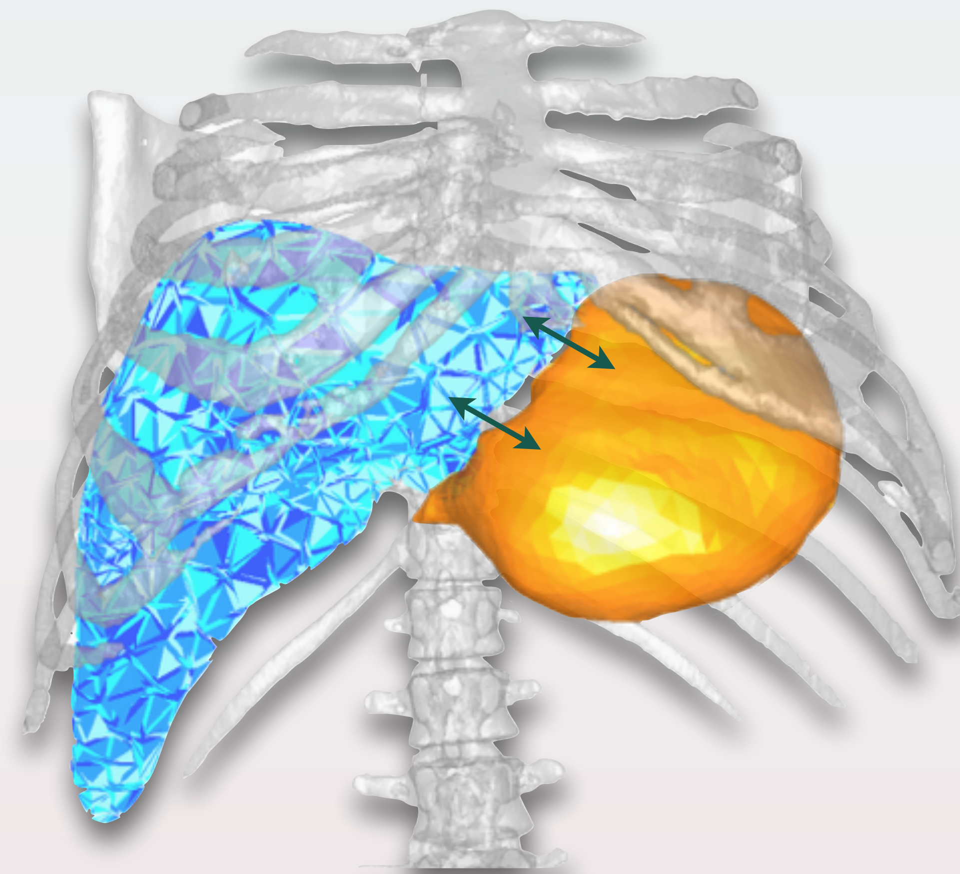
QP approach

INTERACTION BETWEEN TWO DEFORMABLE MODELS

- Indirect approach
  - Free Motion
  - Constraint setting
  - Constraint solving
  - Constraint correction: $x = x^{free} + Dx$

$$A \begin{bmatrix} x^{free} \\ Dx \end{bmatrix} = b + H^T \lambda$$

$$A \begin{bmatrix} x^{free} \\ Dx \end{bmatrix} = b + H^T \lambda$$



$$0 \leq \delta^{free} + \left[\underbrace{\qquad}_{W}{}^{-1} + \underbrace{\qquad}_{W}{}^{-1} \right] \geq 0$$

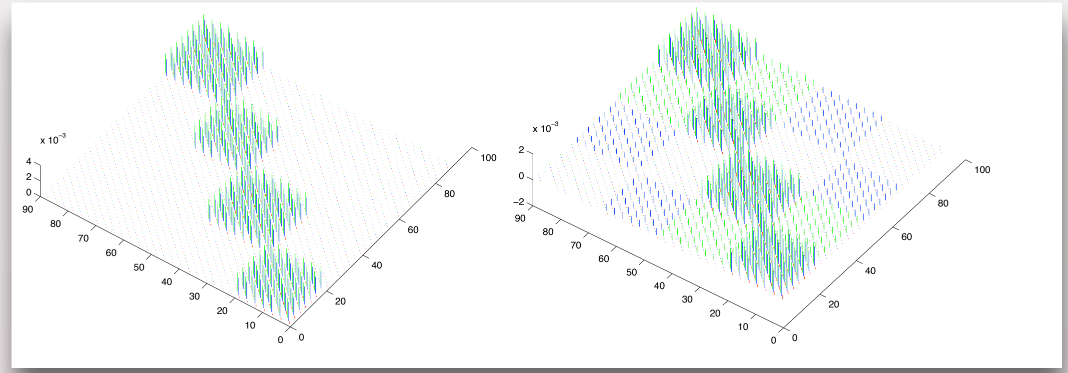
[Compliance in constraint space]

(N)LCP approach

(N)LCP APPROACH

- Build and solve (N)LCP
  - Direct solvers (Lemke) vs. iterative (Gauss-Seidel)
  - Inputs: W, δ^{free} (and constraints law), Output: λ

- What represents W ?
  - Mechanical coupling between constraints
  - Footstool example



- How to compute $W = H A^{-1} H^T$ for non-linear models ?
  - Linear model: A^{-1} can be precomputed
  - A is changing so computing A^{-1} in real-time is challenging !

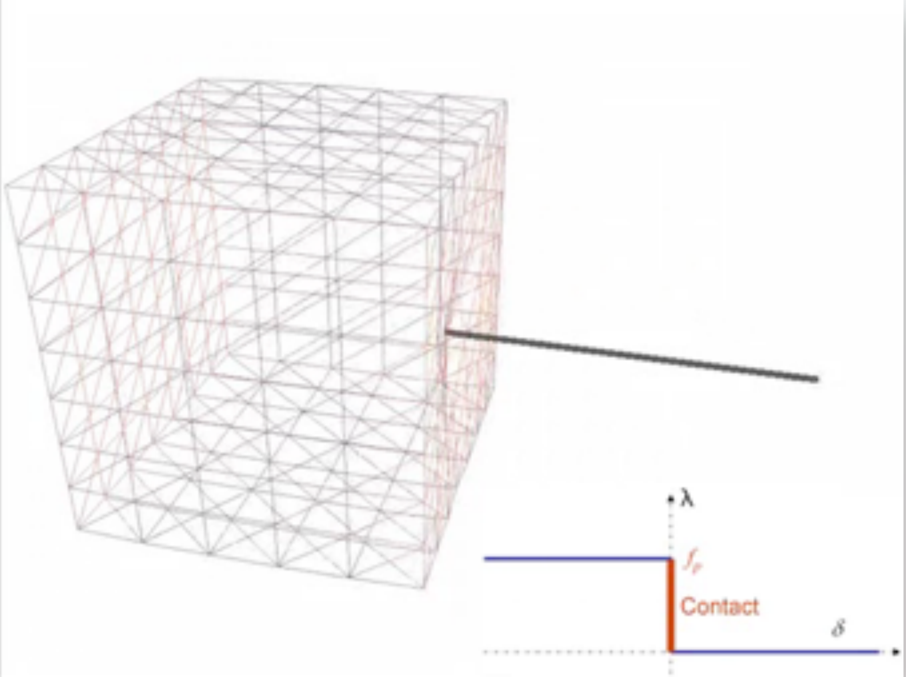
(N)LCP APPROACH

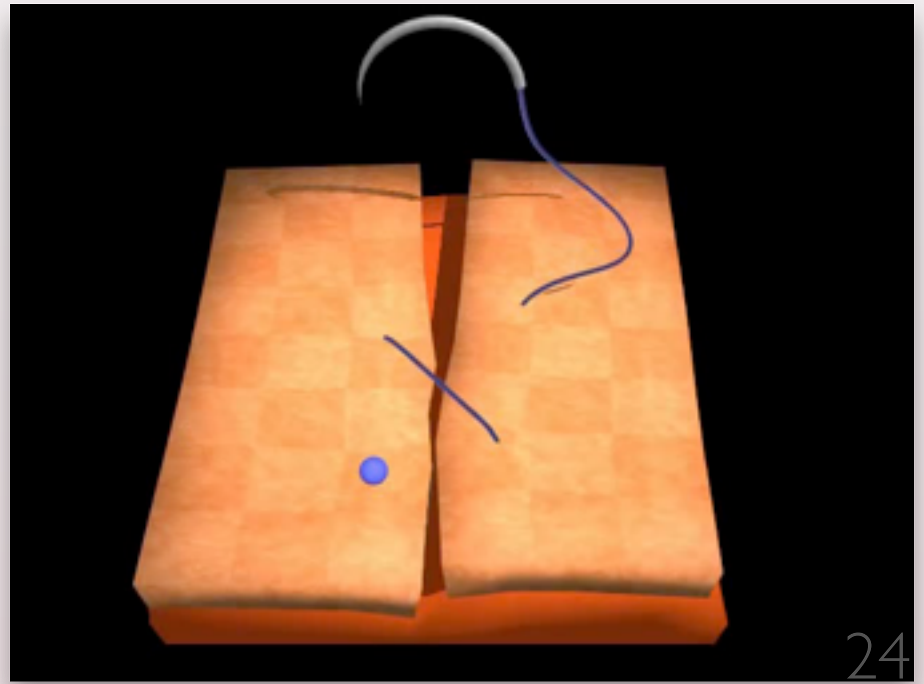
- Solve (N)LCP
  - Inputs: W, δ^{free} (and constraints law), Output: λ
  - Direct solvers
    - pivoting method: Lemke (see Siconos Library)
    - LCP <=> QP and use of QP solver algorithms
  - Iterative (block-Gauss-Seidel)
    - Block contact+friction
    - Iterative solver (slow convergence but it works well !)

$$\underbrace{\delta_\alpha - \mathbf{W}_{\alpha\alpha}\ \lambda_\alpha}_{\text{unknown}} = \underbrace{\sum_{\beta=1}^{\alpha-1} \mathbf{W}_{\alpha\beta}\ \lambda_\beta + \sum_{\beta=\alpha+1}^{m} \mathbf{W}_{\alpha\beta}\ \lambda_\beta}_{\text{frozen}} + \ \delta_\alpha^{\text{free}}$$

CONSTRAINT-BASED NEEDLE-INSERTION MODEL

- Many surgical techniques involve needle insertion (biopsy, brachytherapy, ...)
  - These needles are rigid or flexible
  - Different constraint laws for puncture, cutting, friction... etc...
  - Results validated by comparing with experiments of the literature
- Suturing simulation
  - The beam model can be used for the whole suture (including the needle)
  - Constraint based approach to simulate the suturing
  - Work in progress with DigitalTrainers
- NLCP Generic solver for all type of mechanical interactions

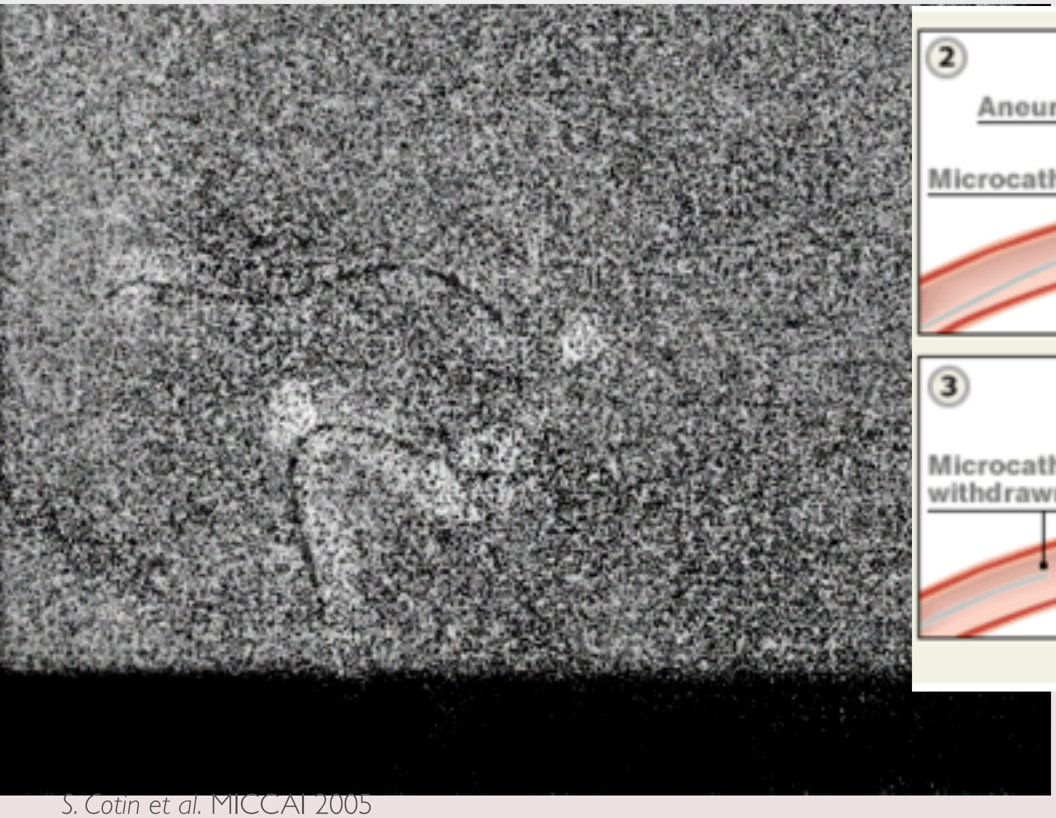


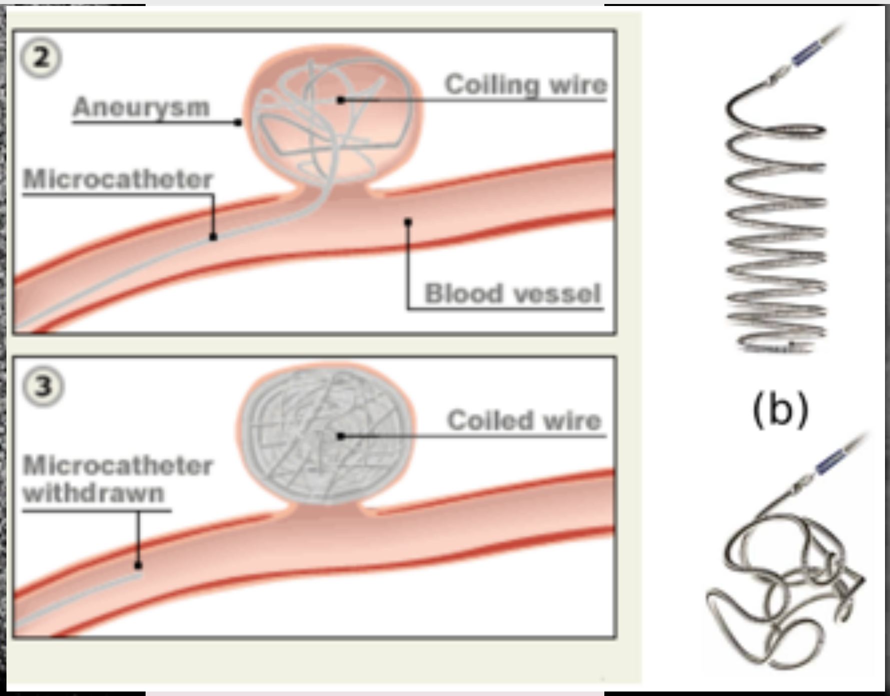


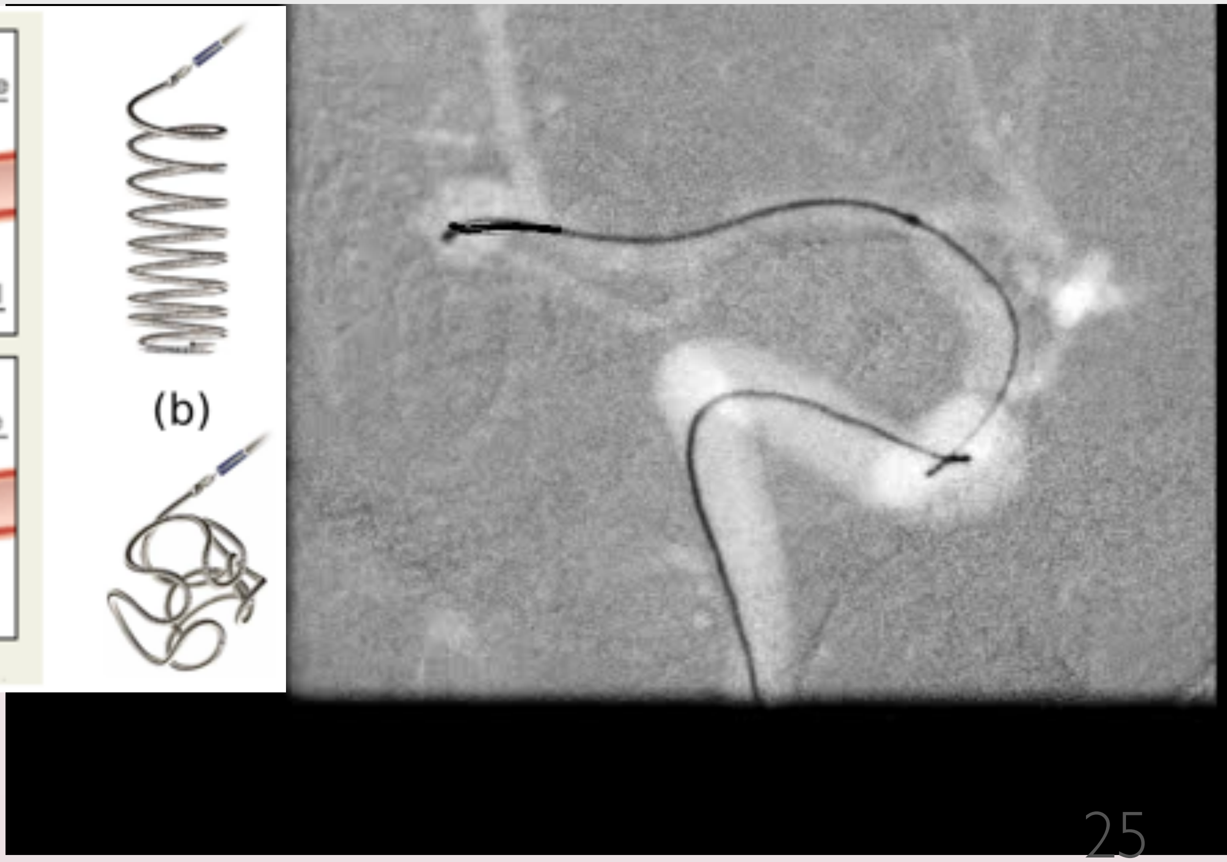
Guébert et al Vriphys 2008
Duriez et al. MICCAI 2009
Guébert et al SCA 2009

COMPLIANCE FOR WIRE-LIKE STRUCTURES

- A is a block tri-diagonal matrix...
  - Order the contact along the curvilinear abscissa
  - Gauss-Seidel NLCP solver using unbuilt matrix $H A^{-1} H^{T}$
  - Interactive simulation of more than 200 beams with hundreds of contacts
  - Frictional contact between coil and vessel wall



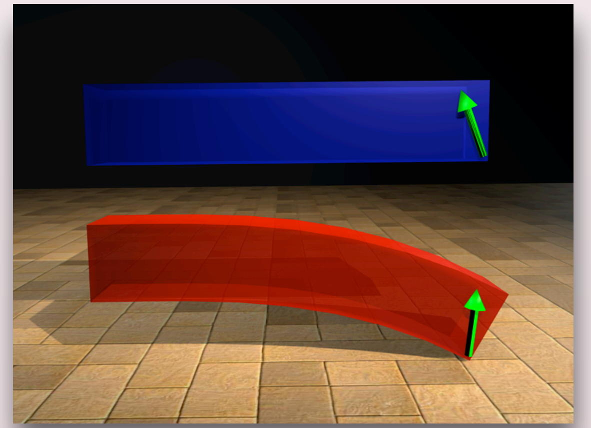


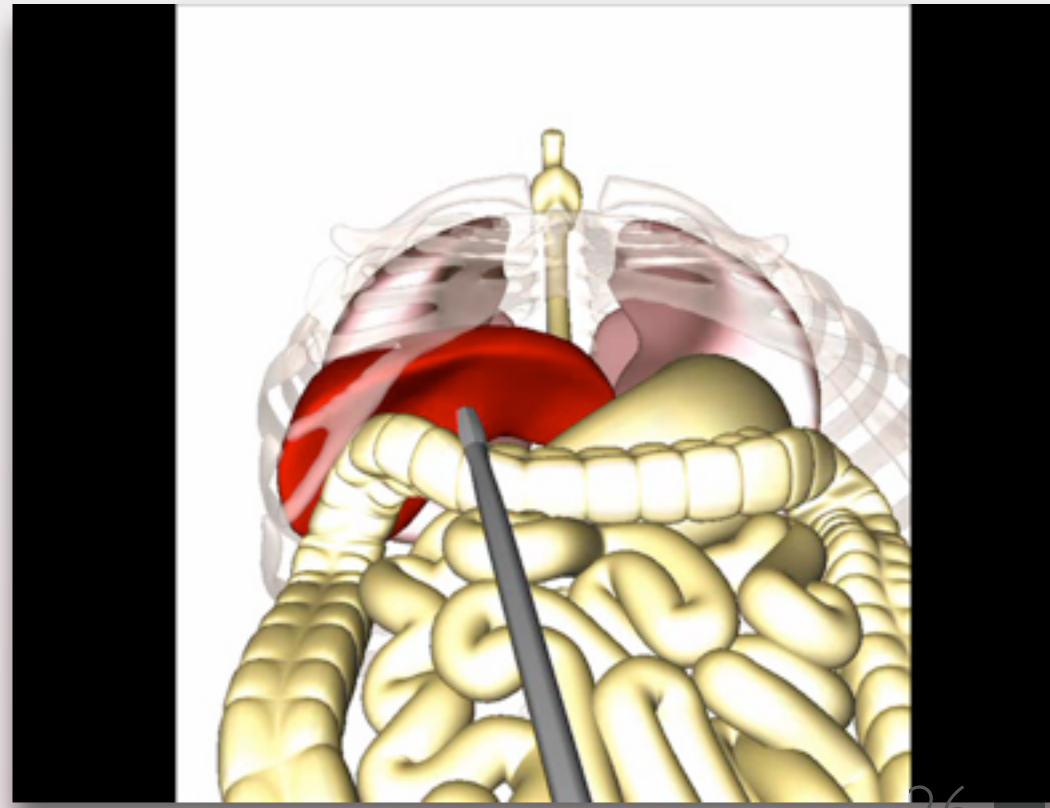


S. *Cotin et al.* MICCAI 2005
C. *Duriez et al.* Computer Aided Surgery Journal 2006
J. *Dequidt et al.* MICCAI 2008 & MICCAI 2009

COMPLIANCE WARPING

- Computation of $W=H A^{-1} H^T$ on a volume deformable object ?
  - In general, too long for real-time
  - The main role of W is to get the mechanical coupling between contacts.
- Could we «precompute» A^{-1}?
  - No... as we are using non-linear model, A is changing
  - Yes... but it is the approximation that the compliance is only «rotated» by deformation: $A^{-1} \sim R A_0^{-1} Rt$

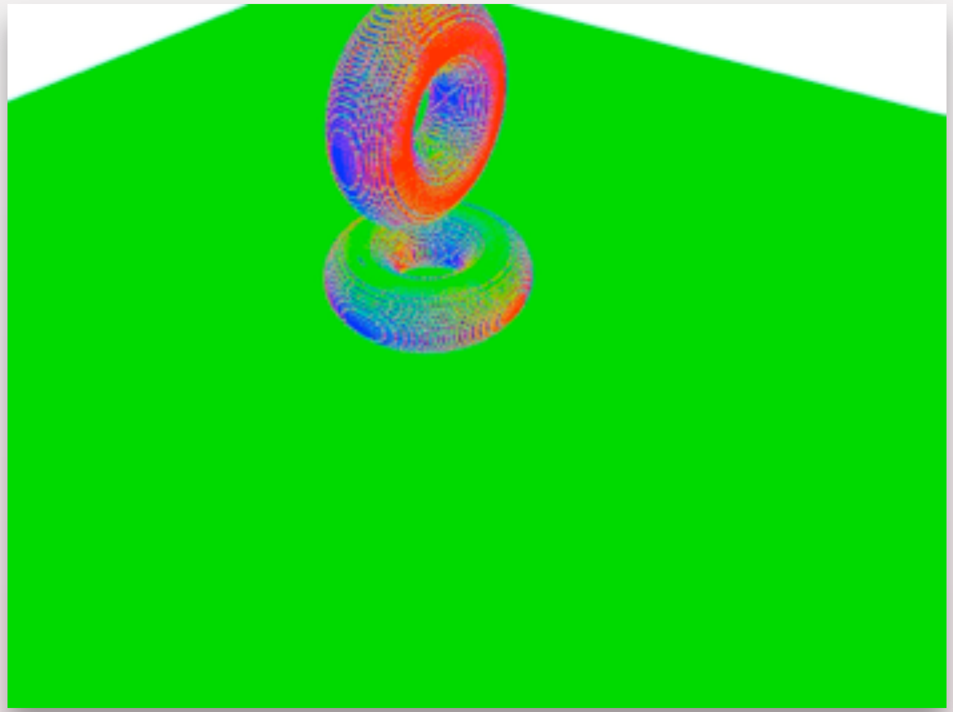




Saupin *et al.* CGI 2008

VOLUME CONTACT MODEL AT ARBITRARY RESOLUTION

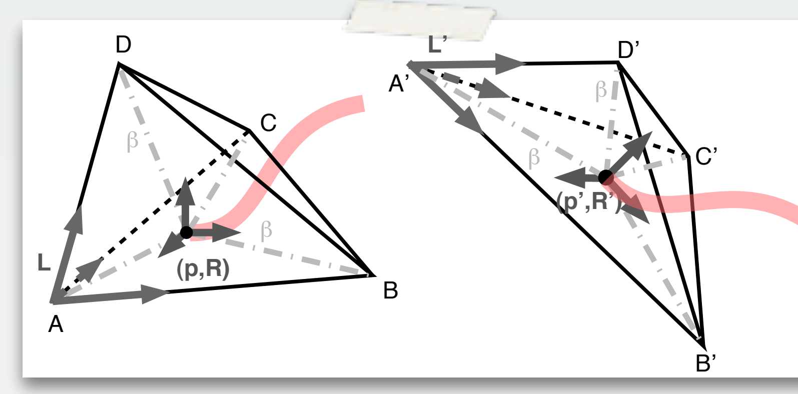
- **Goal: reduce the size of W using less contact constraints**
- Volume of Interpenetration instead of distance
- Algorithm
  - Use GPU to render the simulation scene along 3 directions
  - This generates a volumetric image
  - From this image we identify the intersection volumes
  - These volumes are used to compute the collision response (forces)
- Independent from the mesh resolution
- No additional cost to detect self-collisions

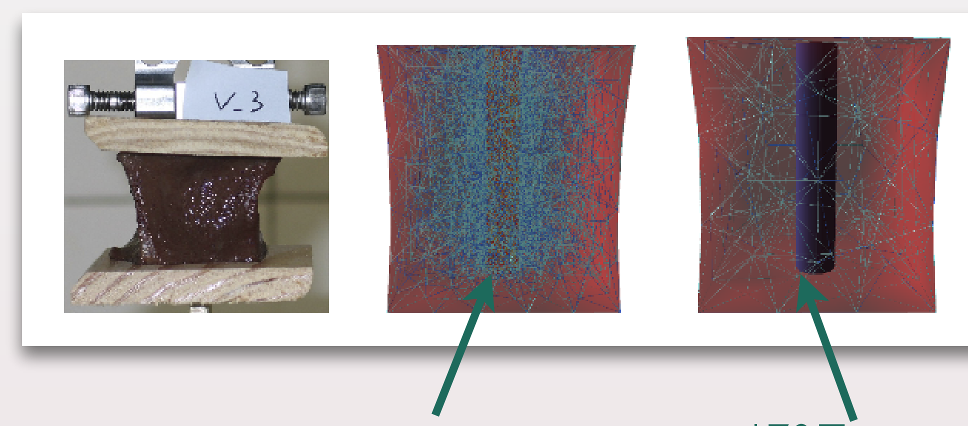


Allard et al. SIGGRAPH 2010

VASCULARIZED ORGAN MODEL

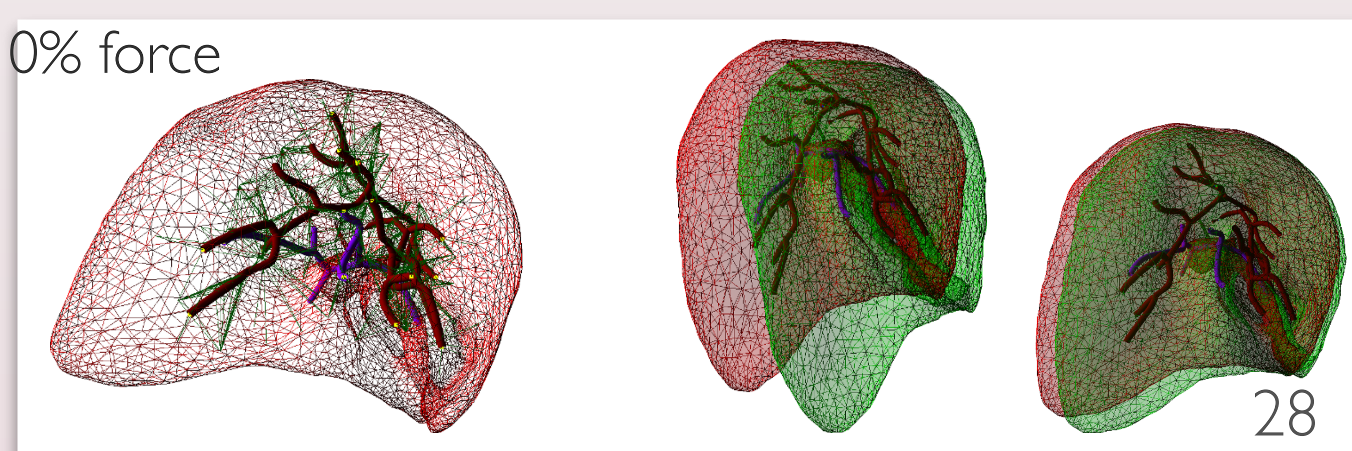
- Mechanical coupling between vessel and parenchyma
  - Corotational frame inside tetra elements
    - Coupled with the beam nodes (displacements & internal forces)
  - *In vivo* measurement of vascularized sample response
    - Significant difference of stiffness with and without the vessel
    - Try to reproduce this difference numerically
  - Numerical Simulation of Vascularized Tissue
    - Numerical validation of the coupling (err <10% force response)





52.000 Tetra

170 Tetra
2 beams



I. Peterlick *et al.* MICCAI 2012

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

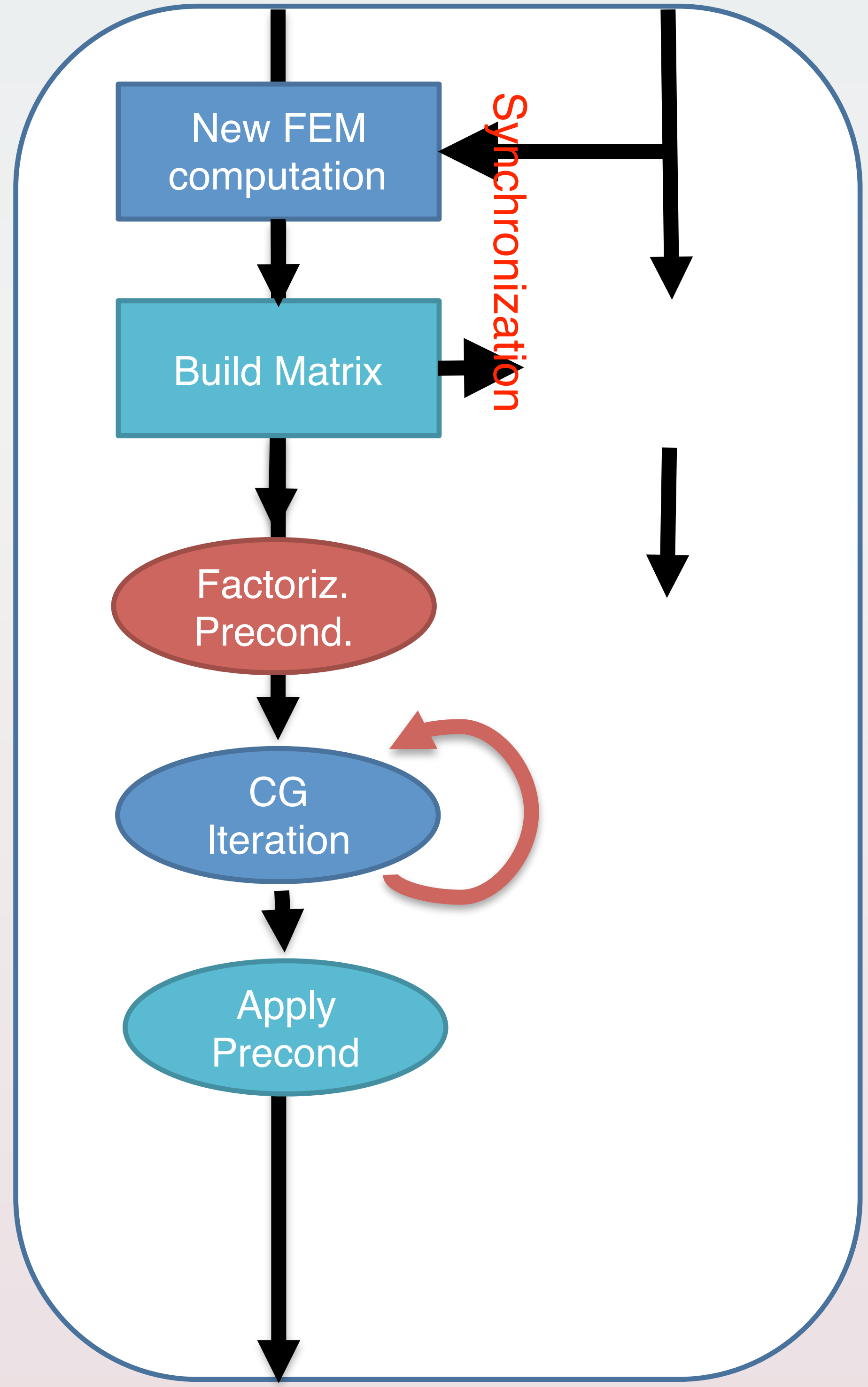
Problématique:

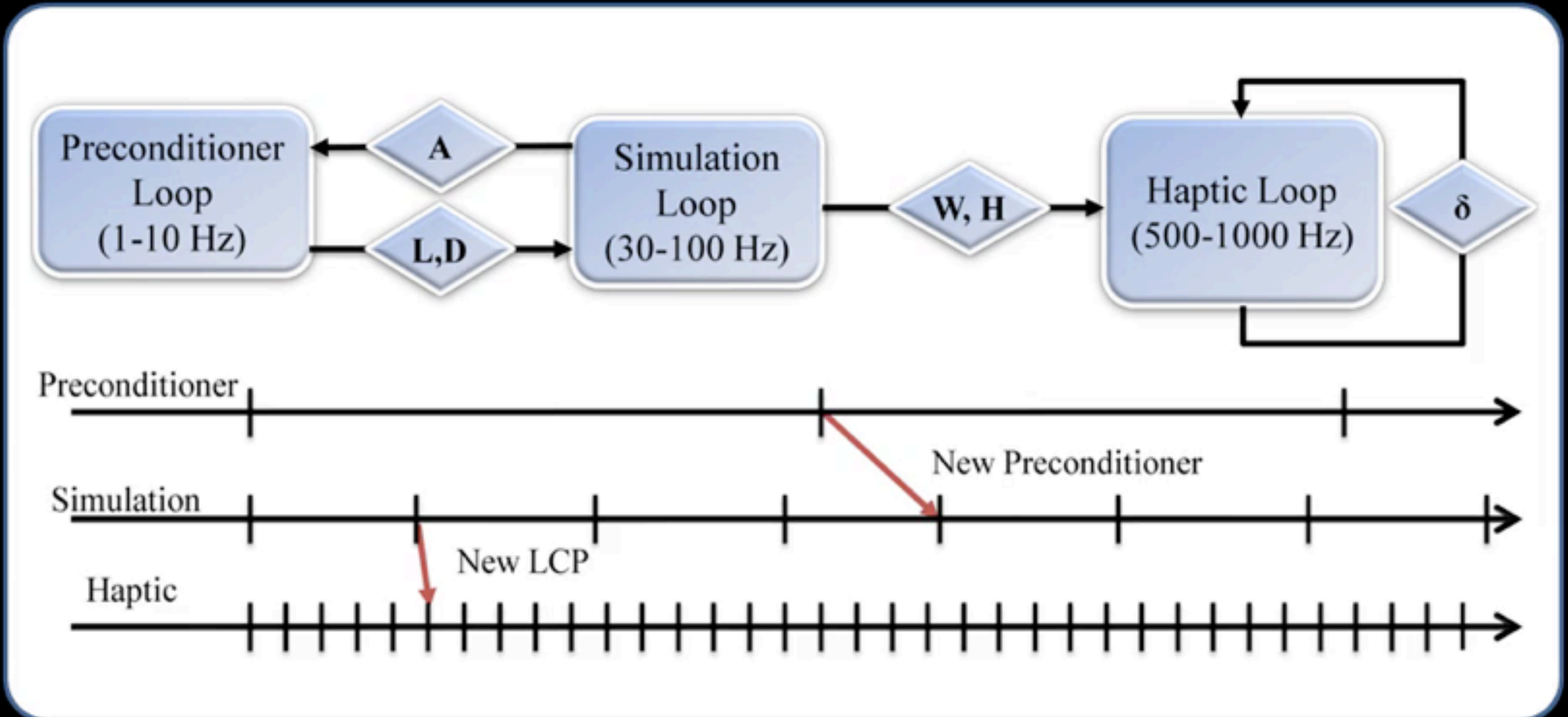
- Grandes déformations = Modèles FEM non-linéaires
  - Linéarisation à chaque pas de la simulation
  - Un système $\mathbf{A}\mathbf{x} = \mathbf{b}$ à résoudre
  - Matrice **A** de grande taille mais creuse
- Hétérogénéités = Systèmes matriciels mal conditionnés
  - Mauvaise convergence algorithmes itératifs
  - Non compatible avec temps-réel

préconditionneur: $\underbrace{\mathbf{P}^{-1}\mathbf{A}}_{\approx \mathbf{I}} x = \mathbf{P}^{-1}b$

Motivations pour le préconditionneur asynchrone:

- Cohérence temporelle de la valeur de **A**
- Factorisation d'une valeur «ancienne» de **A** pour **P**
- Le coût de la factorisation de **P** est déporté





- The MJED method is used to simulate hyperelastic material in real time (30Hz) with an implicit time integration.
- An asynchronous preconditioner is updated at low rates (10 Hz) to provide an evaluation of the compliance matrix necessary to solve contacts.
- A local contact problem is updated and solved at high rates (1KHz) to provide realistic Haptic Feedback.

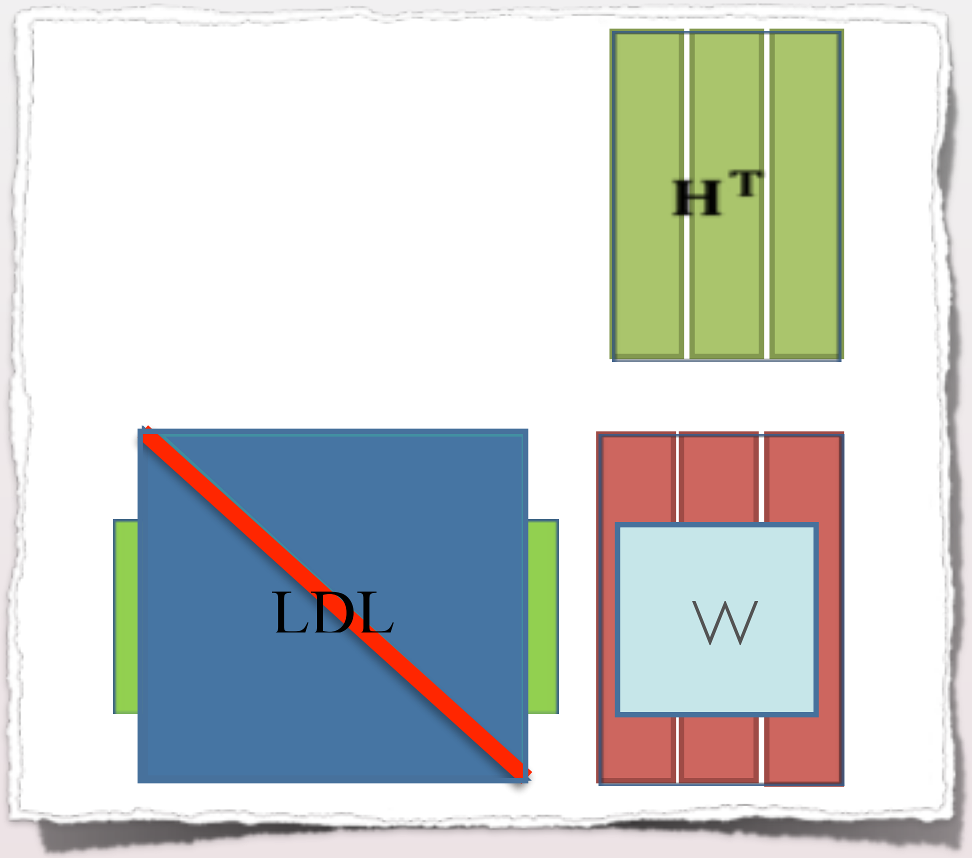
PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$W \approx H\,(LDL)^{-1}H^{T}$$

1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP
2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$
3. Matrice de compliance par multiplication $H\ S$
   On obtient: $W = H\ (LDL)^{-1}H^T$



Courtecuisse et al. MICCAI 2011

PRÉCONDITIONNEUR ASYNCHRONE

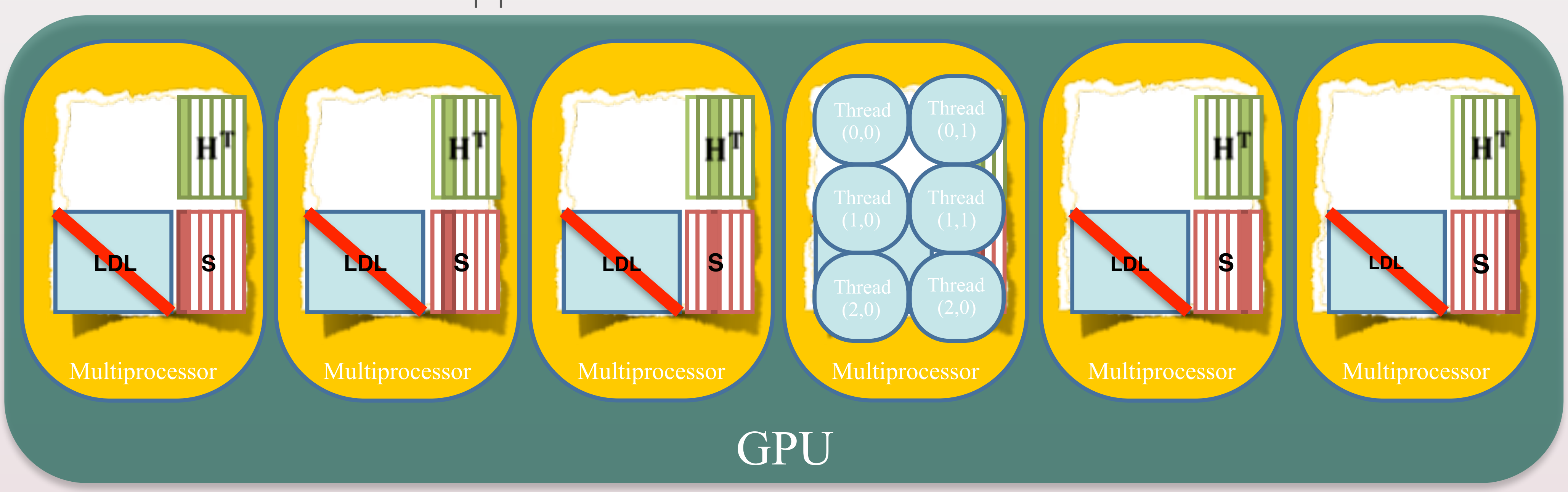
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\left(\mathbf{LDL}^T\right)^{-1}\mathbf{H}^T$$

Parallélisation:

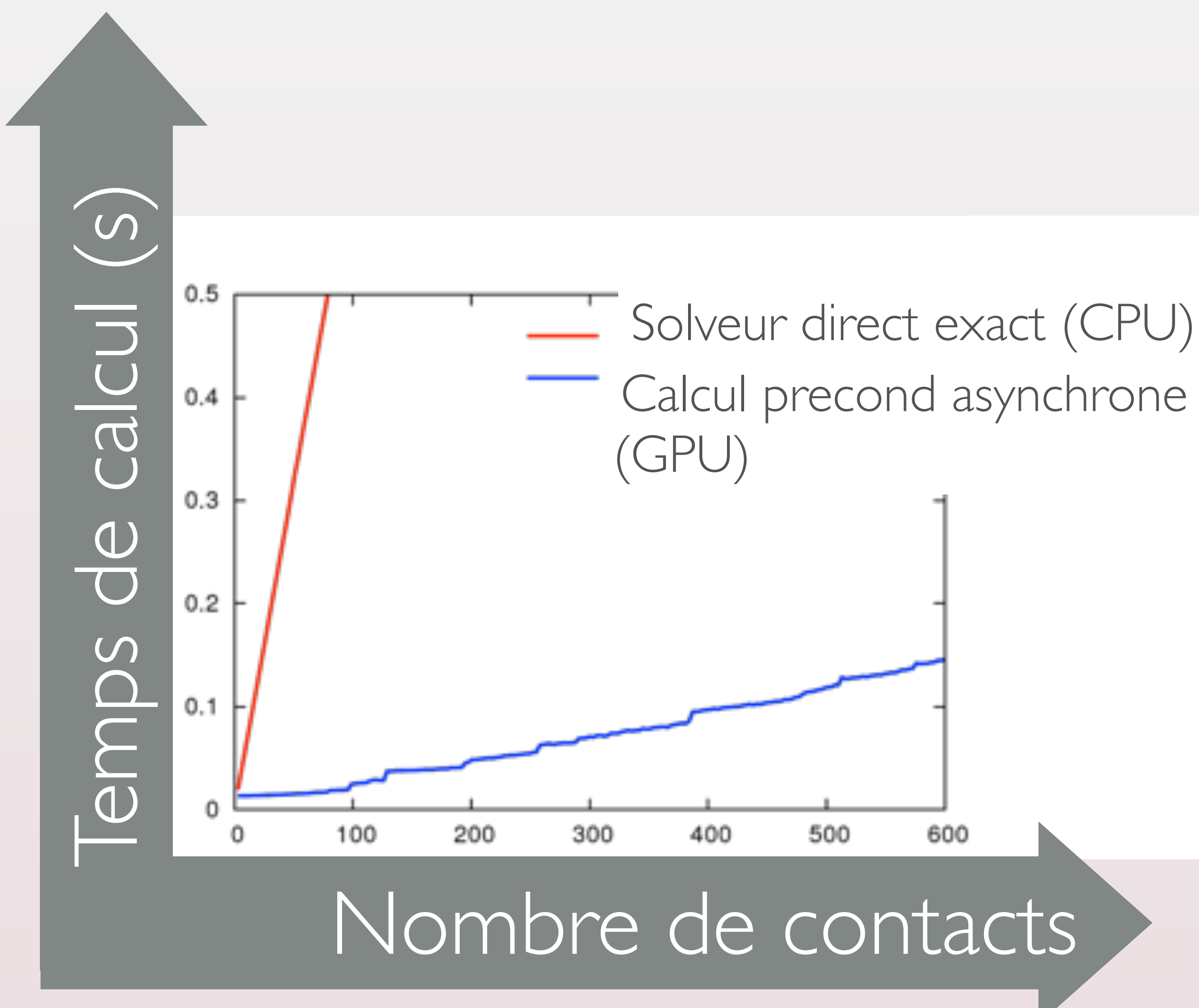
- Premier niveau: Résolution séparée pour chaque contrainte
- Deuxième niveau: Application de la factorisation LDL

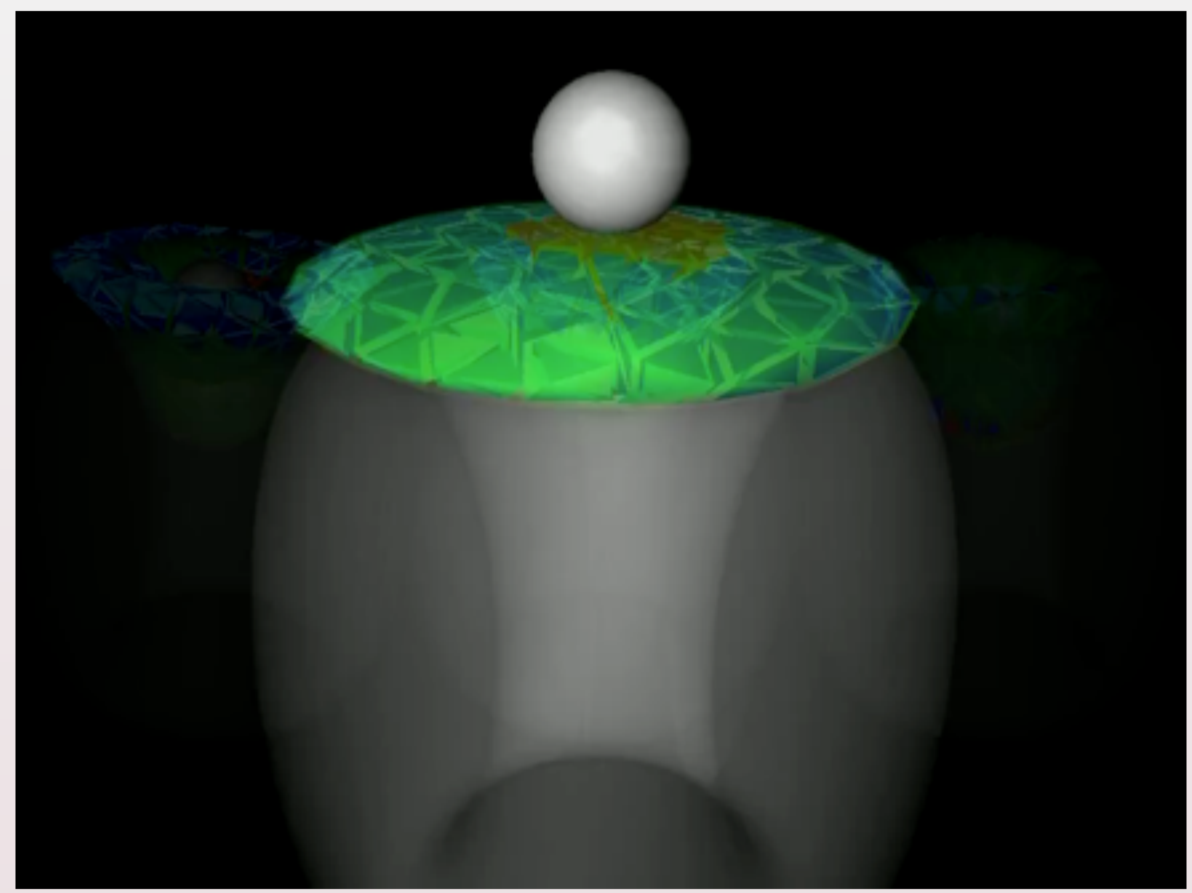


PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

- Contacts sur des objets avec propriétés mécaniques hétérogènes
- Validation par comparaison avec la solution exacte
- Temps de calcul très réduit par rapport au calcul exact sur CPU

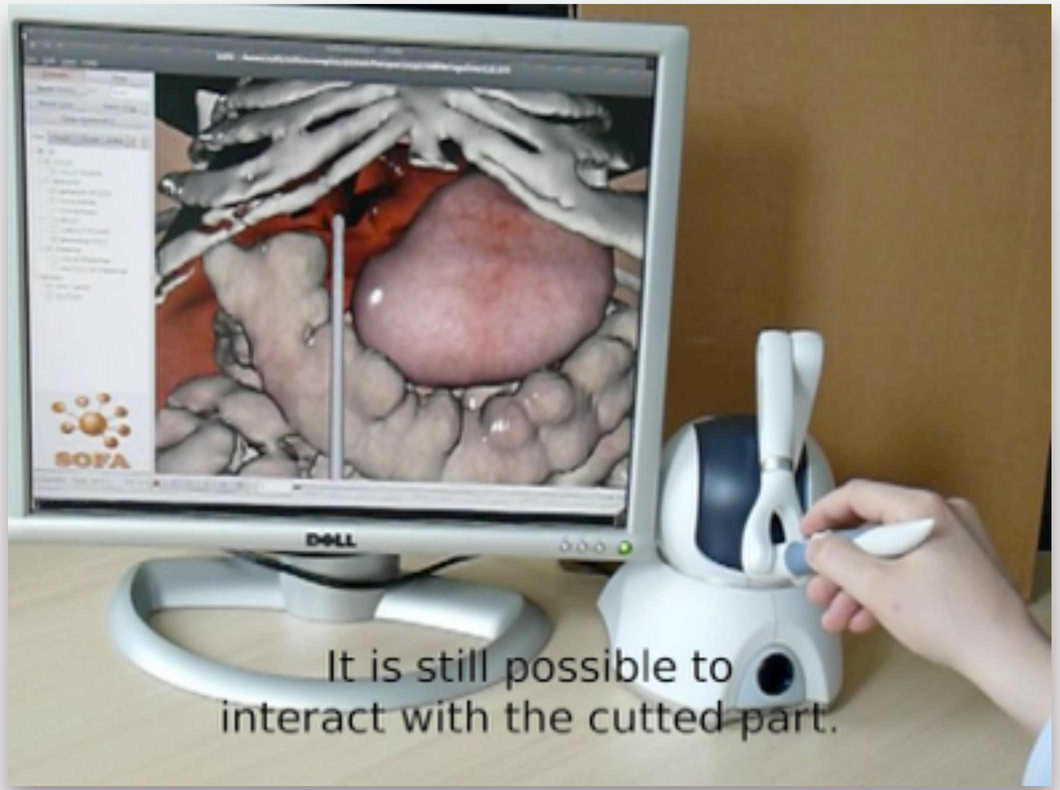


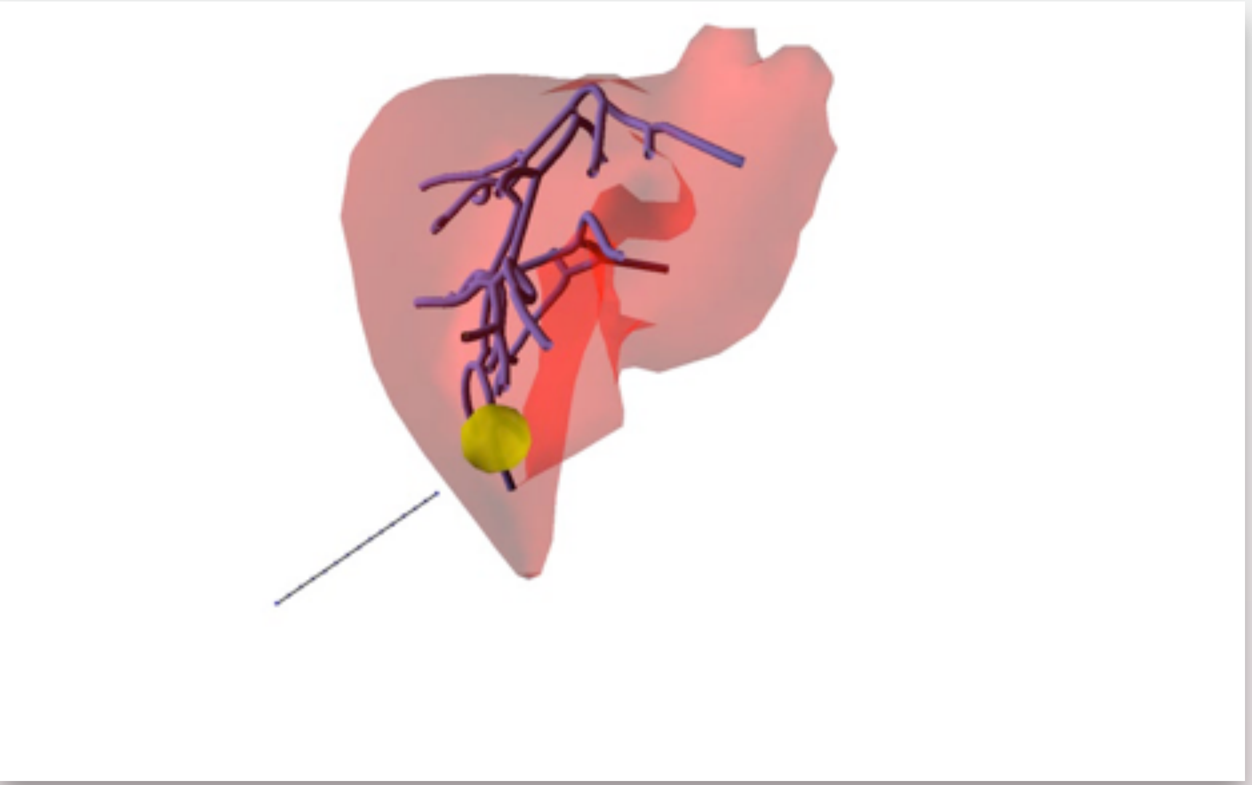


Vert: solution exacte
Bleu/rouge: notre méthode

LIVER SIMULATION

All these method can be combined...

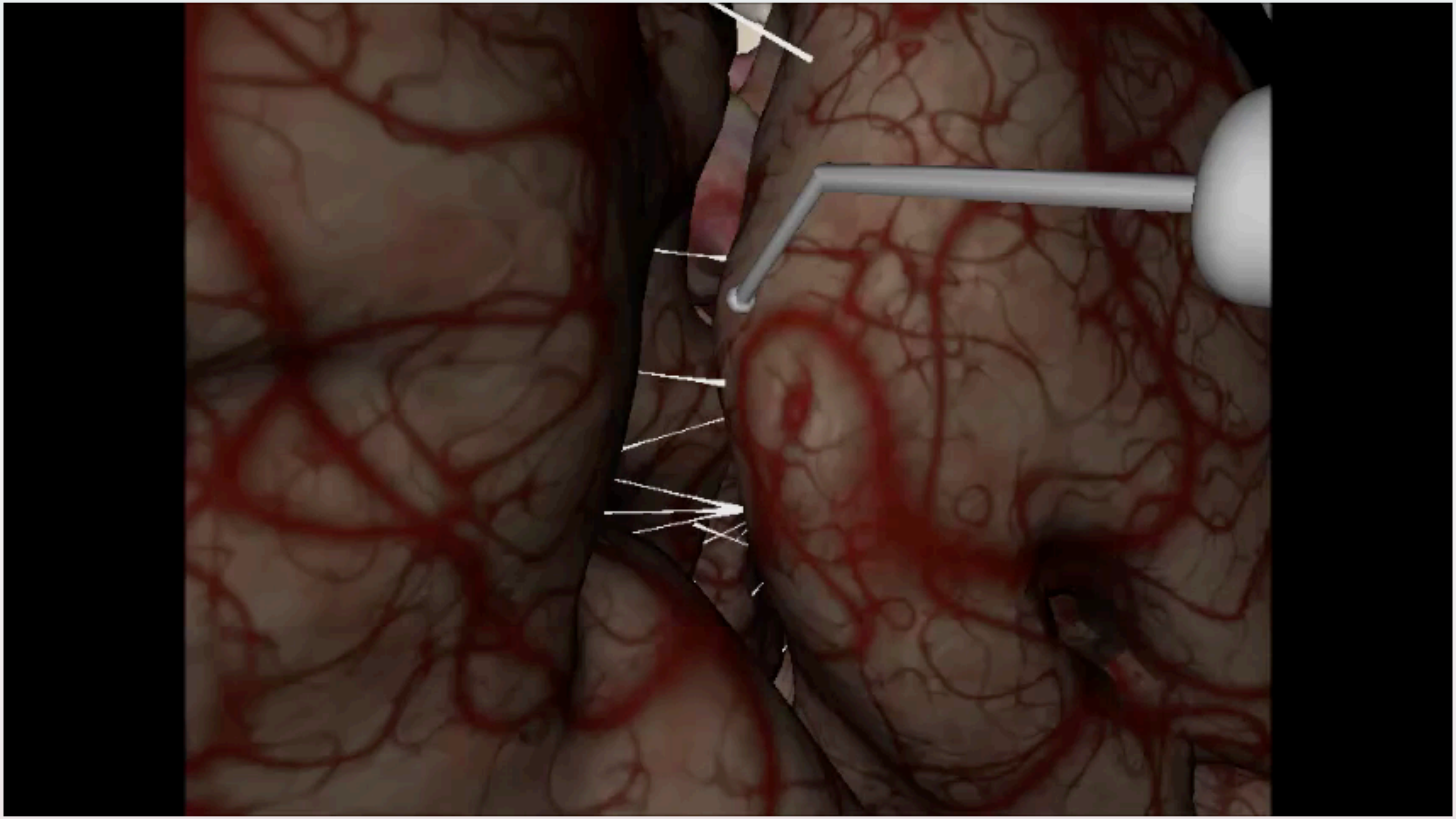


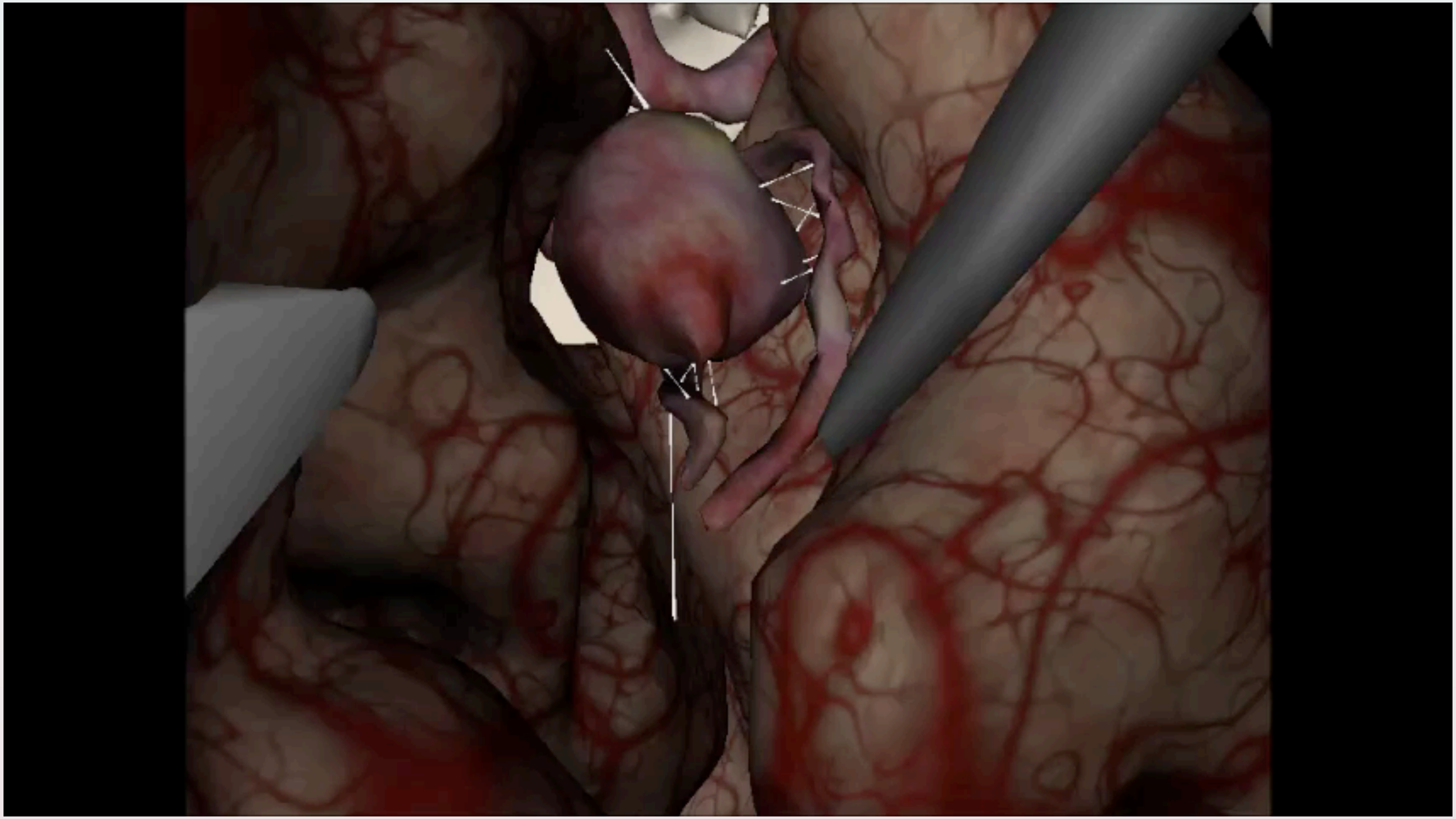


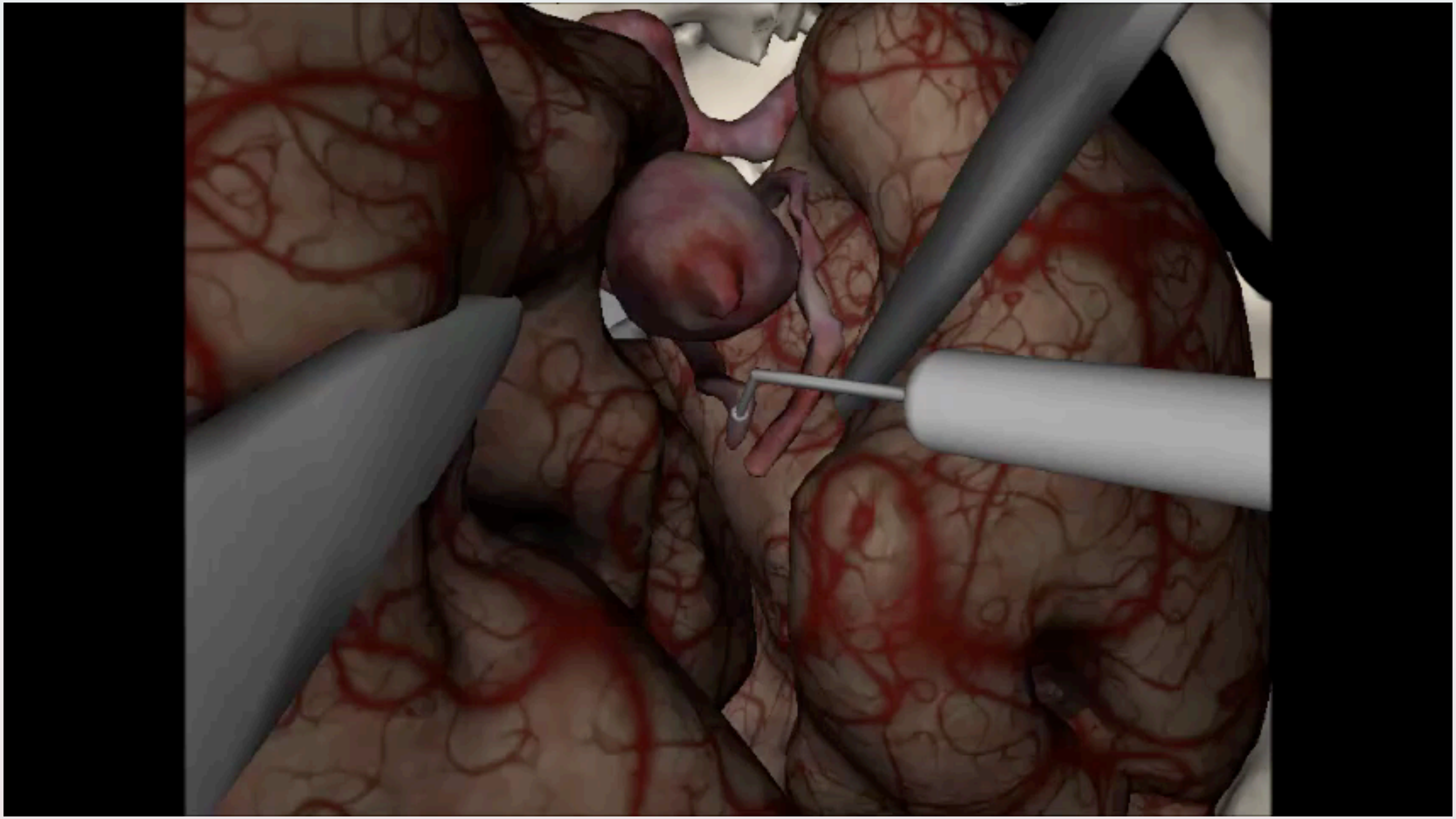
Asynchronous Preconditionner
+ Volume contact model

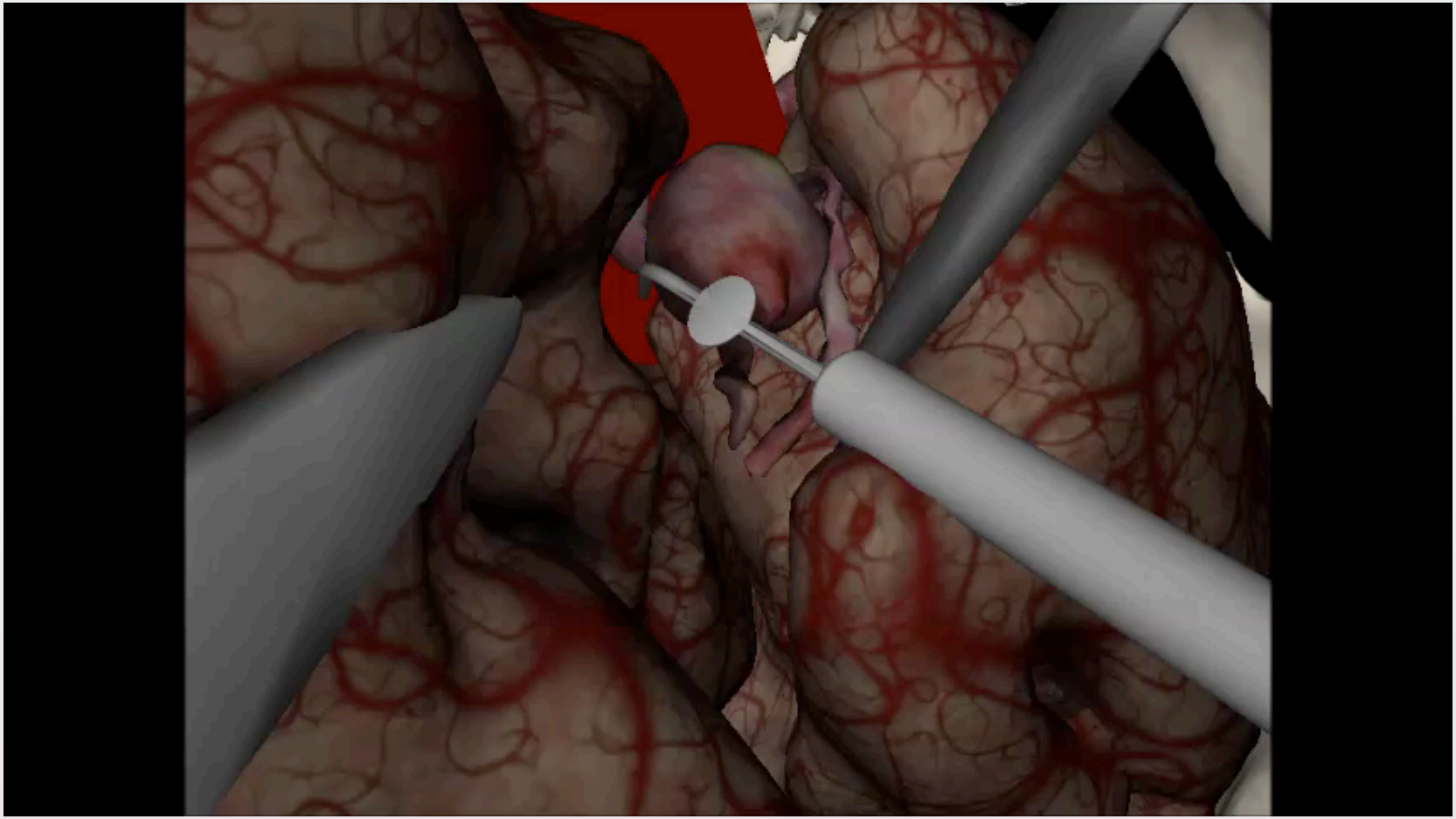
Asynchronous Preconditionner
+ Vascularized Liver
+ Constraint-based needle insertion

Courtecuisse et al. SIGGRAPH demo
Courtecuisse et al. MEDIA (submitted)









PERSPECTIVE AND CONCLUSION

A RETENIR…

- Importance de bien modéliser les interactions
  - Grande influence sur le comportement mécanique
  - Peut générer des instabilités numériques
- Différentes classes d'approches
  - Event driven / Time stepping
  - Lois de contact utilisé
  - Approche par Pénalisation / Contrainte Lagrangienne
  - Optimisation numérique pour obtenir une solution en temps-réel