2.1 $\int_a^b f(x)\,dx \approx \sum_{i=0}^{n-1} c_i f(x_i)$

$$= \frac{(x-x_{i+1})(x-x_{i+2})(x-x_{i+3})}{(x_i-x_{i+1})(x_i-x_{i+2})(x_i-x_{i+3})} f(x_i) +$$

$$\frac{(x-x_i)(x-x_{i+2})(x-x_{i+3})}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})(x_{i+1}-x_{i+3})} f(x_{i+1}) +$$

$$\frac{(x-x_i)(x-x_{i+1})(x-x_{i+3})}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})(x_{i+2}-x_{i+3})} f(x_{i+2}) +$$

$$\frac{(x-x_i)(x-x_{i+1})(x-x_{i+2})}{(x_{i+3}-x_i)(x_{i+3}-x_{i+1})(x_{i+3}-x_{i+2})} f(x_{i+3})$$

tomando: $h = \frac{x_{i+3}-x_i}{3} = x_{i+1}-x_i = x_{i+2}-x_{i+1} = x_{i+3}-x_{i+1}$

entonces: $x_{i+3}-x_i = 3h$ ① $\qquad -x_{i+1}+x_i = -h$ ③

$-x_{i+3}+x_{i+2} = -h$ ② $\qquad x_{i+2}-x_{i+1} = h$ ④

① + ② $\quad x_{i+2}-x_i = 2h$

① + ③ $\quad x_{i+3}-x_{i+1} = 2h$

Con esto:

$$\int_{x_i}^{x_{i+3}} f_3(x)\,dx = \int_{x_i}^{x_{i+3}} \left(\frac{(x-x_{i+1})(x-x_{i+2})(x-x_{i+3})}{(-h)(-2h)(-3h)} f(x_i) + \right.$$

$$\frac{(x-x_i)(x-x_{i+2})(x-x_{i+3})}{(h)(-h)(-2h)} f(x_{i+1}) +$$

$$\frac{(x-x_i)(x-x_{i+1})(x-x_{i+3})}{(2h)(h)(-h)} f(x_{i+2}) +$$

$$\left. \frac{(x-x_i)(x-x_{i+1})(x-x_{i+2})}{(3h)(2h)(h)} f(x_{i+3}) \right)$$

agrupando:

$$\int_{x_i}^{x_{i+3}} f_3(x)\,dx = -\frac{f(x_i)}{6h^3} \int_{x_i}^{x_{i+3}} (x-x_{i+1})(x-x_{i+2})(x-x_{i+3}) +$$

$$\frac{f(x_{i+1})}{2h^3} \int_{x_i}^{x_{i+3}} (x-x_i)(x-x_{i+2})(x-x_{i+3}) -$$

$$\frac{f(x_{i+2})}{2h^3} \int_{x_i}^{x_{i+3}} (x-x_i)(x-x_{i+1})(x-x_{i+3}) +$$

$$\frac{f(x_{i+3})}{6h^3} \int_{x_i}^{x_{i+3}} (x-x_i)(x-x_{i+1})(x-x_{i+2})$$

La solución de las integrales está dada por:

$$I = \int (x-a)(x-b)(x-c)\,dx$$

$$\int (x^2 - ax - bx + ab)(x-c)\,dx$$

	u		v
	$x^2 - ax - bx + ab$	$+$	$x - c$
	$2x - a - b$	$-$	$\frac{(x-c)^2}{2}$
	2	$+$	$\frac{(x-c)^3}{6}$
	0		$\frac{(x-c)^4}{24}$

$$\frac{(x-a)(x-b)(x-c)^2}{2} + \left(-x + \frac{a}{2} + \frac{b}{2}\right)\frac{(x-c)^3}{3} + \frac{(x-c)^4}{12}$$

$$\frac{(x-a)(x-b)(x-c)^2}{2} + \left(-x + \frac{a}{2} + \frac{b}{2}\right)\frac{(x-c)^3}{3} \underbrace{+ \frac{(x-c)^4}{12} + \frac{3(x-c)^4}{12}}$$

$$\frac{(x-c)(x-c)^3}{3} = \frac{4(x-c)^4}{12} \qquad -\frac{3(x-c)^4}{12}$$

$$\frac{(x-a)(x-b)(x-c)^2}{2} + \left(-x + \frac{a}{2} + \frac{b}{2} + x - c\right)\frac{(x-c)^3}{3} - \frac{(x-c)^4}{4}$$

$$\frac{(x-a)(x-b)(x-c)^2}{2} + \left(-c + \frac{a}{2} + \frac{b}{2}\right)\frac{(x-c)^3}{3} - \frac{(x-c)^4}{4}$$

Con esto, reemplazando en las integrales

$I_1 =$

$$-\frac{f(x_i)}{6h^2}\left[\left(\frac{(x-x_{i+1})(x-x_{i+2})(x-x_{i+3})^2}{2}-\frac{(x-x_{i+3})^4}{4}+\left(-x_{i+3}+\frac{x_{i+1}}{2}+\frac{x_{i+2}}{2}\right)\left(\frac{(x-x_{i+3})^3}{3}\right)\right)\right]_{x_i}^{x_{i+3}}$$

Reemplazando los valores de la integral definida y usando

$$x_{i+1}-x_{i+3} = -2h$$

$$x_{i+2}-x_{i+3} = -h$$

$$\frac{x_{i+1}}{2}+\frac{x_{i+2}}{2}-x_{i+3} = \frac{-3h}{2}$$

Se da que

$$-\frac{f(x_i)}{6h^3}\left[\left(-\frac{(-h)(-2h)(-3h)^2}{2}+\frac{(-3h)^4}{4}-\left(\frac{-3h}{2}\right)\left(\frac{(-3h)^3}{3}\right)\right)\right]$$

$$-\frac{f(x_i)}{6h^3}\left[\left(-\frac{9h^4}{4}\right)\right] = \frac{3}{8}f(x_i)h$$

· Con el mismo proceso en las integrales se obtiene que

$$I_2 = 9/8\, f(x_{i+1})\, h$$

$$I_3 = 9/8\, f(x_{i+2})\, h$$

$$I_4 = 3/8\, f(x_{i+3})\, h$$

reemplazando en la ecuación original

$$\int_{x_i}^{x_{i+3}} f_3(x)\,dx \approx \frac{3}{8} f(x_i)\, h + \frac{9}{8} f(x_{i+1}) + \frac{9}{8} f(x_{i+2}) + \frac{3}{8} (x_{i+3})$$

$$= \boxed{\frac{3h}{8}\left(f(x_i) + 3f(x_{i+1}) + 3f(x_{i+2}) + f(x_{i+3}) \right)}$$