Problemas Integración

3. $f(x) \approx P_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m)$

$+ \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)$

$\boxed{h = \frac{b-a}{2} = b - x_m = x_m - a}$ $\boxed{2h = b-a}$

$x = x_0 + th$ $x_2 = x_0 + 2h$ $x - x_0 = th$

$x_1 = x_0 + 1h$ $x - x_1 = x - x_0 + h = th - h$

$x - x_2 = x - x_0 - 2h = th - 2h$

Reemplazando:

$f(x) \approx P_2(x) = \frac{\cancel{h}(t-1)\cancel{h}(t-2)}{(-2\cancel{h})(\cancel{h})} f(a) + \frac{(t\cancel{h})\cancel{h}(t-2)}{(\cancel{h})(-\cancel{h})} f(x_m)$

$+ \frac{(t\cancel{h})\cancel{h}(t-1)}{(2\cancel{h})(\cancel{h})} f(b)$

$= \frac{(t-1)(t-2)}{-2} f(a) + \frac{t(t-2)}{-1} f(x_m)$

$+ \frac{t(t-1)}{2} f(b)$

$P_1(x) = \dfrac{t^2-3t+2}{2}$ $\qquad x = x_0 + th$

$\qquad dx = h\,dt$

$P_2(x) = (t^2-2t)/-1$

$P_3(x) = \dfrac{t^2-t}{2}$

$$A_1 = f(a)\int_0^2 \frac{t^2-3t+2}{2}\,h\,dt + f(x_m)\int_0^2 \frac{t^2-2t}{-1}\,h\,dt$$
$$+ f(b)\int_0^2 \frac{t^2-t}{2}\,h\,dt$$

$$= f(a)\left(\frac{h}{2}\right)\left(\frac{t^3}{3} - \frac{3t^2}{2} + 2t\right)\Big|_0^2 + f(x_m)\left(\frac{h}{-1}\right)\left(\frac{t^3}{3} - \frac{2t^2}{2}\right)\Big|_0^2$$
$$+ f(b)\left(\frac{h}{2}\right)\left(\frac{t^3}{3} - \frac{t^2}{2}\right)\Big|_0^2$$

$$= f(a)\left(\frac{h}{\cancel{2}}\right)\left(\frac{\cancel{2}}{3}\right) + f(x_m)\left(\frac{h}{\cancel{-1}}\right)\left(\cancel{-}\frac{4}{3}\right) + f(b)\left(\frac{h}{2}\right)\left(\frac{2}{3}\right)$$

factorizando

$$\boxed{= \frac{h}{3}\left(f(a) + 4f(x_m) + f(b)\right)}$$