Архипов Максим

1) $M[x] = 300$

$P(x > A) \le \frac{M[x]}{A}$ нер-во Маркова

а) $P(x < 400) \le \frac{300}{400} = \frac{3}{4} = 0,75$

б) $P(x \le 500) \ge 1 - \frac{300}{500} = 0,4$

2) $n = 1600$ η - распределение Бернулли

$p = 0,3$

$\Delta = 50$

$M[\eta] = np = 1600 \cdot 0,3 = 480$

$D[\eta] = np(1-p) = 1600 \cdot 0,3 \cdot 0,7 = 336$

нер-во Чебышева

$P(|\eta - M[\eta]| < \Delta) \ge 1 - \frac{D[\eta]}{\Delta^2}$

$P(|\eta - 480| < 50) \ge 1 - \frac{336}{50^2} = 0,87$

3)

x_1	x_2	x_3	x_4	x_5	x_6
9	5	7	7	4	10

$D[x] = 1$

$\overline{x} = \frac{9+5+7+7+4+10}{6} = 7$

$1 - \frac{\alpha}{2} = 0,995$

$z_\alpha = 2,58$

$\Delta = \frac{\sigma}{\sqrt{n}} \cdot z_\alpha = \frac{1}{\sqrt{6}} \cdot 2,58 \approx 1,05$

Доверительный интервал

$(7 - 1,05; 7 + 1,05)$

$(5,95; 8,05)$

4) $x_i \sim N(\mu, \sigma^2)$

ОМП $\hat{\mu}, \hat{\sigma}$ - ?

$$f(\mu,\sigma^2)(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y-\mu)^2}{2\sigma^2}\right)$$

$$f(x,\mu,\sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right) =$$

$$= \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right)$$

$$L(x,\mu,\sigma^2) = -\ln(2\pi)^{n/2} - \frac{n}{2}\ln\sigma^2 - \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial}{\partial\mu} L = \frac{2\sum_{i=1}^{n}(x_i-\mu)}{2\sigma^2} = \frac{n\bar{x}-n\mu}{\sigma^2} = 0$$

$$\frac{\partial}{\partial\sigma^2} L = -\frac{n}{2\sigma^2} + \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^4} = 0$$

$$\hat{\mu} = \bar{x}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$