**Ex: 13.1** $A = -20\ \log|T|\,[\text{dB}]$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\lvert T\rvert =$ | 1 | 0.99 | 0.9 | 0.8 | 0.7 | 0.5 | 0.1 | 0 |
| $A \simeq$ | 0 | 0.1 | 1 | 2 | 3 | 6 | 20 | $\infty$ |

**Ex: 13.2**

$A_{\max} = 20\ \log 1.05 - 20\ \log 0.95 \simeq 0.9\ \text{dB}$

$A_{\min} = 20\ \log\left[\dfrac{1}{0.01}\right] = 40\ \text{dB}$

**Ex: 13.3**

$$T(s) = k\frac{(s+j2)\,(s-j2)}{\left(s+\dfrac{1}{2}+j\sqrt{\dfrac{3}{2}}\right)\left(s+\dfrac{1}{2}-j\sqrt{\dfrac{3}{2}}\right)}$$

$$= k\frac{\left(s^2+4\right)}{s^2+s+\dfrac{1}{4}+\dfrac{3}{4}}$$

$$= k\frac{\left(s^2+4\right)}{s^2+s+1}$$

$$T(0) = k\frac{4}{1} = 1$$

$$k = \frac{1}{4}$$

$$\therefore T(s) = \frac{1}{4}\frac{\left(s^2+4\right)}{s^2+s+1}$$

**Ex: 13.4**

$$T(s) = a_3\frac{s\left(s^2+4\right)}{(s+0.1+j0.8)\,(s+0.1-j0.8)}\times \frac{1}{(s+0.1+j1.2)\,(s+0.1-j1.2)}$$

$$= a_3\frac{s\left(s^2+4\right)}{\left(s^2+0.2s+0.65\right)\left(s^2+0.2s+1.45\right)}$$

**Ex: 13.5**

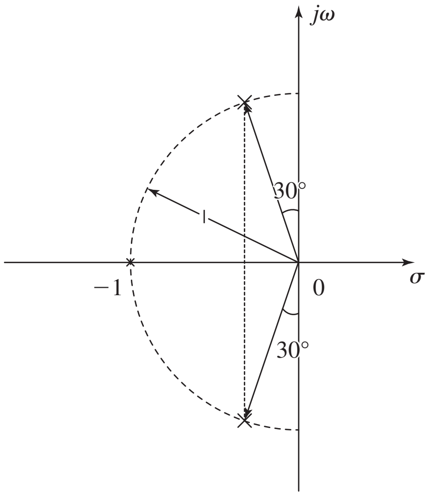



Figure 1

From Fig.1 we see that the real pole is at

$s = -1$

and thus gives rise to a factor $(s+1)$ in the denominator. The pair of complex conjugate poles are at

$-\cos 60° \pm j\ \sin 60°$

$= -0.5 \pm j\sqrt{3}/2$

The corresponding quadratic in the denominator will be

$$= \left(s+0.5+j\frac{\sqrt{3}}{2}\right)\left(s+0.5-j\frac{\sqrt{3}}{2}\right)$$

$$= s^2+s+1$$

Since the filter is of the all-pole type, the transfer function will be

$$T(s) = k\frac{1}{(s+1)(s^2+s+1)}$$

Since the dc gain is unity, $k = 1$ and

$$T(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$= \frac{1}{\sqrt{(1+\omega^2)(1-\omega^2)^2+\omega^2(1+\omega^2)}}$$

$$= \frac{1}{\sqrt{(1-\omega^4)(1-\omega^2)+\omega^2+\omega^4}}$$

$$= \frac{1}{\sqrt{1-\omega^4-\omega^2+\omega^6+\omega^2+\omega^4}}$$

$$= \frac{1}{\sqrt{1+\omega^6}} \qquad \text{Q.E.D.}$$

$|T(j\omega)| = 1/\sqrt{2}$ at $\omega = \omega_{3dB}$, thus

$$\frac{1}{\sqrt{1+\omega_{3dB}^6}} = \frac{1}{\sqrt{2}}$$

$\Rightarrow \omega_{3dB} = 1$ rad/s

At $\omega = 3$ rad/s, we have

$$|T| = \frac{1}{\sqrt{1+3^6}} = \frac{1}{\sqrt{730}}$$

$A = -20\ \log\ |T| = -20\ \log(1/\sqrt{730})$

$= 10\ \log(730) = 28.6\ \text{dB}$

**Ex: 13.6**

$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$

$|T(j\omega)| = \dfrac{1}{\sqrt{1 + \epsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}}}$

$A(\omega_s) = -20\ \log|T(j\omega_s)|$

$= 10\ \log\left[1 + \epsilon^2\left(\dfrac{\omega_s}{\omega_p}\right)^{2N}\right]$

Thus, $10\ \log\left[1 + 0.5088^2 \times 1.5^{2N}\right] \geq 30$

$N = 10$: LHS $= 29.35$ dB

$N = 11$: LHS $= 32.87$ dB

$\therefore$ Use $N = 11$ and obtain

$A_{\min} = 32.87$ dB

For $A_{\min}$ to be exactly 30 dB, we need

$10\ \log\left[1 + \epsilon^2 \times 1.5^{22}\right] = 30$

$\epsilon \Rightarrow 0.3654 \Rightarrow A_{\max} = 20\ \log\sqrt{1 + 0.3654^2}$

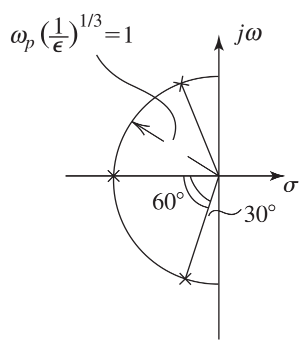
$= 0.54$ dB

**Ex: 13.7** The real pole is at $s = -1$

The complex conjugate poles are at

$s = -\cos 60^\circ \pm j\sin 60^\circ$

$= -0.5 \pm j\sqrt{\dfrac{3}{2}}$



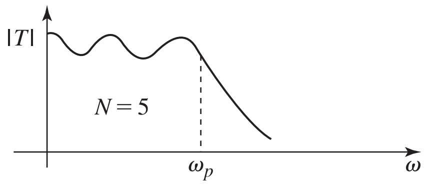

$T(s) =$

$$\frac{1}{(s+1)\left(s + 0.5 + j\sqrt{\dfrac{3}{2}}\right)\left(s + 0.5 - j\sqrt{\dfrac{3}{2}}\right)}$$

$= \dfrac{1}{(s+1)\left(s^2 + s + 1\right)}$

DC gain $= 1$

**Ex: 13.8**




$|T(j\omega)| = \dfrac{1}{\sqrt{1 + \epsilon^2\cos^2\left[N\cos^{-1}\left(\dfrac{\omega}{\omega_P}\right)\right]}}$

for $\omega < \omega_p$.

Peaks are obtained when

$\cos^2\left[N\cos^{-1}\left(\dfrac{\hat{\omega}}{\omega_p}\right)\right] = 0$

$\cos^2\left[5\cos^{-1}\left(\dfrac{\hat{\omega}}{\omega_p}\right)\right] = 0$

$5\cos^{-1}\left(\dfrac{\hat{\omega}}{\omega_p}\right) = (2k+1)\dfrac{\pi}{2},\ k = 0,\ 1,\ 2$

$\therefore \hat{\omega} = \omega_p\cos\left[\dfrac{(2k+1)\pi}{10}\right],\ k = 0, 1, 2$

$\hat{\omega}_1 = \omega_p\cos\left(\dfrac{\pi}{10}\right) = 0.95\omega_p$

$\hat{\omega}_2 = \omega_p\cos\left(\dfrac{3}{10}\pi\right) = 0.59\omega_p$

$\hat{\omega}_3 = \omega_p\cos\left(\dfrac{5}{10}\pi\right) = 0$

Valleys are obtained when

$\cos^2\left[N\cos^{-1}\left(\dfrac{\check{\omega}}{\omega_p}\right)\right] = 1$

$5\cos^{-1}\left(\dfrac{\check{\omega}}{\omega_p}\right) = k\pi,\ k = 0, 1, 2$

$\therefore \check{\omega} = \omega_p\cos\left(\dfrac{k\pi}{5}\right),\ k = 0, 1, 2$

$\check{\omega}_1 = \omega_p\cos 0 = \omega_p$

$\check{\omega}_2 = \omega_p\cos\dfrac{\pi}{5} = 0.81\omega_p$

$\check{\omega}_3 = \omega_p\cos\dfrac{2\pi}{5} = 0.31\omega_p$

**Ex: 13.9**

$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{0.5}{10}} - 1} = 0.3493$

$A(\omega_s) = 10\ \log\left[1 + \epsilon^2\cosh^2\left(N\cosh^{-1}\dfrac{\omega_s}{\omega_p}\right)\right]$

$= 10\ \log\left[1 + 0.3493^2\cosh^2\left(7\cosh^{-1}2\right)\right]$

$= 64.9$ dB

For $A_{\max} = 1$ dB, $\epsilon = \sqrt{10^{0.1} - 1} = 0.5088$

$A(\omega_s) = 10 \log\left[1 + 0.5088^2 \cosh^2\left(7 \cosh^{-1} 2\right)\right]$

$= 68.2$ dB

This is an increase of 3.3 dB

**Ex: 13.10** $\epsilon = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$

(a) For the Chebyshev filter:

$A(\omega_s)$

$= 10 \log\left[1 + 0.5088^2 \cosh^2\left(N \cosh^{-1} 1.5\right)\right]$

$\geq 50$ dB

$N = 7.4 \;\therefore\;$ choose $N = 8$

Excess attenuation =

$10 \log\left[1 + 0.5088^2 \cosh^2\left(8 \cosh^{-1} 1.5\right)\right] - 50$

$= 55 - 50 = 5$ dB

(b) For a Butterworth filter

$\epsilon = 0.5088$

$$A(\omega_s) = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

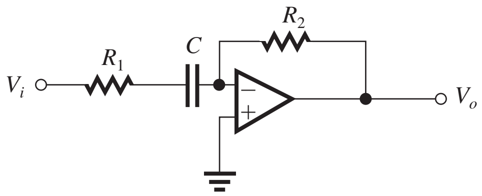
$= 10 \log\left(1 + 0.5088^2 (1.5)^{2N}\right) \geq 50$

$N = 15.9 \;\therefore\;$ choose $N = 16$

Excess attenuation =

$10 \log\left(\left[1 + 0.5088^2 (1.5)^{32}\right] - 50\right) = 0.5$ dB

**Ex: 13.11**




$10^4 = \frac{1}{CR_1}$, $R_1 = 10$ kΩ

$C = 0.01$ μF

H.F. gain = $\frac{-R_2}{R_1} = -10$

$R_2 = 100$ kΩ

**Ex: 13.12** Refer to Fig. 13.14.

$$\omega_0 = \frac{1}{CR} = 10^3 \text{ rad/s}$$

For $R$ arbitrarily selected to be 10 kΩ,

$$C = \frac{1}{10^3 \times 10^4} = 0.1 \ \mu\text{F}$$

The two resistors labeled $R_1$ can also be selected to be a convenient value, say 10 kΩ each.

**Ex: 13.13** $T(s) = \dfrac{\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$

For maximally flat response, $Q = 1/\sqrt{2}$, thus

$$T(s) = \frac{\omega_0^2}{s^2 + s\sqrt{2}\,\omega_0 + \omega_0^2}$$

$$T(j\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j\sqrt{2}\,\omega\omega_0}$$

$$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 2\omega^2\omega_0^2}}$$

$$= \frac{\omega_0^2}{\sqrt{\omega_0^4 + \omega^4}}$$

$$= \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_0}\right)^4}}$$

At $\omega = \omega_0$,

$$|T(j\omega_0)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

which is 3 dB below the value at
$\omega = 0$ (0 dB). Q.E.D.

**Ex: 13.14** $T(s) = \dfrac{sK\left(\dfrac{\omega_0}{Q}\right)}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$

where $K$ is the center-frequency gain. For $\omega_0 = 10^5$ rad/s and 3-dB bandwidth $= 10^3$ rad/s, we have

$$\text{3-dB } BW = \frac{\omega_0}{Q}$$

$$10^3 = \frac{10^5}{Q}$$

$\Rightarrow Q = 100$

Also, for a center-frequency gain of 10, we have

$K = 10$

Thus,

$$T(s) = \frac{10^4 s}{s^2 + 10^3 s + 10^{10}}$$

**Ex: 13.15** (a) $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$

$$|T(j\omega)| = |a_2| \frac{\omega_0^2 - \omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \dfrac{\omega^2\omega_0^2}{Q^2}}}$$

$$= |a_2| \bigg/ \sqrt{1 + \frac{\omega^2\omega_0^2}{Q^2(\omega_0^2 - \omega^2)^2}} \qquad (1)$$

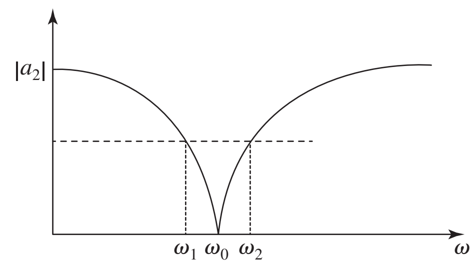



Figure 1

Refer to Fig. 1 and note that at any value of $|T|$ there are two frequencies, $\omega_1$ and $\omega_2$, with this gain value. From Eq. (1) we obtain

$$\frac{\omega_1^2\ \omega_0^2}{Q^2(\omega_0^2 - \omega_1^2)^2} = \frac{\omega_2^2\ \omega_0^2}{Q^2(\omega_2^2 - \omega_0^2)^2}$$

$$\Rightarrow \frac{\omega_1}{\omega_0^2 - \omega_1^2} = \frac{\omega_2}{\omega_2^2 - \omega_0^2}$$

$$\omega_1\omega_2^2 - \omega_1\omega_0^2 = \omega_2\omega_0^2 - \omega_1^2\omega_2$$

$$\omega_1\omega_2(\omega_1 + \omega_2) = (\omega_1 + \omega_2)\omega_0^2$$

$$\Rightarrow \omega_1\omega_2 = \omega_0^2 \qquad \text{Q.E.D.}$$

Now if $\omega_1$ and $\omega_2$ differ by $BW_a$,

$$\omega_2 - \omega_1 = BW_a$$

and if the attenuation over this band of frequencies is to be greater than $A$ dB, then by using Eq. (1) we have

$$10\log_{10}\left[1 + \frac{\omega_0^2\ \omega_1^2}{(\omega_0^2 - \omega_1^2)^2 Q^2}\right] \geq A$$

$$\frac{\omega_1\ \omega_0}{(\omega_0^2 - \omega_1^2)Q} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q}\ \frac{\omega_1\ \omega_0}{(\omega_1\omega_2 - \omega_1^2)} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q}\ \frac{\omega_0}{\omega_2 - \omega_1} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{1}{Q}\ \frac{\omega_0}{BW_a} \geq \sqrt{10^{A/10} - 1}$$

$$Q \leq \frac{\omega_0}{BW_a\sqrt{10^{A/10} - 1}}$$

(b) For $A = 3$ dB we have

$$Q = \frac{\omega_0}{BW_a\sqrt{10^{3/10} - 1}}$$

$$\Rightarrow BW_a = \frac{\omega_0}{Q} \qquad \text{Q.E.D.}$$

**Ex: 13.16** From Fig. 13.16(e) we have

$$\omega_{\max} = \sqrt{\frac{\left(\dfrac{\omega_n}{\omega_0}\right)^2(1 - \dfrac{1}{2Q^2}) - 1}{\left(\dfrac{\omega_n}{\omega_0}\right)^2 + \dfrac{1}{2Q^2} - 1}}$$

$$= \sqrt{\frac{\left(\dfrac{1.2}{1}\right)^2\left(1 - \dfrac{1}{2 \times 100}\right) - 1}{\left(\dfrac{1.2}{1}\right)^2 + \dfrac{1}{2 \times 100} - 1}}$$

$= 0.986$ rad/s

$$T_{\max} = \frac{|a_2|\ |\omega_n^2 - \omega_{\max}^2|}{\sqrt{(\omega_0^2 - \omega_{\max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{\max}^2}}$$

where

$$\text{DC gain} = |a_2|\ \frac{\omega_n^2}{\omega_0^2} = 1$$

$$\Rightarrow |a_2| = \frac{1}{1.44} = 0.694$$

Thus,

$$T_{\max} = \frac{0.694\ |1.44 - 0.972|}{\sqrt{(1 - 0.972)^2 + \dfrac{1}{100} \times 0.972}} = 3.17$$

HF transmission $= |a_2| = 0.694$

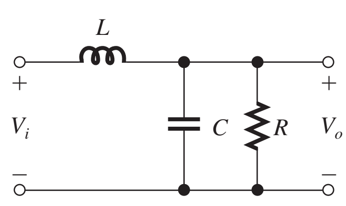
**Ex: 13.17** Maximally flat $\quad \Rightarrow Q = \dfrac{1}{\sqrt{2}}$

$$\omega_0 = 2\pi \times 100 \times 10^3$$

Arbitrarily selecting $R = 1$ k$\Omega$, we get

$$Q = \omega_0 CR \Rightarrow C = \frac{1}{\sqrt{2} \times 2\pi 10^5 \times 10^3}$$

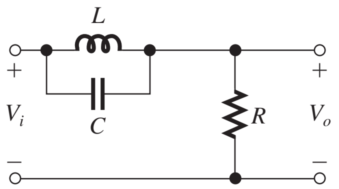
$= 1125$ pF



Also $Q = \dfrac{R}{\omega_0 L}$

$$\therefore L = \frac{R}{\omega_0 Q} = \frac{10^3}{2\pi 10^5 \times \frac{1}{\sqrt{2}}} = 2.25 \text{ mH}$$

**Ex: 13.18**




From Exercise 13.15 above, 3-dB bandwidth $= \omega_0/Q$

$2\pi 10 = 2\pi 60/Q \Rightarrow Q = 6$

$Q = \omega_0 CR$

$6 = 2\pi 60 \times C \times 10^4 \Rightarrow C = 1.6\ \mu\text{F}$

$$Q = \frac{R}{\omega_0 L}$$

$$L = \frac{R}{\omega_0 Q} = \frac{10^4}{2\pi 60 \times 6} = 4.42 \text{ H}$$

**Ex: 13.19** $f_0 = 10$ kHz, $\Delta f_{3\text{dB}} = 500$ Hz

$$Q = \frac{f}{\Delta f_{3\text{dB}}} = \frac{10^4}{500} = 20$$

Using the data at the top of Table 13.1, we get

$C_4 = C_6 = 1.2$ nF

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 13.26 \text{ k}\Omega$$

$$R_6 = Q/\omega_0 C = \frac{20}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 265 \text{ k}\Omega$$

Now using the data in Table 13.1 for the bandpass case, we obtain

$K =$ center-frequency gain $= 10$

Referring to Fig. 13.21(c), we have

$1 + r_2/r_1 = 10$

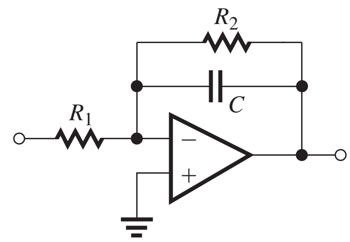
Selecting $r_1 = 10$ kΩ, we obtain $r_2 = 90$ kΩ.

**Ex: 13.20** From Eq. (13.25), we have $\omega_p = 2\pi 10^4$ and

$$T(s) = \frac{{\omega_p}^5}{8.1408\left(s + 0.2895\omega_p\right)} \times \frac{1}{\left(s^2 + s0.4684\omega_p + 0.4293{\omega_p}^2\right)} \times \frac{1}{\left(s^2 + s0.1789\omega_p + 0.9883{\omega_p}^2\right)}$$

The circuit consists of 3 sections in cascade:

(a) First-order section:




$$T(s) = \frac{-0.2895\omega_p}{s + 0.2895\omega_p}$$

where the numerator coefficient is set so that the dc gain $= -1$.

Let $R_1 = 10$ kΩ

dc gain $= R_2/R_1 = 1 = R_2 = 10$ kΩ

$\omega_0 = 0.2895\omega_p$

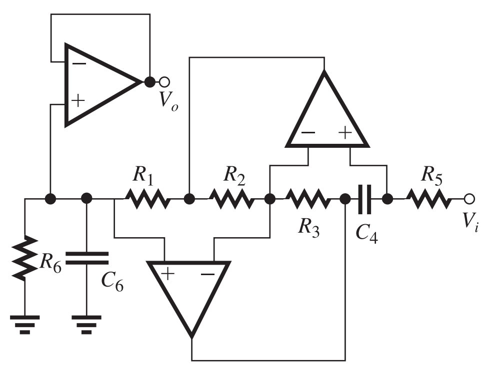
$$\frac{1}{CR_2} = 0.2895\omega_p$$

$$\Rightarrow C = \frac{1}{0.2895 \times 2\pi 10^4 \times 10^4} = 5.5 \text{ nF}$$

(b) Second-order section with transfer function:

$$T(s) = \frac{0.4293\omega_P^2}{s^2 + s0.4684\omega_p + 0.4293\omega_P^2}$$

where the numerator coefficient is selected to yield a dc gain of unity.




Select $R_1 = R_2 = R_3 = R_5 = 10$ kΩ

$$\Rightarrow C = \frac{1}{\sqrt{0.4293} \times 2\pi 10^4 \times 10^4} = 2.43 \text{ nF}$$

$C_4 = C_6 = C = 2.43$ nF

$$Q = \frac{\sqrt{0.4293}\omega_p}{0.4684\omega_p} = 1.4 \Rightarrow R_6 = \frac{Q}{\omega_0 C} = 14 \text{ k}\Omega$$

(c) Second-order section with transfer function:

$$T(s) = \frac{0.9883\omega^2{}_p}{s^2 + s0.1789\omega_p + 0.9883\omega^2{}_p}$$

The circuit is similar to that in (b) above but with

$R_1 = R_2 = R_3 = R_5 = 10\ \text{k}\Omega$

$$C_4 = C_6 = \frac{1}{\omega_0 \times 10^4}$$

$$= \frac{1}{\sqrt{0.9883} \times 2\pi 10^4 \times 10^4}$$

$= 1.6\ \text{nF}$

$$Q = \frac{\sqrt{0.9883}}{0.1789} = 5.56$$

Thus $R_6 = Q/\omega_0 C = 55.6\ \text{k}\Omega$

Placing the three sections in cascade, i.e. connecting the output of the first-order section to the input of the second-order section in (b) and the output of of section (b) to the input of (c) results in the overall transfer function in Eq. (13.25) except for an inversion.

**Ex: 13.21** Refer to the KHN circuit in Fig. 13.24. Choosing $C = 1$ nF, we obtain

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi 10^4 \times 10^{-9}} = 15.9\ \text{k}\Omega$$

Using Eq. (13.62) and selecting $R_1 = 10\ \text{k}\Omega$, we get

$R_f = R_1 = 10\ \text{k}\Omega$

Using Eq. (13.63) and setting $R_2 = 10\ \text{k}\Omega$, we obtain

$R_3 = R_2\,(2Q - 1) = 10\,(2 \times 2 - 1) = 30\ \text{k}\Omega$

$$\text{High-frequency gain} = K = 2 - \frac{1}{Q} = 1.5\ \text{V/V}$$

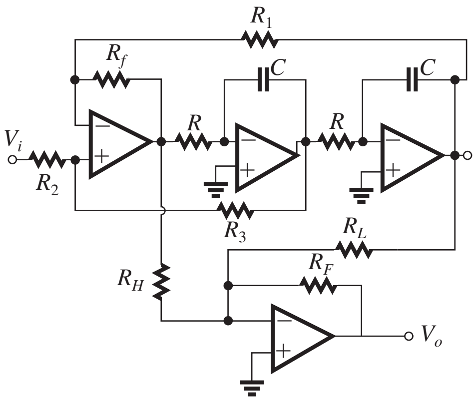
The transfer function to the output of the first integrator is

$$\frac{V_{bp}}{V_i} = -\frac{1}{sCR} \times \frac{V_{hp}}{V_i} = \frac{sK/(CR)}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

Thus the center-frequency gain is given by

$$\frac{K}{CR}\frac{Q}{\omega_0} = KQ = 1.5 \times 2 = 3\ \text{V/V}$$

**Ex: 13.22**




$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)\,s^2 + (R_F/R_L)\,\omega^2{}_0}{s^2 + s\omega_0/Q + \omega_0^2}$$

given $C = 1$ nF, $R_L = 10\ \text{k}\Omega$, then

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi 5 \times 10^3 \times 10^{-9}} = 31.83\ \text{k}\Omega$$

$R_1 = 10\ \text{k}\Omega \Rightarrow R_f = 10\ \text{k}\Omega$

$R_2 = 10\ \text{k}\Omega \Rightarrow R_3 = R_2\,(2Q - 1)$

$= 10\,(10 - 1) = 90\ \text{k}\Omega$

$$\frac{R_H}{R_L}\omega_0^2 = \omega_n^2 \Rightarrow R_H = 10\left(\frac{8}{5}\right)^2 = 25.6\ \text{k}\Omega$$

$$\text{DC gain} = K\frac{R_F}{R_L} = \left(2 - \frac{1}{Q}\right)\frac{R_F}{R_L} = 3$$

$$R_F = \frac{3 \times 10}{2 - 1/5} = 16.7\ \text{k}\Omega$$

**Ex: 13.23** Refer to Fig. 13.25 (b)

$$CR = \frac{1}{\omega_0} \Rightarrow C = \frac{1}{2\pi 10^4 \times 10^4} = 1.59\ \text{nF}$$

$R_d = QR = 20 \times 10 = 200\ \text{k}\Omega$

Center frequency gain $= KQ = 1$

$$\therefore K = \frac{1}{Q} = \frac{1}{20}$$

$R_g = R/K = 20R = 200\ \text{k}\Omega$

**Ex: 13.24** Refer to Fig. 13.26 and Table 13.2 (AP entry).

$C = 10\ \text{nF}$

$$R = \frac{1}{\omega_0 C} = \frac{1}{10^4 \times 10 \times 10^{-9}} = 10\ \text{k}\Omega$$

$QR = 5 \times 10 = 50\ \text{k}\Omega$

$C_1 = C \times \text{flat gain} = 10 \times 1 = 10\ \text{nF}$

$R_1 = \infty$

$$R_2 = \frac{R}{\text{gain}} = R/1 = 10\ \text{k}\Omega$$

$r = 10\ \text{k}\Omega$

$$R_3 = \frac{Qr}{\text{gain}} = \frac{5 \times 10}{1} = 50\ \text{k}\Omega$$

**Ex: 13.25** From Eq. (13.76) we have

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1}{10^4} = 2 \times 10^{-4}\text{s}$$

For $C = C_1 = C_2 = 1$ nF

$$R = \frac{2 \times 10^{-4}}{10^{-9}} = 200\ \text{k}\Omega$$

Thus $R_3 = 200\ \text{k}\Omega$

From Eq. (13.75) we have

$m = 4Q^2 = 4$

Thus, $R_4 = \dfrac{R}{m} = \dfrac{200}{4} = 50\ \text{k}\Omega$

**Ex: 13.26** The transfer function of the feedback network is given in Fig. 13.28(a). The poles are the roots of the denominator polynomial,

$$s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4} = 0$$

For $C_1 = C_2 = 10^{-9}$ F, $R_3 = 2 \times 10^5 \Omega$,

$R_4 = 5 \times 10^4\ \Omega$,

$$s^2 + s\left(\frac{2}{10^{-9} \times 2 \times 10^5} + \frac{1}{10^{-9} \times 5 \times 10^4}\right) + \frac{1}{10^{-18}10^{10}} = 0$$

$$s^2 + s\left(3 \times 10^4\right) + 10^8 = 0$$

$$s = \frac{-3 \times 10^4 \pm \sqrt{9 \times 10^8 - 4 \times 10^8}}{2}$$

$= -0.382 \times 10^4$ and $-2.618 \times 10^4$ rad/s

**Ex: 13.27** Refer to the circuit in Fig. 13.30(a), where the transfer function is given on page 1126 as

$$T(s) = \frac{-s(\alpha/C_1R_4)}{s^2 + s\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)\dfrac{1}{R_3} + \dfrac{1}{C_1C_2R_3R_4}}$$

Now, using the component value obtained in Exercise 13.25, namely

$C_1 = C_2 = 1$ nF

$R_3 = 200\ \text{k}\Omega$

$R_4 = 50\ \text{k}\Omega$

the center-frequency gain is given by (note that $Q = 1$)

$$|T(j\omega_a)| = \alpha\left(\frac{R_3}{R_4}\right)\Big/\left(1 + \frac{C_1}{C_2}\right)$$

$$1 = \alpha \times \frac{4}{1+1} = 2\alpha$$

$\Rightarrow \alpha = 0.5$

Thus,

$$\frac{R_4}{\alpha} = \frac{50}{0.5} = 100\ \text{k}\Omega$$

and

$$\frac{R_4}{1-\alpha} = \frac{50}{0.5} = 100\ \text{k}\Omega$$

**Ex: 13.28**

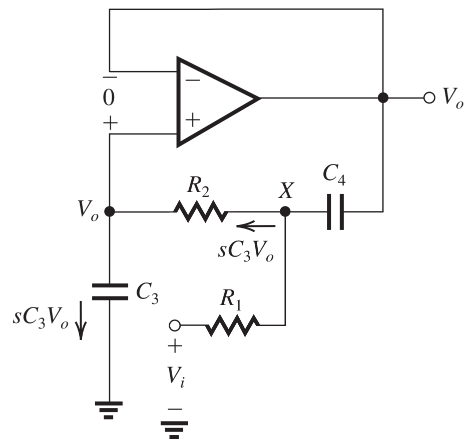



Figure 1

Figure 1 shows the circuit of Fig. 13.34(c) with partial analysis to determine the transfer function. The voltage at node X can be written as

$V_x = V_o + sC_3V_oR_2$

$= (1 + sC_3R_2)V_o \qquad (1)$

Now a node equation at X takes the form

$$\frac{V_i - V_x}{R_1} = sC_3V_o + sC_4(V_x - V_o)$$

Substituting for $V_x$ from Eq. (1) gives

$V_i - V_o(sC_3R_2 + 1) = sC_3R_1V_o + sC_4R_1(sC_3R_2V_o)$

$V_i = V_o[s^2C_3C_4R_1R_2 + sC_3(R_1 + R_2) + 1]$

$$\Rightarrow \frac{V_o}{V_i} = \frac{1/C_3C_4R_1R_2}{s^2 + s\dfrac{1}{C_4}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) + \dfrac{1}{C_3C_4R_1R_2}} \qquad (2)$$

Thus,

$\omega_0 = 1/\sqrt{C_3C_4R_1R_2}$

and

$$Q = \left[\frac{\sqrt{C_3C_4R_1R_2}}{C_4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]^{-1}$$

which are identical to Eqs. (13.77) and (13.78), respectively. Q.E.D.

From Eq. (2) we see that

$\dfrac{V_o}{V_i}(0) = 1$ Q.E.D.

**Ex: 13.29** The design equations are Eqs. (13.79) and (13.80). Thus,

$R_1 = R_2 = R = 10\ \text{k}\Omega$

and

$C_4 = C$

$C_3 = C/4Q^2 = C \Big/ \left(4 \times \frac{1}{2}\right) = 0.5C$

where

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times \frac{1}{\sqrt{2}}}{2\pi \times 4 \times 10^3}$$

$$\Rightarrow C = \frac{\sqrt{2}}{2\pi \times 4 \times 10^3 \times 10 \times 10^3} = 5.63 \text{ nF}$$

Thus,

$C_4 = 5.63$ nF

$C_3 = 2.81$ nF

**Ex: 13.30** Refer to the results in Example 13.3

(a) $\Delta R_3/R_3 = +2\%$

$$S_{R3}^{\omega_o} = -1/2 \Rightarrow \frac{\Delta\omega_O}{\omega_O} = -\frac{1}{2} \times 2 = -1\%$$

$$S_{R3}^{Q} = \frac{1}{2} \Rightarrow \Delta Q/Q = \frac{1}{2} \times 2 = 1\%$$

(b) $\Delta R_4/R_4 = 2\%$

$$S_{R_4}^{\omega_o} = -\frac{1}{2} \Rightarrow \frac{\Delta\omega_O}{\omega_O} = -1\%$$

$$S_{R_4}^{Q} = -\frac{1}{2} \Rightarrow \frac{\Delta Q}{Q} = -\frac{1}{2} \times 2 = -1\%$$

(c) Combining the results in (a) & (b), we get

$$\frac{\Delta\omega_O}{\omega_O} = -1 - 1 = -2\%$$

$$\frac{\Delta Q}{Q} = 1 - 1 = 0\%$$

(d) Using the results in (c) for both resistors being 2% high, we have

$$\frac{\Delta\omega_O}{\omega_O} = S_{C1}^{\omega_o}\frac{\Delta C_1}{C_1} + S_{C2}^{\omega_o}\frac{\Delta C_2}{C_2} - 2$$

$$= -\frac{1}{2}(-2) + \frac{-1}{2}(-2) - 2$$

$$= 2 - 2 = 0\%$$

$$\frac{\Delta Q}{Q} = S_{C1}^{Q}\frac{\Delta C_1}{C} + S_{C2}^{Q}\frac{\Delta C_2}{C_2} + 0$$

$$= 0(-2) + (0)(-2) + 0 = 0\%$$

**Ex: 13.31** $f_0 = f_{3dB} = 20$ MHz

$Q = 1/\sqrt{2}$

DC gain = 1

Using Eq. (13.99), we obtain

$G_m = \omega_0 C = 2\pi \times 20 \times 10^6 \times 2 \times 10^{-12}$

$= 0.251$ mA/V

Thus,

$G_{m1} = G_{m2} = 0.251$ mA/V

$$G_{m3} = \frac{G_m}{Q} = \frac{0.251}{1/\sqrt{2}} = 0.355 \text{ mA/V}$$

$G_{m4} = G_m|\,\text{Gain}\,| = G_m = 0.251$ mA/V

**Ex: 13.32** From Eq. (13.109)

$C_3 = C_4 = \omega_0 T_c C$

$$= 2\pi 10^4 \times \frac{1}{200 \times 10^3} \times 20$$

$= 6.283$ pF

From Eq. (13.112)

$$C_5 = \frac{C_4}{Q} = \frac{6.283}{20} = 0.314 \text{ pF}$$

From Eq. (13.113)

$$\text{Centre-frequency gain} = \frac{C_6}{C_5} = 1$$

$C_6 = C_5 = 0.314$ pF

**Ex: 13.33** $R_p = \omega_0 L Q_0$

$= 2\pi 10^6 \times 3.18 \times 10^{-6} \times 150 = 3\text{ k}\Omega$

$R = R_L \parallel r_o \parallel R_p = 2\text{ k}\Omega \Rightarrow R_L = 15\text{ k}\Omega$

**Ex: 13.34** $Q = (R_1 \parallel R_{\text{in}})/\omega_0 L$

$$= \frac{10^3 \parallel 10^3}{\left(2\pi \times 455 \times 10^3\right) \times 5 \times 10^{-6}} = 35$$

$BW = f_0/Q = 455/35 = 13$ kHz

$$C_1 + C_{\text{in}} = \frac{1}{\omega_0^2 L}$$

$$= \frac{1}{\left(2\pi \times 455 \times 10^3\right)^2 \times 5 \times 10^{-6}}$$

$= 24.47$ nF

$C_1 = 24.47 - 0.2 = 24.27$ nF

**Ex: 13.35** To just meet specifications,

$$Q = \frac{f_0}{BW} = \frac{455}{10} = 45.5$$

$$\therefore \frac{R_1 \parallel n^2 R_{\text{in}}}{\omega_0 L} = 45.5$$

$R_1 \parallel n^2 R_{\text{in}} = 45.5 \times 2\pi \times 455 \times 10^3 \times 5 \times 10^{-6}$

$= 650\ \Omega$

$n^2 R_{\text{in}} = 1.86\ \text{k}\Omega$

$$n = \sqrt{\frac{1.86}{1}} = 1.36$$

$$C_1 + \frac{C_{\text{in}}}{n^2} = \frac{1}{\omega_0^2 L} = 24.47\ \text{nF}$$

$C_1 = 24.36\ \text{nF}$

At resonance, the voltage developed across $R_1$ is $I\left(R_1 \parallel n^2 R_{\text{in}}\right) = IR$. Thus, $V_{be} = IR/n$ & $I_c = g_m V_{be} = g_m IR/n$,

$$\frac{I_c}{I} = g_m R/n = \frac{40 \times 0.65}{1.36} = 19.1 \frac{\text{A}}{\text{A}}$$

**Ex: 13.36** $200 = (f_0/Q)\sqrt{2^{1/2} - 1}$
Eq. (13.123)

$$\frac{f_0}{Q} = 310.8\ \text{kHz}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{\left(2\pi \times 10.7 \times 10^6\right)^2 \times 3 \times 10^{-6}}$$

$= 73.75\ \text{pF}$

$$\frac{\omega_O}{Q} = \frac{1}{CR}$$

$$R = \frac{1}{73.7 \times 10^{-12} \times 2\pi \times 310.8 \times 10^3}$$

$= 6.94\ \text{k}\Omega$

**13.1** $T(s) = \dfrac{\omega_0}{s + \omega_0}$, $T(j\omega) = \dfrac{\omega_0}{j\omega + \omega_0}$

$$|T(j\omega)| = \frac{\omega_0}{\sqrt{\omega_0^2 + \omega^2}} = \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_0}\right)^2}}$$

$$\phi(\omega) \equiv \tan^{-1}\left[\frac{\text{Im}\,(T(j\omega))}{\text{Re}\,(T(j\omega))}\right]$$

$= -\tan^{-1}(\omega/\omega_O)$

$G = 20\log_{10}|T(j\omega)|$

$A = -20\log_{10}|T(j\omega)|$

| $\omega$ | $\|T(j\omega)\|$ [V/V] | $G$ [dB] | $A$ [dB] | $\phi$ [degrees] |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| $0.5\omega_0$ | 0.8944 | –0.97 | 0.97 | –26.57 |
| $\omega_0$ | 0.7071 | –3.01 | 3.01 | –45.0 |
| $2\omega_0$ | 0.4472 | –6.99 | 6.99 | –63.43 |
| $5\omega_0$ | 0.1961 | –14.1 | 14.1 | –78.69 |
| $10\omega_0$ | 0.0995 | –20.0 | 20.0 | –84.29 |
| $100\omega_0$ | 0.010 | –40.0 | 40.0 | –89.43 |

**13.2** $T(s) = \dfrac{2\pi \times 10^4}{s + 2\pi \times 10^4}$

$$T(j\omega) = \frac{2\pi \times 10^4}{2\pi \times 10^4 + j\omega}$$

$$= \frac{1}{1 + j[\omega/2\pi \times 10^4]}$$

$$|T(j\omega)| = 1\bigg/\sqrt{1 + \left(\frac{\omega}{2\pi \times 10^4}\right)^2}$$

$\phi(\omega) = -\tan^{-1}(\omega/2\pi \times 10^4)$

(a) $f = 1$ kHz

$\omega = 2\pi \times 10^3$ rad/s

$|T| = 1/\sqrt{1 + 0.01} \simeq 0.995$ V/V

$\phi = -\tan^{-1}(0.1) = -5.7°$

Peak amplitude of output sinusoid = 0.995 V

Phase of output relative to that of input = −5.7°.

(b) $f = 10$ kHz

$\omega = 2\pi \times 10^4$ rad/s

$|T| = 1/\sqrt{2} = 0.707$ V/V

$\phi = -\tan^{-1}(1) = -45°$

Peak amplitude of output sinusoid = 0.707 V

Phase of output relative to that of input = −45°.

(c) $f = 100$ kHz

$\omega = 2\pi \times 10^5$ rad/s

$|T| = 1/\sqrt{1 + 100} \simeq 0.1$ V/V

$\phi = -\tan^{-1}(10) = -84.3°$

Peak amplitude of output sinusoid = 0.1 V

Phase of output relative to that of input = −84.3°.

(d) $f = 1$ MHz

$\omega = 2\pi \times 10^6$ rad/s

$|T| = 1/\sqrt{1 + 10^4} = 0.01$ V/V

$\phi = -\tan^{-1}(100) = -89.4°$

Peak amplitude of output sinusoid = 0.01 V

Phase of output relative to that of input = −89.4°.

**13.3** $T(s) = \dfrac{1}{(s+1)\left(s^2 + s + 1\right)}$

$$= \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$T(j\omega) = \left[j\left(2\omega - \omega^3\right) + \left(1 - 2\omega^2\right)\right]^{-1}$$

$$|T(j\omega)| = \left[\left(2\omega - \omega^3\right)^2 + \left(1 - 2\omega^2\right)^2\right]^{-\frac{1}{2}}$$

$$= \left[4\omega^2 - 4\omega^4 + \omega^6 + 1 - 4\omega^2 + 4\omega^4\right]^{-\frac{1}{2}}$$

$$= \left[1 + \omega^6\right]^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{1 + \omega^6}} \qquad \text{Q.E.D.}$$

For phase angle:

$$\phi(\omega) = \tan^{-1}\left[\frac{\text{Im}\,(T(j\omega))}{\text{Re}\,(T(j\omega))}\right]$$

$$= -\tan^{-1}\left[\frac{2\omega - \omega^3}{1 - 2\omega^2}\right]$$

For $\omega = 0.1$ rad/s:

$|T(j\omega)| = \left(1 + 0.1^6\right)^{-1/2} \simeq 1$

$\phi(\omega) = -11.5° = -0.20$ rad

For $\omega = 1$ rad/s:

$|T(j\omega)| = \left(1 + 1^6\right)^{-1/2} = 1/\sqrt{2} = 0.707$

$$\phi = -\tan^{-1}\left(\frac{1}{-1}\right) = -135° = -2.356 \text{ rad}$$

Note: $G = -3$ dB

For $\omega = 10$ rad/s:

$|T(j\omega)| = \left(1 + 10^6\right)^{-1/2} = 0.001$

$\phi = -\tan^{-1}\left[\dfrac{2(10) - 10^3}{1 - 2\left(10^2\right)}\right]$

$= -\tan^{-1}\left[\dfrac{-980}{-199}\right]$

$= -\left[180° + \tan^{-1}\left(\dfrac{980}{199}\right)\right]$

$= -258.5°$

$= -4.512$ rad

Now consider an input of $A \sin \omega t$ to $T(s)$. The output is then given by

$A\,|T(j\omega)| \sin(\omega t + \phi(\omega))$

Using this result, the output to each of the following inputs will be:

| INPUT | OUTPUT |
|---|---|
| $10 \sin(0.1t)$ | $10 \sin(0.1t - 0.2)$ |
| $10 \sin(1t)$ | $7.07 \sin(t - 2.356)$ |
| $10 \sin(10t)$ | $0.01 \sin(10t - 4.512)$ |

**13.4** Refer to Fig. 13.3.

$A_{\max} = 20 \log 1.05 = 0.42$ dB

$A_{\min} = 20 \log\left(\dfrac{1}{0.0005}\right)$

$= 66$ dB

Selectivity factor $\equiv \dfrac{f_s}{f_p}$

$= \dfrac{5}{4} = 1.25$

**13.5** At $\omega = 0$, we have

$20 \log|T| = 0$ dB

$\Rightarrow |T| = 1$ V/V

At $\omega = \omega_p$, we have

$20 \log|T| = -A_{\max} = -0.2$ dB

$\Rightarrow |T| = 0.977$ V/V

At $\omega = \omega_s$, we have

$20 \log|T| = -A_{\min} = -60$ dB

$\Rightarrow |T| = 0.001$ V/V

**13.6**

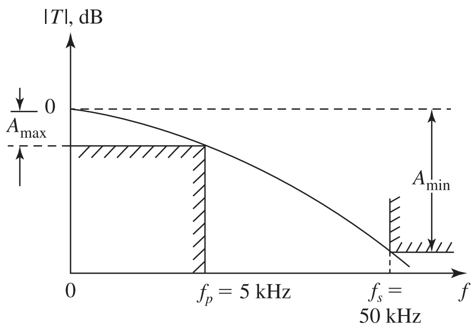



Figure 1

Refer to Fig. 1.

$T(s) = \dfrac{2\pi \times 10^4}{s + 2\pi \times 10^4}$

$T(j\omega) = \dfrac{1}{1 + j\dfrac{\omega}{2\pi \times 10^4}}$

$|T| = 1\Big/\sqrt{1 + \left(\dfrac{f}{10^4}\right)^2}$

At $f = f_p = 5$ kHz, we have

$|T| = 1\Big/\sqrt{1 + \left(\dfrac{5 \times 10^3}{10^4}\right)^2} = 0.894$

Thus,

$A_{\max} = -20 \log 0.894 = 0.97$ dB

At $f = f_s = 10\, f_p = 50$ kHz, we have

$|T| = 1\Big/\sqrt{1 + \left(\dfrac{50 \times 10^3}{10^4}\right)^2} = 0.196$

$A_{\min} = 20 \log\left(\dfrac{1}{0.196}\right) = 14.15$ dB

**13.7** See Fig. 1.

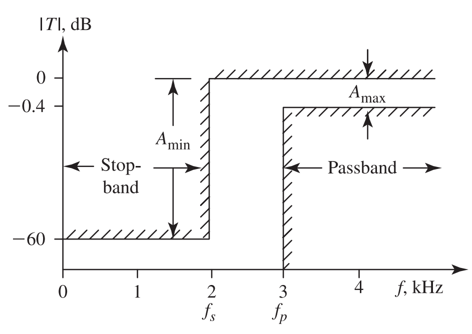



Figure 1

**13.8** See Fig. 1.

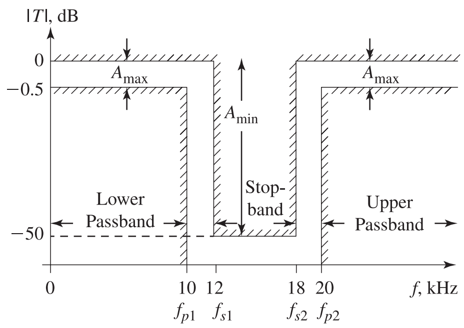



Figure 1

**13.9**

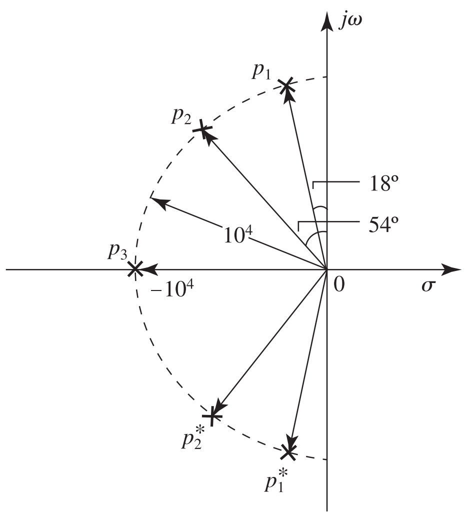



Figure 1

Figure 1 shows the location of the five poles in the $s$ plane. The real-axis pole at $s = -10^4$ gives rise to a factor $(s + 10^4)$. The pair of conjugate poles $p_1$ and $p_1^*$ are at

$s = -10^4 \sin 18° \pm j10^4 \cos 18°$

$= -0.309 \times 10^4 \pm j0.951 \times 10^4$

This pole pair gives rise to a factor

$= (s + 0.309 \times 10^4 + j\,0.951 \times 10^4)(s + 0.309 \times 10^4 - j0.951 \times 10^4)$

$= s^2 + 0.618 \times 10^4 s + 10^8$

The pair of complex conjugate poles $p_2$ and $p_2^*$ are at

$s = -10^4 \sin 54° \pm j10^4 \cos 54°$

$= -0.809 \times 10^4 \pm j0.588 \times 10^4$

This pole pair gives rise to a factor

$= (s + 0.809 \times 10^4 + j\,0.588 \times 10^4)(s + 0.809 \times 10^4 - j0.588 \times 10^4)$

$= (s^2 + 1.618 \times 10^4 s + 10^8)$

Thus the denominator polynomial of $T(s)$ is

$D(s) = (s + 10^4)(s + 0.618 \times 10^4 s + 10^8)(s^2 + 1.618 \times 10^4 s + 10^8)$

(a) The filter is a low-pass of the all-pole type,

$$T(s) = \frac{k}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

Since the dc gain is unity, we have

$k = 10^{20}$

Thus,

$$T(s) = \frac{10^{20}}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

(b) The filter is a high pass,

$$T(s) = \frac{ks^5}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

Since the high-frequency ($s \to \infty$) gain is unity, $k$ must be unity, thus

$$T(s) = \frac{s^5}{(s + 10^4)(s^2 + 0.618 \times 10^4 s + 10^8) \times (s^2 + 1.618 \times 10^4 s + 10^8)}$$

**13.10** $T(s) = \dfrac{k(s^2 + 4)}{(s + 0.25 + j)(s + 0.25 - j)}$

$$= \frac{k(s^2 + 4)}{s^2 + 0.5s + 1.0625}$$

$$T(0) = \frac{4k}{1.0625} = 1$$

$$\Rightarrow k = \frac{1.0625}{4} = 0.2656$$

$$T(s) = \frac{0.2656(s^2 + 4)}{s^2 + 0.5s + 1.0625}$$

$T(\infty) = 0.2656$

**13.11** $T(s)$

$$= \frac{k(s^2 + 4)}{(s + 1)(s + 0.5 - j\,0.8) \times (s + 0.5 - j\,0.8)}$$

$$= \frac{k(s^2 + 4)}{(s + 1)(s^2 + s + 0.89)}$$

$$T(0) = \frac{4k}{0.89} = 1$$

$$\Rightarrow k = \frac{0.89}{4} = 0.2225$$

Thus,

$$T(s) = \frac{0.2225(s^2+4)}{(s+1)(s+s+0.89)}$$

**13.12**

$$T(s) = \frac{s(s^2+10^6)(s^2+9\times10^6)}{s^6+b_5s^5+b_4s^4+b_3s^2+b_1s+b_0}$$

Note that we started with the numerator factors (which represent the given transmission zeros). We used the fact that there is one transmission zero at $s=\infty$ to write the denominator sixth-order polynomial. Thus,

$N = 6$

A sketch of the magnitude response, $|T|$, is given in Fig. 1.

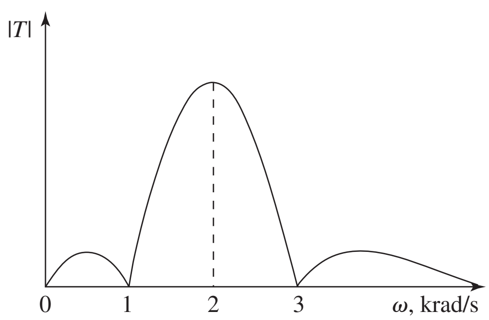



Figure 1

**13.13**

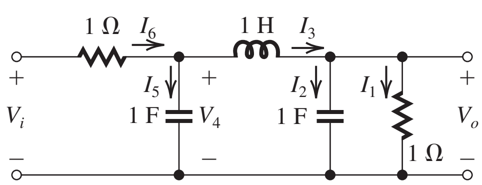



Figure 1

Refer to Fig. 1.

$I_1 = \dfrac{V_o}{1} = V_o$

$I_2 = sCV_o = s\times1\times V_o = sV_o$

$I_3 = I_1 + I_2 = (s+1)V_o$

$V_4 = V_o + sLI_3$

$= V_o + s\times1(s+1)V_o$

$= (s^2+s+1)V_o$

$I_5 = s\times1\times V_4$

$= s(s^2+s+1)V_o$

$I_6 = I_3 + I_5 = (s+1)V_o + s(s^2+s+1)V_o$

$= (s^3+s^2+2s+1)V_o$

$V_i = V_4 + I_6\times1$

$= (s^2+s+1)V_o + (s^3+s^2+2s+1)V_o$

$= (s^3+2s^2+3s+2)V_o$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^3+2s^2+3s+2}$$

All the transmission zeros are at $s=\infty$. To find the poles, we have to factor the third-order denominator polynomial. Toward this end, we find by inspection that one of the zeros of the denominator polynomial is at $s=-1$. Thus, the polynomial will have a factor $(s+1)$ and can be written as

$s^3+2s^2+3s+2 = (s+1)(s^2+as+b)$

where by equating corresponding terms on both sides we find that

$b = 2$

$a+1 = 2 \Rightarrow a = 1$

Thus,

$s^3+2s^2+3s+2 = (s+1)(s^2+s+2)$

and the poles are at

$s = -1$

and at the roots of

$s^2+s+2 = 0$

which are

$s = \dfrac{-1\pm\sqrt{1-8}}{2}$

$= -0.5 \pm j\left(\sqrt{7}/2\right)$

$= -0.5 \pm j1.323$

**13.14** $A_{\max} = 0.5$ dB, $A_{\min} \geq 20$ dB, $\dfrac{\omega_s}{\omega_p} = 1.7$

Using Eq. (13.14), we obtain

$\epsilon = \sqrt{10^{A_{\max}/10}-1}$

$= \sqrt{10^{0.05}-1} = 0.3493$

Using Eq. (13.15), we have

$A(\omega_s) = 10\log[1+\epsilon^2(\omega_s/\omega_p)^{2N}]$

$= 10\log[1+0.3493^2\times1.7^{2N}]$

For $N=5$, $A(\omega_s) = 14.08$ dB

For $N=6$, $A(\omega_s) = 18.58$ dB

For $N=7$, $A(\omega_s) = 23.15$ dB

Thus, to meet the $A_s \geq 20$ dB specification, we use

$N = 7$

in which case the actual minimum stopband attenuation realized is

$A_{\min} = 23.15$ dB

If $A_{\min}$ is to be exactly 20 dB, we can use Eq. (13.15) to obtain the new value of $\epsilon$ as follows:

$$20 = 10 \log[1 + \epsilon^2 \times 1.7^{14}]$$

$$\epsilon = \sqrt{\frac{100 - 1}{1.7^{14}}} = 0.2425$$

Now, using Eq. (13.13) we can determine the value to which $A_{\max}$ can be reduced as

$$A_{\max} = 20 \log\sqrt{1 + \epsilon^2}$$

$$A_{\max} = 20 \log\sqrt{1 + 0.2425^2}$$

$$= 0.25 \text{ dB}$$

**13.15** Using Eq. (13.15), we have

$$A(\omega_s) = 10 \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$$

For large $A(\omega_s)$, we can neglect the unity term in this expression to obtain

$$A(\omega_s) \simeq 10 \log [\epsilon^2(\omega_s/\omega_p)^{2N}]$$

Substituting $A(\omega_s) \geq A_{\min}$ we obtain

$$20 \log \epsilon + 20N \log(\omega_s/\omega_p) \geq A_{\min}$$

$$\Rightarrow N \geq \frac{A_{\min} - 20 \log \epsilon}{20 \log(\omega_s/\omega_p)} \qquad \text{Q.E.D.}$$

**13.16** For an $N$th-order Butterworth filter, we have from Eq. (13.11)

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}}}$$

At the 3-dB frequency $\omega_{3\,\text{dB}}$ we have:
$\epsilon^2 \left(\dfrac{\omega_{3\,\text{dB}}}{\omega_p}\right)^{2N} = 1 \Rightarrow \omega_{3\,\text{dB}} = \left(\dfrac{1}{\epsilon^2}\right)^{1/2N} \omega_p$. Thus, from Eq. (13.15) the attenuation at $\omega = 1.8\omega_{3\,\text{dB}}$ is:

$$A = 10 \log\left[1 + \epsilon^2 \left(\frac{1.8\omega_{3\,\text{dB}}}{\omega_p}\right)^{2N}\right]$$

$$= 10 \log\left[1 + \epsilon^2 \left(1.8\frac{1}{\epsilon^2}\right)^{1/2N}\right]^{2N}$$

$$= 10 \log(1 + 1.8^{2N})$$

For the case $N = 7$,

$$A = 10 \log(1 + 1.8^{14}) = 35.7 \text{ dB}$$

**13.17** $A_{\max} = 0.5$ dB, $N = 5$, $\omega_p = 10^3$ rad/s

Using Eq. (13.14), we obtain

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1} = \sqrt{10^{0.05} - 1}$$

$$= 0.3493$$

The natural modes can be determined by reference to Fig. 13.10(a):

$$\omega_0 = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N}$$

$$\omega_0 = 10^3 \times \left(\frac{1}{0.3493}\right)^{1/5}$$

$$= 1.234 \times 10^3 \text{ rad/s}$$

$$p_1,\ p_1^* = \omega_0\left[\sin\left(\frac{\pi}{10}\right) \pm j\cos\left(\frac{\pi}{10}\right)\right]$$

$$= \omega_0(-0.309 \pm j0.951)$$

$$= 1.234 \times 10^3(0.309 \pm j0.951)$$

$$p_2,\ p_2^* = \omega_0\left[-\sin\left(\frac{3\pi}{10}\right) \pm j\cos\left(\frac{3\pi}{10}\right)\right]$$

$$= 1.234 \times 10^3(-0.809 \pm j0.588)$$

$$p_3 = -\omega_0 = -1.234 \times 10^3$$

**13.18**

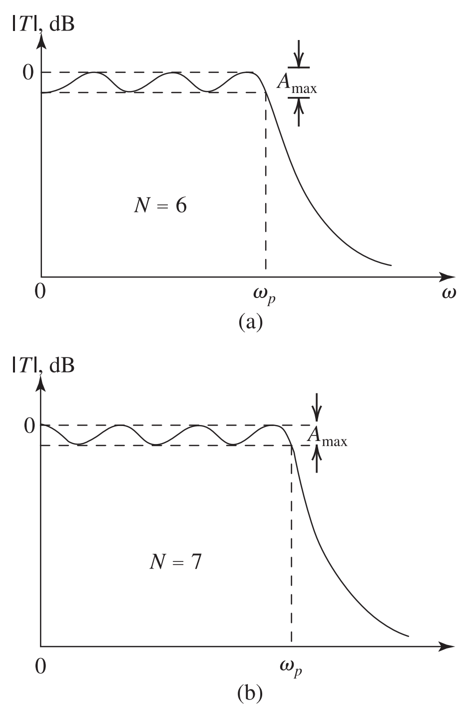



Figure 1

(a) See previous page: Fig. 1(a).

(b) See previous page: Fig. 1(b).

**13.19** $f_p = 10$ kHz, $A_{max} = 3$ dB,
$f_s = 20$ kHz, $A_{min} = 20$ dB

Using Eq. (13.14), we obtain

$\epsilon = \sqrt{10^{A_{max}/10} - 1}$

$= \sqrt{10^{3/10} - 1} = 1$

Using Eq. (13.15), we have

$A(\omega_s) = 10 \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$

Thus,

$A_{min} \geq 10 \log[1 + \epsilon^2(\omega_s/\omega_p)^{2N}]$

$20 \geq 10 \log\ (1 + 2^{2N})$

For $N = 3$,

$10 \log(1 + 2^6) = 18.1$ dB

For $N = 4$,

$10 \log(1 + 2^8) = 24.1$ dB

Thus,

$N = 4$

The poles can be determined by reference to Fig. 13.10(a):

$$\omega_0 = \omega_p\left(\frac{1}{1}\right)^{1/4} = \omega_p = 2\pi \times 10^4 \text{ rad/s}$$

$$p_1,\ p_1^* = \omega_0\left[-\sin\left(\frac{\pi}{8}\right) \pm j\cos\left(\frac{\pi}{8}\right)\right]$$

$$= 2\pi \times 10^4(-0.383 \pm j0.924)$$

$$p_2,\ p_2^* = \omega_0\left[-\sin\left(\frac{3\pi}{8}\right) \pm j\cos\left(\frac{3\pi}{8}\right)\right]$$

$$= 2\pi \times 10^4(-0.924 \pm j0.383)$$

Thus,

$$T(s) = \frac{\omega_0^4}{(s^2 + 0.765\omega_0 s + \omega_0^2)(s^2 + 1.848\omega_0 s + \omega_0^2)}$$

where $\omega_0 = 2\pi \times 10^4$ and where we have assumed the dc gain to be unity.

Using Eq. (13.11), we obtain

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_p}\right)^8}}$$

At $f = 30$ kHz $= 3\,f_p$, the attenuation is

$A = -20 \log|T| = 10 \log(1 + 3^8)$

$= 38.2$ dB

**13.20**

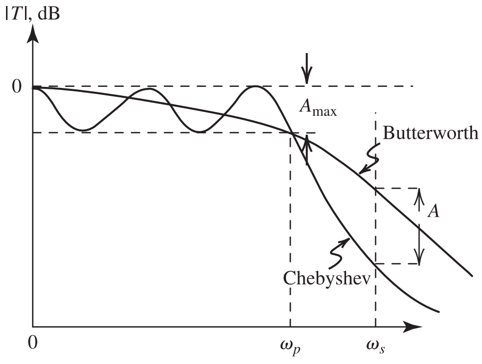



Figure 1

From Fig. 1 we see that at the stopband edge, $\omega_s$, the Chebyshev filter provides $A$(dB) greater attenuation than the Butterworth filter of the same order and having the same $A_{max}$.

**13.21** $N = 7,\ \omega_p = 1$ rad/s, $A_{max} = 0.5$ dB

Using Eq. (13.21), we obtain

$\epsilon = \sqrt{10^{A_{max}/10} - 1}$

$= \sqrt{10^{\,0.05} - 1} = 0.3439$

From Eq. (13.18), we have

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cos^2[N \cos^{-1}(\omega/\omega_p)]}}$$

$$= \frac{1}{\sqrt{1 + \epsilon^2 \cos^2(7 \cos^{-1}\omega)}} \qquad (1)$$

$|T(j\omega)|$ will be equal to unity at the values of $\omega$ that make

$$\cos(7 \cos^{-1}\omega) = 0 \qquad (2)$$

Since the cosine function is 0 for angles that are odd multiples of $\pi/2$, the solutions to Eq. (2) are

$$7 \cos^{-1}\omega_k = (2k + 1)\frac{\pi}{2}$$

where

$k = 0,\ 1,\ 2, ....$

Thus,

$$\omega_k = \cos\frac{(2k + 1)\pi}{14}$$

We now can compute the values of $\omega_k$ as

$$\omega_1 = \cos\frac{\pi}{14} = 0.975 \text{ rad/s}$$

$$\omega_2 = \cos\frac{3\pi}{14} = 0.782 \text{ rad/s}$$

$$\omega_3 = \cos\frac{5\pi}{14} = 0.434 \text{ rad/s}$$

$$\omega_4 = \cos\frac{7\pi}{14} = 0 \text{ rad/s}$$

Next, we determine the passband frequencies at which maximum deviation from 0 dB occurs. From Eq. (1) we see that these are the values of $\omega$ that make

$$\cos^2(7\cos^{-1}\omega) = 1 \tag{3}$$

Since the magnitude of the cosine function is unity for angles that are multiples of $\pi$, the solutions to Eq. (3) are given by

$$7\cos^{-1}\omega_m = m\pi$$

or

$$\omega_m = \cos\frac{m\pi}{7}$$

where

$$m = 0,\ 1,\ 2, \ldots.$$

We now can compute the values of $\omega_m$ as

$$\omega_0 = \cos 0 = 1 \text{ rad/s}$$

$$\omega_1 = \cos\frac{\pi}{7} = 0.901 \text{ rad/s}$$

$$\omega_2 = \cos\frac{2\pi}{7} = 0.623 \text{ rad/s}$$

$$\omega_3 = \cos\frac{3\pi}{7} = 0.223 \text{ rad/s}$$

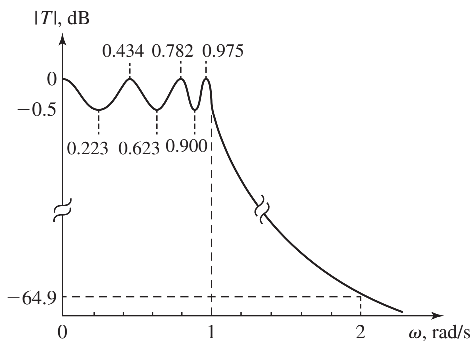



Figure 1

Figure 1 shows a sketch of $|T|$ for this 7th-order Chebyshev filter, with the passband maxima and minima identified. Note that the frequencies of the maxima and minima do *not* depend on the value of $A_{\max}$.

To determine the attenuation at the stopband frequency $\omega = 2$ rad/s, we utilize the expression in Eq. (13.22), thus

$$A(2) = 10\log[1 + 0.3493^2\cosh^2(7\cosh^{-1}2)]$$
$$= 64.9 \text{ dB}$$

This point also is indicated on the sketch in Fig. 1.

Finally, we note that since this is a 7th-order all-pole filter, for $s \to \infty$, $T$ will be proportional to $1/s^7$; that is, $|T|$ will be proportional to $1/\omega^7$, thus the asymptotic response will be $7 \times 6 = 42$ dB/octave.

**13.22**

$$A_{\max} = 1 \text{ dB} \Rightarrow \epsilon = \sqrt{10^{1/10} - 1} = 0.5088$$

$$f_p = 3.4 \text{ kHz},\ f_s = 4 \text{ kHz} \Rightarrow \frac{f_s}{f_p} = 1.176$$

$$A_{\min} = 35 \text{ dB}$$

(a) To obtain the required order $N$, we use Eq. (13.22),

$$A(\omega_s) = 10\log[1 + \epsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$$

Thus,

$$10\log[1 + 0.5088^2\cosh^2(N\cosh^{-1}1.176)] \geq 35$$

We attempt various values for $N$ as follows:

| $N$ | $A(\omega_s)$ |
|---|---|
| 8 | 28.8 dB |
| 9 | 33.9 dB |
| 10 | 39.0 dB |

Use $N = 10$.

Excess attenuation $= 39 - 35 = 4$ dB

(b) The poles can be determined using Eq. (13.23), namely

$$p_k/\omega_p = -\sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$
$$+j\cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$
$$k = 1,\ 2, \ldots, N$$

First we determine

$$\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$
$$= \sinh\left(\frac{1}{10}\sinh^{-1}\frac{1}{0.5088}\right) = 0.1433$$

and

$$\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) = 1.0102$$

Thus,

$$p_1/\omega_p = -0.1433\sin\left(\frac{\pi}{20}\right) + j1.0102\cos\left(\frac{\pi}{20}\right)$$
$$= -0.0224 + j0.9978$$

$$p_2/\omega_p = -0.1433\sin\left(\frac{3\pi}{20}\right) + j1.0102\cos\left(\frac{3\pi}{20}\right)$$

$$= -0.0651 + j0.9001$$

$$p_3/\omega_p = -0.1433 \sin\left(\frac{5\pi}{20}\right) + j1.0102 \cos\left(\frac{5\pi}{20}\right)$$

$$= -0.1013 + j0.7143$$

$$p_4/\omega_p = -0.1433 \sin\left(\frac{7\pi}{20}\right) + j1.0102 \cos\left(\frac{7\pi}{20}\right)$$

$$= -0.1277 + j0.4586$$

$$p_5/\omega_p = -0.1433 \sin\left(\frac{9\pi}{20}\right) + j1.0102 \cos\left(\frac{9\pi}{20}\right)$$

$$= -0.1415 + j0.1580$$

$$p_6 = p_5^*,\ p_7 = p_4^*,\ p_8 = p_3^*,\ p_9 = p_2^*,\ p_{10} = p_1^*$$

Each pair of complex conjugate poles,

$$p_k,\ p_k^* = \omega_p(-\Sigma_k \pm j\ \Omega_k)$$

gives rise to a quadratic factor in the denominator of $T(s)$ given by

$$s^2 + s\omega_p(2\Sigma_k) + \omega_p^2(\Sigma_k^2 + \Omega_k^2)$$

where

$$\omega_p = 2\pi \times 3.4 \times 10^3 \text{ rad/s}$$

Thus, we obtain for the five pole pairs:

$$p_1,\ p_1^*\colon (s^2 + s\ 0.0448\omega_p + 0.9961\omega_p^2)$$

$$p_2,\ p_2^*\colon (s^2 + s\ 0.1302\omega_p + 0.8144\omega_p^2)$$

$$p_3,\ p_3^*\colon (s^2 + s\ 0.2026\omega_p + 0.5205\omega_p^2)$$

$$p_4,\ p_4^*\colon (s^2 + s\ 0.2554\omega_p + 0.2266\omega_p^2)$$

$$p_5,\ p_5^*\colon (s^2 + s\ 0.2830\omega_p + 0.0450\omega_p^2)$$

The transfer function $T(s)$ can now be written as

$$T(s) = \frac{B}{\text{Product of five quadratic terms}}$$

The value of $B$ determines the required dc gain, specifically

$$\text{DC gain} = \frac{B}{\omega_p^{10} \times 0.9961 \times 0.8144 \times 0.5205 \times 0.2266 \times 0.0450}$$

$$= \frac{B}{4.31 \times 10^{-3}\omega_p^{10}}$$

$$\text{For DC gain} = \frac{1}{\sqrt{1+\epsilon^2}} = \frac{1}{\sqrt{1+0.5088^2}}$$

$$= 0.891$$

we select

$$B = 4.31 \times 10^{-3} \times \omega_p^{10} \times 0.891$$

$$= 3.84 \times 10^{-3} \times (2\pi \times 3.4 \times 10^3)^{10}$$

$$= 7.60 \times 10^{40}$$

**13.23** Refer to Fig. 13.13 (row a).

Input resistance $= R_1$

Thus, to obtain an input resistance of 12 kΩ, we select

$$R_1 = 12 \text{ k}\Omega$$

$$|\text{DC gain}| = 10 = \frac{R_2}{R_1}$$

$$\Rightarrow R_2 = 10\ R_1 = 120 \text{ k}\Omega$$

$$CR_2 = \frac{1}{\omega_0} = \frac{1}{2\pi \times 5 \times 10^3}$$

$$\Rightarrow C = \frac{1}{2\pi \times 5 \times 10^3 \times 120 \times 10^3} = 265 \text{ pF}$$

**13.24** Refer to Fig. 13.13 (row b).

H.F. Input resistance $= R_1$

Thus,

$$R_1 = 120 \text{ k}\Omega$$

$$CR_1 = \frac{1}{\omega_0} = \frac{1}{2\pi \times 200}$$

$$\Rightarrow C = \frac{1}{2\pi \times 200 \times 120 \times 10^3}$$

$$= 6.63 \text{ nF}$$

$$\text{High-frequency gain} = -\frac{R_2}{R_1} = -1$$

$$\Rightarrow R_2 = R_1 = 120 \text{ k}\Omega$$

**13.25** Refer to the op-amp-RC circuit in Fig. 13.13 (row c). Assuming an ideal op amp, we have

$$T(s) = \frac{V_o}{V_i} = -\frac{Z_2(s)}{Z_1(s)}$$

Since both $Z_1$ and $Z_2$ have a parallel structure, it is far more convenient to work in terms of $Y_1(s)$ and $Y_2(s)$, thus

$$T(s) = -\frac{Y_1(s)}{Y_2(s)}$$

$$= -\frac{\dfrac{1}{R_1} + s\ C_1}{\dfrac{1}{R_2} + s\ C_2}$$

$$= -\left(\frac{C_1}{C_2}\right) \frac{s + \dfrac{1}{C_1R_1}}{s + \dfrac{1}{C_2R_2}}$$

Thus,

$$\omega_Z = \frac{1}{C_1R_1}$$

$$\omega_P = \frac{1}{C_2R_2}$$

$$\text{DC gain} = T(0) = -\frac{R_2}{R_1}$$

$$\text{HF gain} = T(\infty) = -\frac{C_1}{C_2}$$

**13.26** We use the op-amp-RC circuit of Fig. 13.13(c).

Low-frequency input resistance $= R_1$

Thus,

$R_1 = 10\ \text{k}\Omega$

$$\text{DC gain} = -\frac{R_2}{R_1} = -1$$

$\Rightarrow R_2 = R_1 = 10\ \text{k}\Omega$

$$f_Z = \frac{\omega_Z}{2\pi} = \frac{1}{2\pi C_1R_1}$$

$$100 = \frac{1}{2\pi C_1 \times 10 \times 10^3}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 10^6} = 0.16\ \mu\text{F}$$

$$f_P = f_0 = \frac{1}{2\pi C_2R_2}$$

$$10 \times 10^3 = \frac{1}{2\pi C_2 \times 10 \times 10^3}$$

$$C_2 = \frac{1}{2\pi \times 10^8} = 1.6\ \text{nF}$$

Figure 1 shows a sketch of the magnitude of the transfer function. Note that the magnitude of the high-frequency gain is $C_1/C_2 = 100$ or 40 dB.

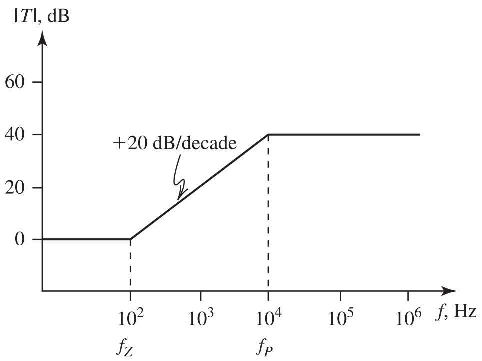



Figure 1

**13.27** Figure 1 (below) shows the bandpass filter realized as the cascade of a first-order low-pass filter and a first-order high-pass filter. The component values are determined as follows:

$R_{\text{in}} = R_1$

To make $R_{\text{in}}$ as large as possible while satisfying the constraint that no resistance is larger than 100 kΩ, we select

$R_1 = 100\ \text{k}\Omega$

The low-frequency gain of the low-pass circuit is $(-R_2/R_1)$. With $R_1 = 100\ \text{k}\Omega$, the maximum gain obtained is unity and is achieved by selecting

$R_2 = 100\ \text{k}\Omega$

This implies that the required gain of 12 dB or 4 V/V must be all realized in the high-pass circuit.

This figure belongs to Problem 13.27.

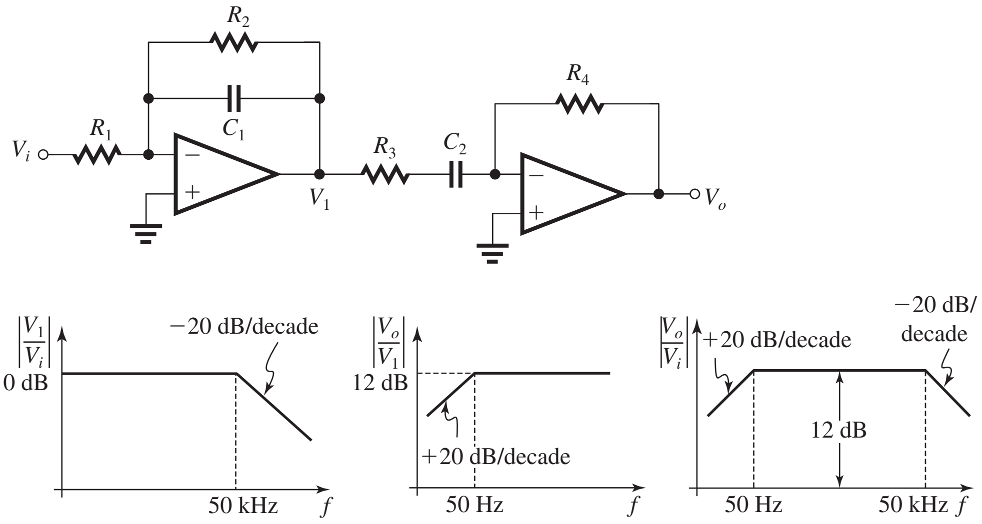



Figure 1

The upper 3-dB frequency of the bandpass filter is the 3-dB frequency of the low-pass circuit, that is,

$$50 \times 10^3 = \frac{1}{2\pi C_1 R_2}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 50 \times 10^3 \times 100 \times 10^3}$$

$$= 31.8 \text{ pF}$$

Next, we consider the high-pass circuit. The high-frequency ($f \gg 50$ Hz) gain of this circuit is $(-R_4/R_3)$. To obtain a gain of $-4$ V/V, we select

$$R_4 = 100 \text{ k}\Omega$$

$$R_3 = \frac{100}{4} = 25 \text{ k}\Omega$$

The lower 3-dB frequency of the bandpass filter (50 Hz) is the 3-dB frequency of the high-pass circuit, thus

$$50 = \frac{1}{2\pi C_2 R_3}$$

$$\Rightarrow C_2 = \frac{1}{2\pi \times 50 \times 25 \times 10^3}$$

$$= 0.127 \text{ } \mu\text{F}$$

**13.28**

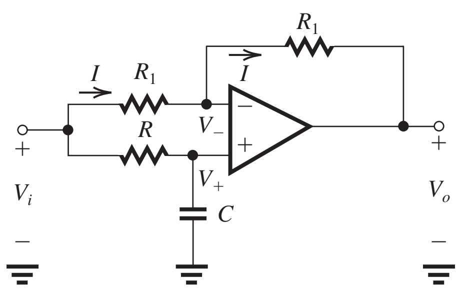



Figure 1

Refer to Fig. 1.

$$V_+ = V_i \frac{1/sC}{R + (1/sC)}$$

$$= V_i \frac{1}{sCR + 1}$$

$$V_- = V_+ = V_i \frac{1}{sCR + 1}$$

$$I = \frac{V_i - V_-}{R_1} = \frac{V_i}{R_1}\left(1 - \frac{1}{sCR + 1}\right)$$

$$= \frac{V_i}{R_1} \frac{sCR}{sCR + 1}$$

$$V_o = V_- - IR_1$$

$$= \frac{V_i}{sCR + 1} - \frac{sRCV_i}{sCR + 1}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{sCR - 1}{sCR + 1}$$

or

$$T(s) = \frac{V_o}{V_i} = -\frac{s - 1/CR}{s + 1/CR} = -\frac{s - \omega_0}{s + \omega_0}$$

where

$$\omega_0 = \frac{1}{CR}$$

$$T(j\omega) = -\frac{j\omega - \omega_0}{j\omega + \omega_0}$$

$$= \frac{\omega_0 - j\omega}{\omega_0 + j\omega}$$

$$= \frac{1 - j(\omega/\omega_0)}{1 + j(\omega/\omega_0)}$$

$$|T(j\omega)| = \sqrt{\frac{1 + (\omega/\omega_0)^2}{1 + (\omega/\omega_0)^2}} = 1$$

$$\phi(\omega) = -2 \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\Rightarrow \frac{\omega}{\omega_0} = \tan\left[-\frac{1}{2}\phi(\omega)\right] \qquad (1)$$

Thus, for a given phase shift $\phi$, we can use Eq. (1) to determine $(\omega/\omega_0)$. For $\omega = 5 \times 10^3$ rad/s, we can then determine the required value of $\omega_0$. Finally, for $C = 10$ nF, the required value for $R$ can be found from

$$R = \frac{1}{\omega_0 \times 10 \times 10^{-9}} = \frac{10^8}{\omega_0}$$

The results obtained are as follows:

| $\boldsymbol{\phi}$ | −30° | −60° | −90° | −120° | −150° |
|---|---|---|---|---|---|
| $\boldsymbol{\omega/\omega_0}$ | 0.268 | 0.577 | 1 | 1.732 | 3.732 |
| $\boldsymbol{\omega_0}$ **(krad/s)** | 18.66 | 8.66 | 5 | 2.89 | 1.34 |
| $\boldsymbol{R}$ **(kΩ)** | 5.36 | 11.55 | 20 | 34.60 | 74.63 |

**13.29**

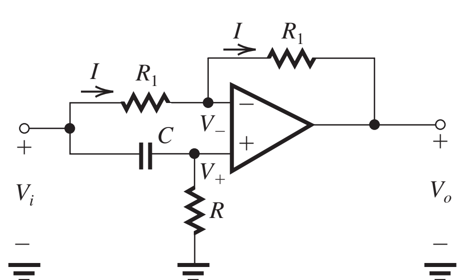



Figure 1

Figure 1 (see previous page) shows the circuit of Fig. 13.14 with $R$ and $C$ interchanged. To determine the transfer function $T(s) = V_o(s)/V_i(s)$, we analyze the circuit as follows:

$$V_+ = V_i \frac{R}{R + \frac{1}{sC}}$$

$$= V_i \frac{s}{s + \frac{1}{CR}}$$

$$V_- = V_+ = V_i \frac{s}{s + \frac{1}{CR}}$$

$$I = \frac{V_i - V_-}{R_1} = \frac{V_i}{R_1}\left(1 - \frac{s}{s + \frac{1}{CR}}\right)$$

$$= \frac{V_i}{R_1} \frac{1/CR}{s + \frac{1}{CR}}$$

$$V_o = V_- - IR_1$$

$$= V_i \left[\frac{s}{s + \frac{1}{CR}} - \frac{1/CR}{s + \frac{1}{CR}}\right]$$

Thus,

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{s - (1/CR)}{s + (1/CR)}$$

$$= \frac{s - \omega_0}{s + \omega_0}$$

where

$$\omega_0 = 1/CR$$

$$T(j\omega) = \frac{-\omega_0 + j\omega}{\omega_0 + j\omega}$$

$$\phi(\omega) = \phi_N(\omega) - \phi_D(\omega)$$

where $\phi_N$ is the phase angle of the numerator and $\phi_D$ is the phase angle of the denominator.

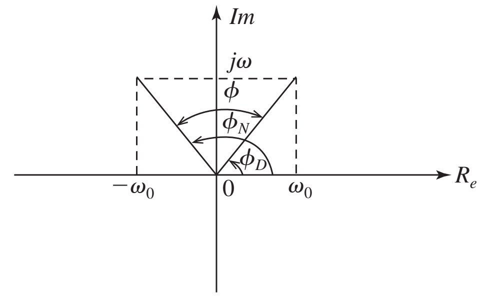


Figure 2

A graphical construction showing $\phi_N$ and $\phi_D$ and their difference $\phi$ is depicted in Fig. 2 above.

Observe that the difference, $\phi$, is a positive angle whose value ranges from 180° (at $\omega = 0$) to 0° (at $\omega = \infty$). The two end points should also be obvious from $T(j\omega)$ which is $-1$ at $\omega = 0$ and $+1$ at $\omega = 0$.

**13.30** Figure 1 shows a circuit composed of the cascade connection of two all-pass circuits of the type shown in Fig. 13.14. We require that each circuit provide $-120°$ phase shift at $\omega = 2\pi \times 60$ rad/s. From the data in Fig. 13.14(a),

$$\phi(\omega) = -2\tan^{-1}(\omega CR)$$

Thus,

$$-120° = -2\tan^{-1}(\omega CR)$$

$$\omega CR = \tan 60°$$

$$2\pi \times 60 \times 1 \times 10^{-6} \times R = \frac{\sqrt{3}}{2}$$

$$R = \frac{\sqrt{3}}{2\pi \times 60 \times 10^{-6} \times 2} = 2.3\ \text{k}\Omega$$

The value of $R_1$ can be selected arbitrarily, say

$$R_1 = 10\ \text{k}\Omega$$

This figure belongs to Problem 13.30.

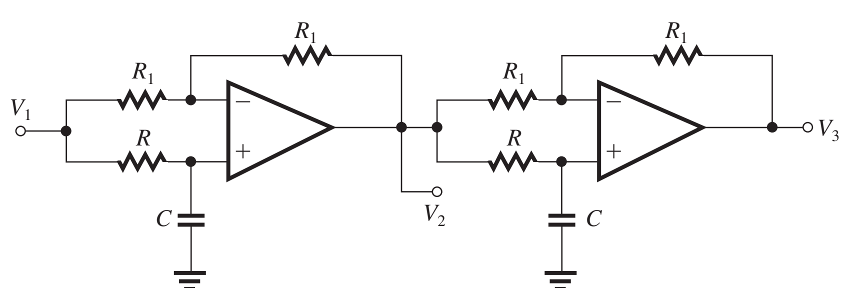


Figure 1

**13.31** Refer to Fig. 13.16(a).

$$T(s) = \frac{\omega_0^2}{s^2 + s\,\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{10^8}{s^2 + 5000\,s + 10^8}$$

$$\omega_{\max} = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$$

$$= 10^4\sqrt{1 - \frac{1}{2 \times 4}} = 9354 \text{ rad/s}$$

Since the dc gain is unity, we have

$$|a_o/\omega_0^2| = 1$$

$$|T_{\max}| = \frac{Q}{\sqrt{1 - \dfrac{1}{4Q^2}}} = \frac{2}{\sqrt{1 - \dfrac{1}{16}}} = 2.066$$

**13.32** There are many possibilities, but only two are optimal. The first is the Butterworth filter which exhibits maximum flatness of $|T|$ at $\omega = 0$. The second is the Chebyshev for which $|T|$ exhibits equiripple response in the passband.

See Figure 1 below. Figure 1(a) shows the second-order Butterworth filter that just meets the given passband specifications. Figure 1(b) shows the second-order Chebyshev filter that just meets the given passband specifications. Note that no stopband specifications were given (otherwise the problem would be overspecified); we are simply asked to calculate the attenuation at $\omega_s = 2\omega_p = 2$ rad/s.

For both filters, since $A_{\max} = 3$ dB, we have

$$\epsilon = 1$$

(a) Butterworth filter:

For a second-order Butterworth, we have $Q = 1/\sqrt{2} = 0.707$ and $\omega_0 = \omega_p = 1$ rad/s. The dc gain is unity. Thus,

$$T(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \omega^{2N}}}$$

Thus,

$$|T(j2)| = \frac{1}{\sqrt{1 + 2^4}}$$

$$A_{\min} = A(\omega_s) = 20\log_{10}\sqrt{17} = 12.3 \text{ dB}$$

(b) Chebyshev filter:

For a second-order Chebyshev filter, the poles are given by Eq. (13.23), which for $N = 2$, $\omega_p = 1$, and $\epsilon = 1$ yields

$$p_1,\ p_1^* = -\sin\left(\frac{\pi}{4}\right)\ \sinh\left(\frac{1}{2}\sinh^{-1}1\right)$$

$$\pm j\cos\left(\frac{\pi}{4}\right)\ \cosh\left(\frac{1}{2}\sinh^{-1}1\right)$$

$$= -0.3218 \pm j0.7769$$

Thus, for this pair of complex-conjugate poles, we have

$$\frac{\omega_0}{Q} = 2 \times 0.3218 = 0.6436$$

and

$$\omega_0^2 = 0.3218^2 + 0.7769^2 = 0.7071$$

Thus,

$$T(s) = \frac{K}{s^2 + 0.6436\,s + 0.7071}$$

From Fig. 1(b) we see that the dc gain is $1/\sqrt{1 + \epsilon^2} = 0.7071$

Thus,

$$K = 0.7071 \times 0.707 = 0.5$$

This figure belongs to Problem 13.32.

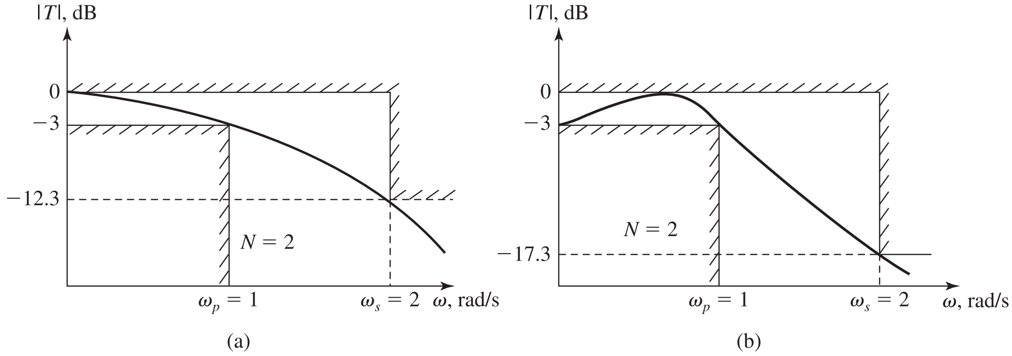



Figure 1

and

$$T(s) = \frac{0.5}{s^2 + 0.6436\,s + 0.7071}$$

At $s = j2$, we have

$$|T(j2)| = \frac{0.5}{\sqrt{(0.7071 - 4)^2 + (0.6436 \times 2)^2}}$$

$= 0.1414$

$A_{\min} = A(2) = -20 \log 0.1414$

$= 17.0$ dB

**13.33** Refer to Fig. 13.16(b). For a maximally flat response, we have

$Q = 1/\sqrt{2}$

and

$\omega_{3dB} = \omega_0$

Thus,

$\omega_0 = 1$ rad/s

If the high-frequency gain is unity, then we have

$a_2 = 1$

and

$$T(s) = \frac{s^2}{s^2 + \sqrt{2}\,s + 1}$$

The two zeros are at $s = 0$. The poles are complex conjugate and given by

$$s = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{\left(1 - \frac{1}{4Q^2}\right)}$$

$$= -\frac{1}{2 \times \frac{1}{\sqrt{2}}} \pm j\sqrt{\left(1 - \frac{1}{4 \times \frac{1}{2}}\right)}$$

$$= -\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}$$

$= -0.707 \pm j0.707$

**13.34** $f_0 = 10$ kHz $\Rightarrow \omega_0 = 2\pi \times 10^4$ rad/s

$BW = \dfrac{f_0}{Q} = 500$ Hz $\Rightarrow Q = 20$

Center-frequency gain $= 10$

Thus,

$$T(s) = \frac{10\left(\dfrac{\omega_0}{Q}\right)s}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$$

$$= \frac{\pi \times 10^4 s}{s^2 + s\pi \times 10^3 + (2\pi \times 10^4)^2}$$

Poles are at $\dfrac{-\omega_0}{2Q} \pm j\omega_0\sqrt{1 - \dfrac{1}{4Q^2}}$

$$= -\frac{\pi}{2} \times 10^3 \pm j2\pi \times 10^4\sqrt{1 - \frac{1}{4 \times 20^2}}$$

$$= \frac{\pi}{2} \times 10^3[-1 \pm j40 \times 0.9997]$$

$= 1.57 \times 10^3(-1 \pm j39.988)$

Zeros are at $s = 0$ and $s = \infty$.

**13.35** (a) A second-order bandpass filter with a center-frequency gain of unity (arbitrary) has the transfer function

$$T(s) = \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

Thus,

$$|T(j\omega)| = \frac{\omega\omega_0/Q}{\sqrt{(\omega_0^2 - \omega^2) + (\omega\omega_0/Q)^2}}$$

$$= 1/\sqrt{1 + Q^2\,\frac{(\omega_0^2 - \omega^2)^2}{\omega^2\omega_0^2}} \qquad (1)$$

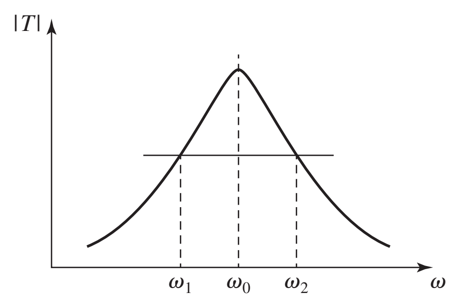



Figure 1

From Fig. 1 we see that at each $|T|$ (below the peak value) there are two frequencies $\omega_1 < \omega_0$ and $\omega_2 > \omega_0$ with the same $|T|$. The relationship between $\omega_1$ and $\omega_2$ can be determined by considering the second term in the denominator of Eq. (1), as follows:

$$\frac{\omega_0^2 - \omega_1^2}{\omega_1\omega_0} = \frac{\omega_2^2 - \omega_0^2}{\omega_2\omega_0}$$

Cross multiplying and collecting terms results in

$\omega_1\omega_2 = \omega_0^2$ Q.E.D.

(b) Since the two edges of the passband must be geometrically symmetric around $\omega_0$, we have

$\omega_0 = \sqrt{\omega_{P1}\omega_{P2}}$

$= \sqrt{8100 \times 10{,}000}$

$= 9000$ rad/s

This figure belongs to Problem 13.35, part (b).

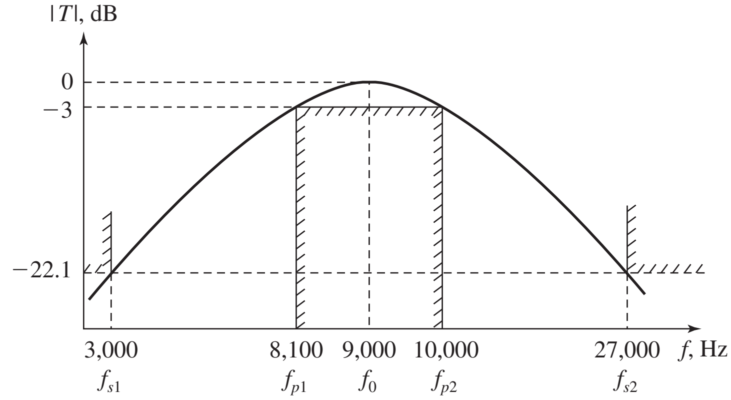



Figure 2

The $Q$ factor can now be found from

$$3\text{-dB } BW = \frac{\omega_0}{Q}$$

$$10{,}000 - 8100 = \frac{9000}{Q}$$

$$\Rightarrow Q = \frac{9000}{1900} = 4.74$$

The geometric symmetry of $|T|$ enables us to find $\omega_{s2}$ from

$$\omega_{s1}\omega_{s2} = \omega_0^2$$

$$\Rightarrow \omega_{s2} = \frac{(9000)^2}{3000} = 27{,}000 \text{ rad/s}$$

Using Eq. (1), we obtain

$$A_{\min} = A(\omega_{s1}) = 10 \log\left[1 + Q^2 \frac{(\omega_0^2 - \omega_{s1}^2)^2}{\omega_{s1}^2\ \omega_0^2}\right]$$

$$= 10 \log\left[1 + 4.74^2 \frac{(9000^2 - 3000^2)^2}{3000^2 \times 9000^2}\right]$$

$$= 22.1 \text{ dB}$$

Figure 2 shows a sketch of $|T|$.

**13.36** An increase in $Q_p$ increases the magnitude of the peak. On the other hand, an increase in $Q_z$ increases the magnitude of the dip. Thus, the results shown in Fig. 1 are obtained.

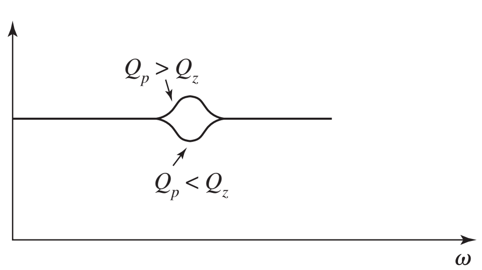



Figure 1

**13.37** Since near the poles, $|T|$ exhibits a peak, and near the zeros it exhibits a dip, the results shown in Fig. 1 are obtained.

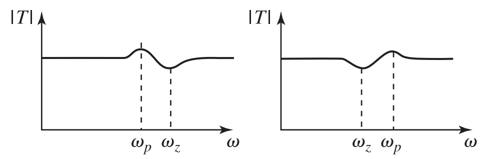



Figure 1

**13.38** Refer to Fig. 13.17(c). Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_{RC}}{Z_{RC} + sL}$$

$$= \frac{1}{1 + sLY_{RC}}$$

$$= \frac{1}{1 + sL\left(sC + \dfrac{1}{R}\right)}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + s\dfrac{L}{R} + 1}$$

$$= \frac{1/LC}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{LC}}$$

Thus,

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{[which is Eq. (13.34)]}$$

$$\frac{\omega_0}{Q} = \frac{1}{CR} \Rightarrow Q = \omega_0 CR$$

[which is Eq. (13.35)] Q.E.D.

**13.39** $Q = \omega_0 CR$

$$5 = 10^5 \times C \times 10 \times 10^3$$

$$\Rightarrow C = 5 \text{ nF}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow L = 1/\omega_0^2 C$$

$$L = \frac{1}{10^{10} \times 5 \times 10^{-9}} = 20 \text{ mH}$$

**13.40** (a) If $L$ became $1.01L$, we obtain

$$\omega_0 = \frac{1}{\sqrt{1.01LC}} \simeq \frac{1}{1.005\sqrt{LC}} \simeq \frac{0.995}{\sqrt{LC}}$$

Thus, $\omega_0$ decreases by 0.5%.

(b) If $C$ increases by 1%, similar analysis as in (a) results in $\omega_0$ decreasing by 0.5%.

(c) $\omega_0$ is independent of the value of $R \Rightarrow$ no change.

**13.41** (a) As $\omega$ reaches 0 (dc), we get

$$\frac{V_o}{V_i} = \frac{1/sC_2}{\dfrac{1}{sC_1} + \dfrac{1}{sC_2}} = \frac{C_1}{C_1 + C_2}$$

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

As $\omega$ reaches $\infty$,

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

No transmission zero are introduced.

(b) $$\frac{V_o}{V_i} = \frac{Z_{R,C_2}}{Z_{R,C_2} + \dfrac{1}{sC_1}}$$

$$= \frac{1}{1 + \dfrac{1}{sC_1} Y_{R,C_2}}$$

$$= \frac{1}{1 + \dfrac{1}{sC_1}\left(\dfrac{1}{R} + sC_2\right)}$$

$$= \frac{1}{\left(1 + \dfrac{C_2}{C_1}\right) + \dfrac{1}{sC_1 R}}$$

$$= \frac{s}{s\left(1 + \dfrac{C_1}{C_2}\right) + \dfrac{1}{C_1 R}}$$

$$\frac{V_o}{V_i}(0) = 0$$

$$\frac{V_o}{V_i}(\infty) = \frac{1}{1 + \dfrac{C_2}{C_1}} = \frac{C_1}{C_1 + C_2}$$

There is a transmission zero at $s = 0$, due to $C_1$.

(c) $$\frac{V_o}{V_i} = \frac{sL_2}{s(L_1 + L_2)} = \frac{L_2}{L_1 + L_2}$$

$$\frac{V_o}{V_i}(0) = \frac{L_2}{L_1 + L_2}$$

$$\frac{V_o}{V_i}(\infty) = \frac{L_2}{L_1 + L_2}$$

No transmission zeros.

(d) $$\frac{V_o}{V_i} = \frac{sL_2}{s(L_1 + L_2) + R}$$

$$\frac{V_o}{V_i}(0) = 0$$

$$\frac{V_o}{V_i}(\infty) = \frac{L_2}{L_1 + L_2}$$

There is a transmission zero at 0 (dc) due to $L_2$.

**13.42** Refer to the circuit in Fig. 13.18(b).

$$Q = \omega_0 CR$$

$$\frac{1}{\sqrt{2}} = 10^6 \times 1 \times 10^{-9} \times R$$

$$\Rightarrow R = 707\ \Omega$$

$$\omega_0^2 LC = 1$$

$$10^{12} \times L \times 1 \times 10^{-9} = 1$$

$$\Rightarrow L = 1 \text{ mH}$$

**13.43** Refer to Fig. 13.18(c). Using the voltage divider rule, we obtain

$$\frac{V_o}{V_i} = \frac{Z_{LR}}{Z_{LR} + \dfrac{1}{sC}}$$

$$= \frac{1}{1 + \dfrac{1}{sC} Y_{LR}}$$

$$= \frac{1}{1 + \dfrac{1}{sC}\left(\dfrac{1}{R} + \dfrac{1}{sL}\right)}$$

$$= \frac{1}{1 + \dfrac{1}{sCR} + \dfrac{1}{s^2 LC}}$$

$$= \frac{s^2}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{LC}}$$

**13.44** Using superposition, we find the resulting $T(s)$ in each of the three cases and then sum the results. Thus (see Figure 1 on the next page):

(a) When $x$ is lifted off ground and connected to $V_x$, the circuit becomes as shown in Fig. 1(a), and the transfer function becomes that of a low-pass

This figure belongs to Problem 13.44.

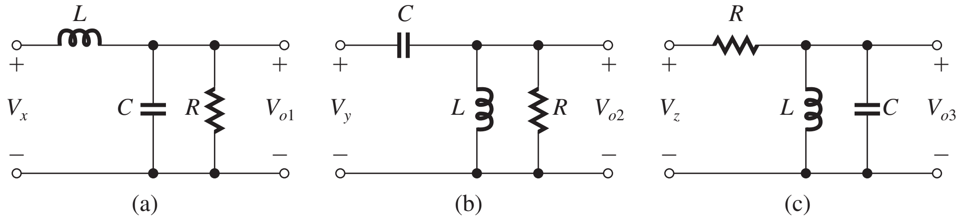



Figure 1

filter with $\omega_0 = 1/\sqrt{LC}$, $Q = \omega_0\, CR$, and dc gain of unity, thus

$$\frac{V_{o1}}{V_x} = \frac{\omega_0^2}{s^2 + s\,\dfrac{\omega_0}{Q} + \omega_0^2} \qquad (1)$$

(b) With $y$ lifted off ground and connected to $V_y$, the resulting circuit [shown in Fig. 1(b)], is that of a high-pass filter with $\omega_0 = 1/\sqrt{LC}$, $Q = \omega_0 CR$, and a high-frequency gain of unity, thus

$$\frac{V_{o2}}{V_y} = \frac{s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} \qquad (2)$$

(c) With $z$ lifted off ground and connected to $V_z$, the resulting circuit shown in Fig. 1(c) is that of a bandpass filter with $\omega_0 = 1/\sqrt{LC}$, $Q = \omega_0\, CR$, and a center-frequency gain of unity, thus

$$\frac{V_{o3}}{V_c} = \frac{s\dfrac{\omega_0}{Q}}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2} \qquad (3)$$

Now summing (1), (2), and (3) gives

$$V_o = \frac{s^2 V_y + s\left(\dfrac{\omega_0}{Q}\right) V_z + \omega_0^2 V_x}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$$

**13.45** $L = C_4 R_1 R_3 R_5 / R_2$

Selecting $R_1 = R_2 = R_3 = R_5 = R = 10\ \text{k}\Omega$, we have

$L = C_4 R^2$

$= C_4 \times 10^8$

Thus,

$C_4 = L \times 10^{-8}$

(a) For $L = 15$ H, we have

$C_4 = 15 \times 10^{-8}\ \text{F} = 0.15\ \mu\text{F}$

(b) For $L = 1.5$ H, we have

$C_4 = 0.015\ \mu\text{F} = 15\ \text{nF}$

(c) For $L = 0.15$ H, we have

$C_4 = 1.5\ \text{nF}$

**13.46** (a) Figure 1 on the next page shows the analysis. It is based on assuming ideal op amps that exhibit virtual short circuits between their input terminals, and draw zero currents into their input terminals. Observe that:

(1) The circuit is fed at port-1 with a voltage $V_1$. The virtual short circuit between the input terminals of each of $A_2$ and $A_1$ cause the voltage at port-2 to be

$$V_2 = V_1 \qquad (1)$$

(2) With port-2 terminated in an impedance $Z_5$, the current that flows out of port-2 is

$$I_2 = \frac{V_2}{Z_5}$$

(3) Following the analysis indicated, we find the input current into port-1 as

$$I_1 = \frac{Z_2 Z_4}{Z_1 Z_3 Z_5}$$

This current could have been written as

$$I_1 = V_1 \frac{Z_2 Z_4}{Z_1 Z_3} I_2 \qquad (2)$$

(4) Equations (1) and (2) describe the operation of the circuit: It propagates the voltage applied to port-1, $V_1$, and to port-2, $V_2 = V_1$; and whatever current is drawn out of port-2 is multiplied by the function $(Z_2 Z_4 / Z_1 Z_3)$ and appears at port-1.

(5) The result of the two actions in (4) is that the input impedance looking into port-1 becomes

$$Z_{11} \equiv \frac{V_1}{I_1} = \frac{V_2}{I_2 (Z_2 Z_4 / Z_1 Z_3)}$$

$$= \left(\frac{Z_1 Z_3}{Z_2 Z_4}\right)\left(\frac{V_2}{I_2}\right)$$

or

$$Z_{11} = \left(\frac{Z_1 Z_3}{Z_2 Z_4}\right) Z_5$$

(b) If port-1 is terminated in an impedance $Z_6$, the input impedance looking into port-2 can be obtained by invoking the symmetry of the circuit, thus

$$Z_{22} = \left(\frac{Z_2 Z_4}{Z_1 Z_3}\right) Z_6$$

This figure belongs to Problem 13.46.

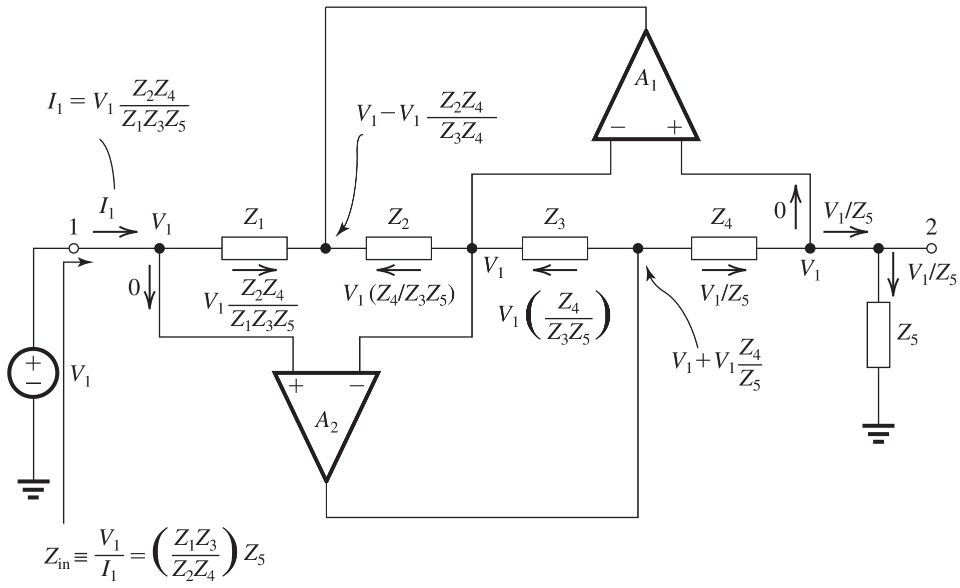


Figure 1

Thus, the circuit behaves as an impedance transformer with the transformation ratio from port-2 to port-1 being $\left(\frac{Z_1 Z_3}{Z_2 Z_4}\right)$ and from port-1 to port-2 being $\left(\frac{Z_2 Z_4}{Z_1 Z_3}\right)$.

Since $Z_1$, $Z_2$, $Z_3$, and $Z_4$ can be arbitrary functions of $s$, the transformation ratio can be an arbitrary function of $s$. Thus the circuit can be used as an impedance converter. For instance, the particular selection of impedances in Fig. 13.20(a) results in the transformation ratio from port-2 to port-1 being $(sC_4R_1R_3/R_2)$. As a result, the circuit in Fig. 13.22(a) converts a resistance $R_5$ into an inductance $L = C_4R_1R_3R_5/R_2$. Other conversion functions are possible and the circuit is known as a Generalized Impedance Converter or GIC.

**13.47** Figure 1 below shows the suggested circuit together with the analysis. The input impedance looking into port-2 is

$$Z_{22}(s) \equiv \frac{V_2}{I_2} = \frac{R_2}{s^2 C_4 C_6 R_1 R_3}$$

For $s = j\omega$ we have

$$Z_{22}(j\omega) = -\frac{R_2}{\omega^2 C_4 C_6 R_1 R_2}$$

This figure belongs to Problem 13.47.

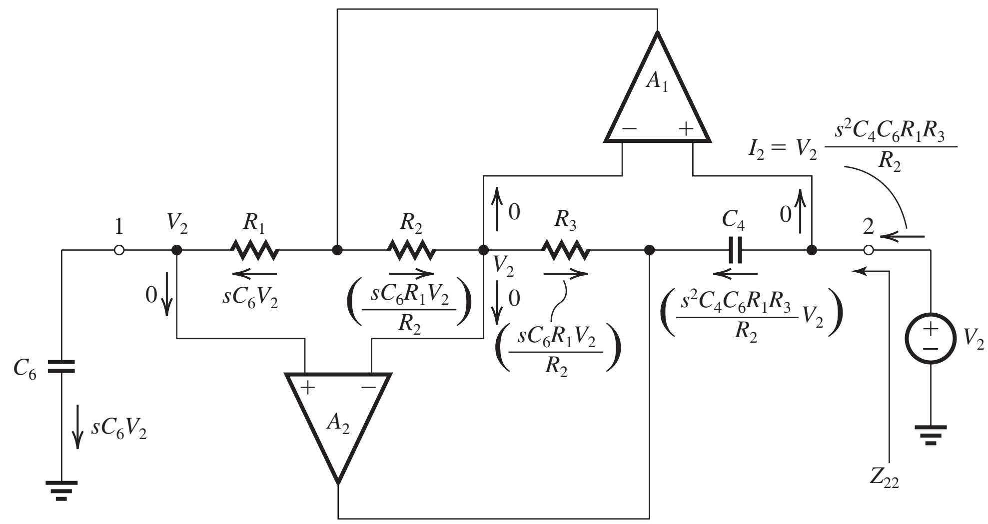


Figure 1

which is a negative resistance whose magnitude depends on frequency $\omega$; thus it is called a Frequency Dependent Negative Resistance or FDNR.

**13.48** Refer to Fig. 13.22(e) and the LPN entry in Table 13.1.

Select

$C_4 = C = 10$ nF

Thus,

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}} = 1.59\ \text{k}\Omega$$

$$R_6 = Q \times 1.59 = 15.9\ \text{k}\Omega$$

$$C_{61} = C\left(\frac{\omega_0}{\omega_n}\right)^2 = 10\left(\frac{10}{12}\right)^2 = 6.94\ \text{nF}$$

$$C_{62} = C - 6.94 = 3.06\ \text{nF}$$

$$K = 1$$

**13.49** For a fifth-order Butterworth filter with $A_{\max} = 3$ dB, and thus $\epsilon = 1$, and $\omega_p = 10^3$ rad/s, the poles can be determined using the graphical construct of Fig. 13.10(a), thus

The pole pair $p_1,\ p_1^*$ has $\omega_0 = 10^3$ rad/s and

$$\frac{\omega_0}{2Q} = 10^3 \sin\left(\frac{\pi}{10}\right) = 10^3 \times 0.309$$

$$\Rightarrow Q = \frac{1}{2 \times 0.309} = 1.618$$

The pole pair $p_2,\ p_2^*$ has $\omega_0 = 10^3$ rad/s and

$$\frac{\omega_0}{2Q} = 10^3 \sin\left(\frac{3\pi}{10}\right) = 10^3 \times 0.809$$

$$\Rightarrow Q = \frac{1}{2 \times 0.809} = 0.618$$

The real-axis pole $p_3$ is at a

$s = -10^3$ rad/s

Thus, the transfer function $T(s)$ is

$$T(s) = \frac{10^3}{s + 10^3} \times \frac{10^6}{s^2 + s\dfrac{10^3}{1.618} + 10^6} \times \frac{10^6}{s^2 + s\dfrac{10^3}{0.618} + 10^6}$$

The first-order factor,

$$T_1(s) = \frac{10^3}{s + 10^3}$$

can be realized by the circuit in Fig. 13.13(a) with $R_1 = R_2 = 100$ kΩ (arbitrary but convenient value).

$$CR_1 = \frac{1}{\omega_0}$$

$$\Rightarrow C = \frac{1}{10^3 \times 100 \times 10^3} = 0.01\ \mu\text{F} = 10\ \text{nF}$$

The second-order transfer function

$$T_2(s) = \frac{10^6}{s^2 + s\dfrac{10^3}{1.618} + 10^6}$$

can be realized by the circuit in Fig. 13.22(a) with

$C_4 = C_6 = C = 10$ nF (practical value)

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{10^3 \times 10 \times 10^{-9}} = 100\ \text{k}\Omega$$

$$R_6 = Q/\omega_0 C = Q \times 100 = 161.8\ \text{k}\Omega$$

$$K = 1$$

The second-order transfer function

$$T_3(s) = \frac{10^6}{s^2 + s\dfrac{10^3}{0.618} + 10^6}$$

can be realized using the circuit in Fig. 13.22(a) with

$C_4 = C_6 = C = 10$ nF (practical value)

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C} = 100\ \text{k}\Omega$$

$$R_6 = Q\left(\frac{1}{\omega_0 C}\right) = 0.618 \times 100 = 61.8\ \text{k}\Omega$$

$$K = 1$$

The complete filter circuit is obtained as the cascade connection of the three filter sections, as shown in Fig. 1 on the next page.

**13.50** $f_0 = 2$ kHz

Selecting,

$C_4 = C_6 = C = 1.0$ nF, we obtain

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{2\pi \times 2 \times 10^3 \times 10 \times 10^{-9}}$$

$$= 79.6\ \text{k}\Omega$$

$$R_6 = \frac{Q}{\omega_0 C} = 2 \times 7.96 = 159.2\ \text{k}\Omega$$

$r_1 = r_2 = 10$ kΩ (arbitrary)

This figure belongs to Problem 13.49.

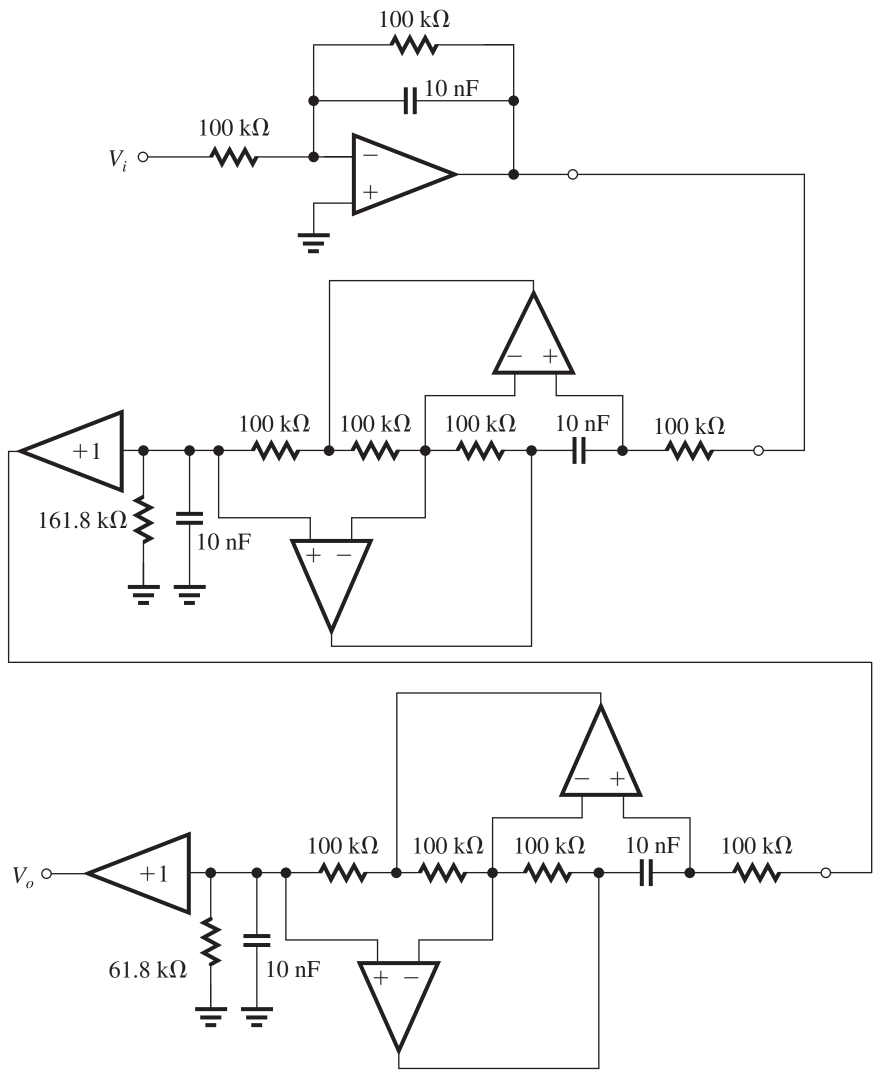



Figure 1

**13.51** Refer to Fig. 13.22(f) and to the HPN entry in Table 13.1.

$T(s) =$

$$K\frac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$$

$$\omega_n^2 = \frac{R_2}{C_4C_6R_1R_3R_{51}}$$

$$\omega_0^2 = \frac{R_2}{C_4C_6R_1R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)$$

Thus,

$$\frac{\omega_0^2}{\omega_n^2} = \frac{R_{51}}{R_5}$$

where

$$R_5 = R_{51} \parallel R_{52}$$

Thus,

$$R_{51} = R_5\left(\frac{\omega_0^2}{\omega_n^2}\right)$$

$$R_{52} = R_5/\left[1 - \left(\frac{\omega_n}{\omega_0}\right)^2\right]$$

Choosing $C_4 = C_5 = C$ (practical value) and $R_1 = R_2 = R_3 = R_4 = R$, we get

$$\omega_0^2 = \frac{R}{C^2R^3} = \frac{1}{C^2R^2}$$

$$\Rightarrow R = 1/\omega_0 C$$

$$\frac{\omega_0}{Q} = \frac{1}{C_6R_6} = \frac{1}{CR_6}$$

$$\Rightarrow R_6 = \frac{Q}{\omega_0 C}$$

Finally,

$K =$ High-frequency gain

**13.52**

(a) $T(s) = \dfrac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}$

Replacing $s$ by $s/10^5$, we obtain

$$T(s) = \frac{0.4508\left(\dfrac{s^2}{10^{10}} + 1.6996\right)}{\left(\dfrac{s}{10^5} + 0.7294\right)\left(\dfrac{s^2}{10^{10}} + \dfrac{s}{10^5}0.2786 + 1.0504\right)}$$

$$= \frac{0.4508 \times 10^5(s^2 + 1.6996 \times 10^{10})}{(s + 0.7294 \times 10^5) \times (s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10})}$$

(b) First-order section:

$$T_1(s) = \frac{0.7294 \times 10^5}{s + 0.7294 \times 10^5}$$

where the dc gain is made unity. This function can be realized using the circuit of Fig. 13.13(a) with

$C = 1$ nF (arbitrary but convenient value)

$$R_2 = \frac{1}{\omega_0 C} = \frac{1}{0.7294 \times 10^5 \times 1 \times 10^{-9}}$$

$= 13.71\ \text{k}\Omega$

For dc gain of unity, we have

$R_1 = R_2 = 13.71\ \text{k}\Omega$

Second-order LPN section:

$$T_2(s) = \frac{0.618(s^2 + 1.6996 \times 10^5)}{s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10}}$$

Selecting, we obtain

$C_4 = C_{61} + C_{62} = C = 1$ nF

and

$R_1 = R_2 = R_3 = R_5 = R$

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{\sqrt{1.0504 \times 10^5} \times 1 \times 10^{-9}}$$

$= 9.76\ \text{k}\Omega$

$$C_{61} = C\left(\frac{\omega_0}{\omega_n}\right)^2$$

$$= 1 \times 10^{-9}\ \frac{1.0504 \times 10^{10}}{1.6996 \times 10^{10}} = 0.618 \text{ nF}$$

$= 618$ pF

$C_{62} = 1 - 0.618 = 0.382 \text{ nF} = 382$ pF

$$Q = \frac{\sqrt{1.0504}}{0.2786} = 3.679$$

$$R_6 = \frac{Q}{\omega_0 C} = 3.679 \times 9.76 = 35.9\ \text{k}\Omega$$

Finally,

$K = \text{DC gain} = 1$

The complete circuit is shown in Fig. 1 below.

**13.53** Bandpass with $f_0 = 2$ kHz, 3-dB bandwidth of 50 Hz, thus

$$Q = \frac{f_0}{BW} = \frac{2 \text{ kHz}}{50 \text{ Hz}} = 40$$

Refer to the circuit in Fig. 13.24(a). Using

$C = 10$ nF

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi \times 2 \times 10^3 \times 10 \times 10^{-9}}$$

$= 7.96\ \text{k}\Omega$

This figure belongs to Problem 13.52, part (b).

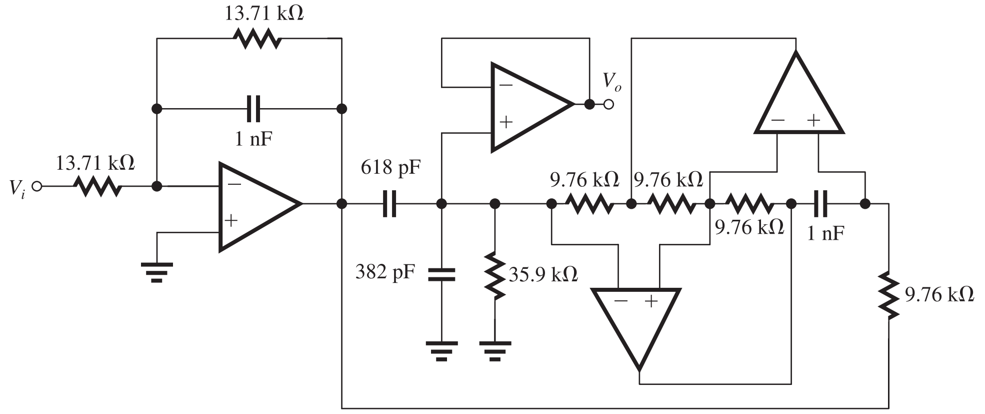



Figure 1

Select

$R_1 = 10\ \text{k}\Omega$

then

$R_f = R_1 = 10\ \text{k}\Omega$

Select

$R_2 = 1\ \text{k}\Omega$

then

$R_3 = (2Q - 1)R_2 = (80 - 1) \times 1 = 79\ \text{k}\Omega$

$K = 2 - \dfrac{1}{Q} = 2 - \dfrac{1}{40} = 1.975$

Center-frequency gain $= KQ = 1.975 \times 40 = 79$

**13.54** (a) Refer to the circuits in Fig. 13.24 and the transfer function in Eq. (13.66), that is,

$$T(s) = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= -K\left(\frac{R_F}{R_H}\right)\frac{s^2 - s(R_H/R_B)\omega_0 + (R_H/R_L)\omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

For this to be an all-pass function, that is,

$$T(s) = -\text{Flat gain} \times \frac{s^2 - s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

then

$$\frac{R_H}{R_L} = 1 \qquad (1)$$

and

$$\frac{R_H}{R_B} = \frac{1}{Q} \qquad (2)$$

That is,

$R_L = R_H = \dfrac{R_B}{Q}$ Q.E.D.

Flat gain $= -K\dfrac{R_F}{R_H}$ Q.E.D.

(b) $\omega_0 = 10^5$ rad/s, $Q = 4$ and flat gain $=$ 10, thus

Selecting

$C = 1\ \text{nF}$

then

$R = \dfrac{1}{\omega_0 C} = \dfrac{1}{10^5 \times 1 \times 10^{-9}} = 10\ \text{k}\Omega$

Selecting

$R_1 = 10\ \text{k}\Omega$

then

$R_f = R_1 = 10\text{k}\Omega$

Selecting

$R_2 = 10\ \text{k}\Omega$

then

$R_3 = R_2(2Q - 1) = 70\ \text{k}\Omega$

Selecting

$R_L = R_H = 10\ \text{k}\Omega$

then

$R_B = QR_H = 4 \times 10 = 40\ \text{k}\Omega$

Now,

$K = 2 - \dfrac{1}{Q} = 2 - \dfrac{1}{4} = 1.75$

Flat gain $= -10 = -K\dfrac{R_F}{R_H}$

$\Rightarrow R_F = \dfrac{10R_H}{K} = \dfrac{10 \times 10}{1.75} = 57.1\ \text{k}\Omega$

**13.55** Consider Fig. 13.24 and Eq. (13.66), that is,

$$T(s) = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (1)$$

For this to be the transfer function of a notch filter, that is,

$$T(s) = -G\frac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} \qquad (2)$$

where $G$ is the high-frequency gain, then by equating the coefficients of the corresponding numerator terms, we obtain

$$R_B = \infty \qquad (3)$$

$\omega_n^2 = \dfrac{R_H}{R_L}\omega_0^2$

$$\Rightarrow \frac{R_H}{R_L} = \left(\frac{\omega_0}{\omega_n}\right)^2 \qquad (4)$$

$G = K\dfrac{R_F}{R_H}$

where

$K = 2 - \dfrac{1}{Q}$

thus

$G = \left(2 - \dfrac{1}{Q}\right)\dfrac{R_F}{R_H}$

$$\Rightarrow \frac{R_F}{R_H} = \frac{G}{2 - (1/Q)} \qquad (5)$$

Equations (3), (4), and (5) are the design equations for the resistors associated with the summer. Observe that the value of one of the three resistors, $R_L$, $R_H$, and $R_F$ can be arbitrarily selected.

**13.56** Refer to Fig. 13.26 and Table 13.2. Using

$C = 10$ nF,

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{10^5 \times 10 \times 10^{-9}}$$

$= 1$ k$\Omega$

$R_d = QR = 10 \times 1 = 10$ k$\Omega$

Select

$r = 20$ k$\Omega$

$R_1 = R_3 = \infty$

If the dc gain is unity, then

$$1 = \text{HF gain} \times \frac{\omega_n^2}{\omega_0^2}$$

$$\Rightarrow \text{HF gain} = \left(\frac{10^5}{1.3 \times 10^5}\right)^2$$

$= 0.5917$

$C_1 = C \times$ high-frequency gain

$= 10 \times 0.5917 = 5.92$ nF

$$R_2 = R\frac{(\omega_0/\omega_n)^2}{\text{HF gain}} = 1 \text{ k}\Omega$$

**13.57** Using Eq. (13.68) with $R_1 = \infty$, we have

$$\frac{V_o}{V_i} = -\frac{C_1}{C}\,\frac{s^2 - s\left(\dfrac{r}{R_3}\right)\left(\dfrac{1}{C_1 R}\right) + \dfrac{1}{CC_1RR_2}}{s^2 + s\dfrac{1}{QCR} + \dfrac{1}{C^2R^2}}$$

Thus,

$$\omega_z = 1/\sqrt{CC_1RR_2} \qquad (1)$$

$$Q_z = \frac{1}{\sqrt{CC_1RR_2}}\left(\frac{R_3}{r}\right)C_1R$$

$$= \sqrt{\left(\frac{C_1}{C}\right)\left(\frac{R}{R_2}\right)}\left(\frac{R_3}{r}\right) \qquad (2)$$

From Eqs. (1) and (2) we see that trimming $\omega_z$ and $Q_z$ can proceed in the following sequence:

(a) Trim $R_2$ to adjust $\omega_z$. This will affect $Q_z$.

(b) Trim $R_3$ to adjust $Q_z$. This will not affect $\omega_z$

**13.58**

$$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + s0.2786 + 1.0504)}$$

(a) Replacing $s$ by $s/10^5$, we obtain

$$T(s) = \frac{0.4508 \times 10^5(s^2 + 1.6996 \times 10^{10})}{(s + 0.7294 \times 10^5)(s^2 + s\,0.2786 \times 10^5 + 1.0504 \times 10^{10})}$$

(b) First-order section:

$$T_1(s) = \frac{0.7294 \times 10^5}{s + 0.7294 \times 10^5}$$

which is made to have a dc gain of unity, as required. This function can be realized by the circuit in Fig. 13.13(a). Selecting

$C = 1$ nF (arbitrary but convenient)

This figure belongs to Problem 13.58, part (b).

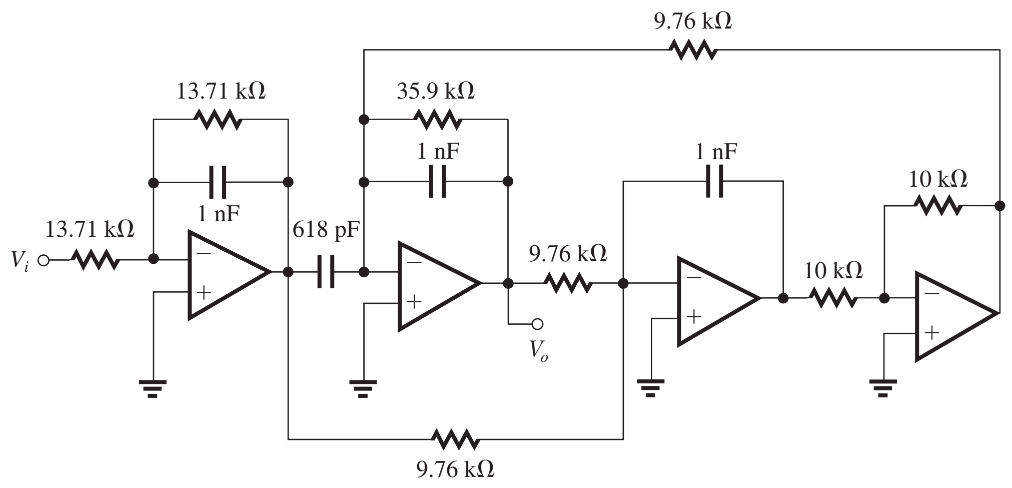



Figure 1

we have

$$R_2 = \frac{1}{\omega_0 C}$$

$$= \frac{1}{0.7294 \times 10^5 \times 1 \times 10^{-9}}$$

$= 13.71\ \text{k}\Omega$

For a dc gain of unity, we have

$R_1 = R_2 = 13.71\ \text{k}\Omega$

Second-order LPN section:

$$T_2(s) = \frac{0.618(s^2 + 1.6996 \times 10^{10})}{s^2 + s0.2786 \times 10^5 + 1.0504 \times 10^{10}}$$

where the dc gain is unity. Refer to Fig. 13.26 and Eq. (13.68).

Selecting

$C = 1\ \text{nF}$

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{\sqrt{1.0504} \times 10^5 \times 1 \times 10^{-9}}$$

$= 9.76\ \text{k}\Omega$

$$R_d = QR = \frac{\sqrt{1.0504}}{0.2786} \times 9.76 = 35.9\ \text{k}\Omega$$

Select

$r = 10\ \text{k}\Omega$

Now,

$C_1 = C \times \text{high-frequency gain}$

$= 1 \times 0.618 = 0.618\ \text{nF} = 618\ \text{pF}$

$R_1 = \infty$

$R_3 = \infty$

$$R_2 = R\frac{(\omega_0/\omega_n)^2}{\text{HF Gain}}$$

$$= 9.76 \times \frac{1.0504}{1.6996} \times \frac{1}{0.618}$$

$= 9.76\ \text{k}\Omega$

The complete circuit is shown in Fig. 1.

**13.59** Refer to Fig. 13.29 and Eqs. (13.75) and (13.76):

$C_1 = C_2 = C = 1\ \text{nF}$

$$R_3 = R \text{ and } R_4 = R/4Q^2 = R\Big/\left(4 \times \frac{1}{2}\right) = \frac{R}{2}$$

$$CR = \frac{2Q}{\omega_0} = \frac{2/\sqrt{2}}{10^5}$$

$$\Rightarrow R = \frac{\sqrt{2}}{10^5 \times 1 \times 10^{-9}} = 14.14\ \text{k}\Omega$$

Thus,

$R_3 = 14.14\ \text{k}\Omega$

$R_4 = 7.07\ \text{k}\Omega$

**13.60** Refer to Fig. 13.28(a).

$$t(s) = \frac{s^2 + s\dfrac{2}{CR} + \dfrac{1}{C^2R^2}}{s^2 + s\dfrac{3}{CR} + \dfrac{1}{C^2R^2}}$$

$$= \frac{s^2 + s(2/\tau) + (1/\tau^2)}{s^2 + s(3/\tau) + (1/\tau^2)}$$

If the network is placed in the negative-feedback path of an ideal infinite-gain op amp, as in Fig. 13.24, the poles will be given by the roots of the numerator polynomial, thus

$$\omega_0 = \frac{1}{\tau}$$

and

$$Q = \frac{1/\tau}{2/\tau} = 0.5$$

Thus, the poles will be coincident at

$s = -\omega_0 = -1/\tau$

**13.61**

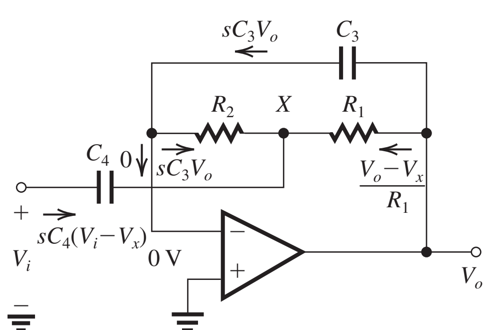



Figure 1

The circuit is shown in Fig. 1. The voltage at node $X$ can be found as

$V_x = 0 - sC_3V_oR_2$

$$= -sC_3R_2V_o \tag{1}$$

A node equation at $X$ can be written as

$$sC_3V_o + sC_4(V_i - V_x) + \frac{V_o - V_x}{R_1} = 0 \tag{2}$$

Substituting for $V_x$ from Eq. (1), we obtain

$$sC_3V_o + sC_4V_i - (-sC_3R_2V_o)\left(sC_4 + \frac{1}{R_1}\right) + \frac{V_o}{R_1} = 0$$

$$V_o\left(sC_3+s^2C_3C_4R_2+sC_3\frac{R_2}{R_1}+\frac{1}{R_1}\right)=-sC_4V_i$$

$$\frac{V_o}{V_i}=\frac{-sC_4}{s^2C_3C_4R_2+sC_3\left(1+\frac{R_2}{R_1}\right)+\frac{1}{R_1}}$$

$$=\frac{-s/C_3R_2}{s^2+s\frac{1}{C_4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{C_3C_4R_1R_2}}$$

For $R_1 = R_2 = R$, $C_4 = C$, and $C_3 = C/36$, we obtain

$$\frac{V_o}{V_i}=\frac{-s(36/CR)}{s^2+s\frac{2}{CR}+\frac{36}{C^2R^2}}$$

This is a bandpass function with

$$\omega_0=\frac{6}{CR}$$

$$Q=\frac{6/CR}{2/CR}=3$$

and

Center-frequency gain $= -18$

**13.62**

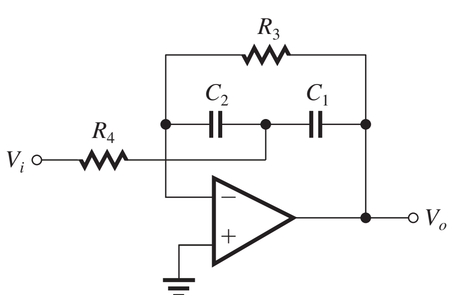



Figure 1

The circuit is shown in Fig. 1. The transfer function can be obtained using the equation on page 1126 with $\alpha = 1$, thus

$$\frac{V_o}{V_i}=\frac{-s/C_1R_4}{s^2+s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}+\frac{1}{C_1C_2R_3R_4}}$$

Using the design equations (13.75) and (13.76), we obtain

$$R_3 = R$$

$$R_4 = R/4Q^2$$

$$CR=\frac{2Q}{\omega_0}$$

where

$$\omega_0 = 2\pi \times 10 \times 10^3$$

$$Q=\frac{f_0}{BW}=\frac{10}{2}=5$$

Thus,

$$CR=\frac{2\times 5}{2\pi\times 10^4}$$

Selecting

$$C = 10\ \text{nF}$$

we obtain

$$R=\frac{10}{2\pi\times 10^4\times 10\times 10^{-9}}=15.92\ \text{k}\Omega$$

Thus,

$$C_1 = C_2 = 10\ \text{nF}$$

$$R_3 = 15.92\ \text{k}\Omega$$

$$R_4=\frac{R}{4Q^2}=\frac{15.92}{100}=159.2\ \Omega$$

$$\text{Center-frequency gain} = -\frac{1/C_1R_4}{\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}}$$

$$=-\frac{1}{2}\frac{R_3}{R_4}=-\frac{1}{2}\times 100=-50\ \text{V/V}$$

**13.63**

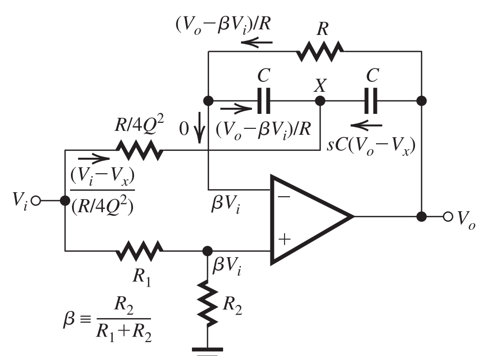



Figure 1

The circuit is shown in Fig. 1. The voltage at node $X$ is given by

$$V_x=\beta V_i-\frac{1}{sC}\frac{V_o-\beta V_i}{R}$$

$$=\beta V_i\left(1+\frac{1}{sCR}\right)-\frac{V_o}{sCR} \qquad (1)$$

Writing a node equation at $X$ gives

$$\frac{V_o-\beta V_i}{R}+\frac{V_i-V_x}{R/4Q^2}+sC(V_o-V_x)=0$$

Substituting for $V_x$ from Eq. (1) and collecting terms gives

$$\frac{V_i}{V_x} = \beta\,\frac{s^2 + s\dfrac{2}{CR}\left[1 + 2Q^2\left(1 - \dfrac{1}{\beta}\right)\right] + \dfrac{4Q^2}{C^2R^2}}{s^2 + s\dfrac{2}{CR} + \dfrac{4Q^2}{C^2R^2}}$$

We observe that, as expected,

$$\omega_0 = \frac{2Q}{CR}$$

(a) To obtain an all-pass function, we set

$$\frac{2}{CR}\left[1 + 2Q^2\left(1 - \frac{1}{\beta}\right)\right] = -\frac{2}{CR}$$

$$\Rightarrow \frac{1}{\beta} = 1 + \frac{1}{Q^2}$$

But,

$$\beta = \frac{R_2}{R_1 + R_2}$$

Thus,

$$\frac{R_2}{R_1} = Q^2$$

(b) To obtain a notch function, we set

$$\frac{2}{CR}\left[1 + 2Q^2\left(1 - \frac{1}{\beta}\right)\right] = 0$$

$$\Rightarrow \frac{1}{\beta} = 1 + \frac{1}{2Q^2}$$

or, equivalently,

$$\frac{R_2}{R_1} = 2Q^2$$

**13.64**

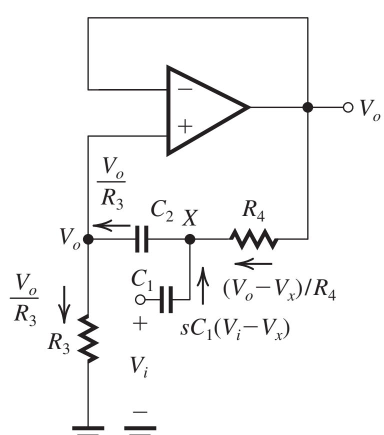



Figure 1

The analysis is shown in Fig. 1, below, left. The voltage at node $X$ is given by

$$V_x = V_o + \frac{V_o}{R_3}\frac{1}{sC_2}$$

$$= V_o\left(1 + \frac{1}{sC_2R_3}\right) \qquad (1)$$

A node equation at $X$ provides

$$\frac{V_o - V_x}{R_4} + sC_1(V_i - V_x) = \frac{V_o}{R_3}$$

$$V_o\left(\frac{1}{R_4} - \frac{1}{R_3}\right) + sCV_i - V_x\left(\frac{1}{R_4} + sC_1\right) = 0$$

Substituting for $V_x$ from Eq. (1) and collecting terms, we obtain

$$\frac{V_o}{V_i} = \frac{s^2}{s^2 + s\dfrac{1}{R_3}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right) + \dfrac{1}{C_1C_2R_3R_4}}$$

This is a high-pass function with a high-frequency gain of unity. To obtain a maximally flat response with $\omega_{3dB} = 10^4$ rad/s and using

$$C_1 = C_2 = C = 10 \text{ nF}$$

then

$$\omega_0 = \omega_{3dB} = 10^4 \text{ rad/s}$$

$$Q = \frac{1}{\sqrt{2}} = \omega_0 R_3 \Big/ \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{1}{\sqrt{2}} = 10^4 R_3 \Big/ \left(\frac{2}{10 \times 10^{-9}}\right)$$

$$\Rightarrow R_3 = \frac{1}{\sqrt{2}} \times \frac{2}{10^{-8}} \times 10^{-4} = 14.14 \text{ k}\Omega$$

$$\omega_0^2 = \frac{1}{C_1C_2R_3R_4}$$

$$10^8 = \frac{1}{10^{-8} \times 10^{-8} \times 14.4 \times 10^3 \times R_4}$$

$$\Rightarrow R_4 = \frac{100}{14.14} \text{ k}\Omega = 7.07 \text{ k}\Omega$$

**13.65**

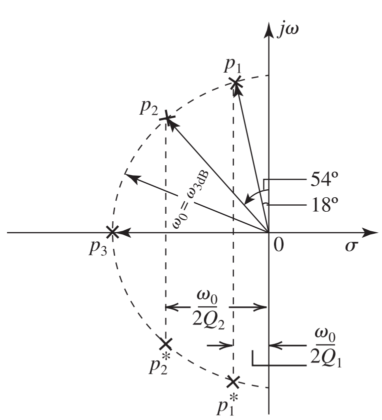



Figure 1

Figure 1 (see previous page) shows a graphical construct to determine the poles of the fifth order Butterworth filter. The pair of complex conjugate poles $p_1$ and $p_1^*$ have a frequency

$$\omega_{01} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a $Q$ factor

$$Q_1 = \frac{1}{2 \sin 18^\circ} = 1.618$$

The pair of complex conjugate poles $p_2$ and $p_2^*$ have

$$\omega_{02} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a $Q$ factor,

$$Q_2 = \frac{1}{2 \sin 54^\circ} = 0.618$$

The real-axis pole $p_3$ is at

$$s = -\omega_0 = -2\pi \times 10^4 \text{ rad/s}$$

The first second-order section can be realized using the circuit in Fig. 13.34(c). The design equations are (13.77)–(13.80).

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C_4 = C$$

$$C_3 = C/4Q^2$$

Here, $Q = Q_1 = 1.618$, thus

$$C_3 = \frac{C}{4 \times 1.618^2} = 0.095C$$

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 1.618}{2\pi \times 10^4 \times 10 \times 10^3} = 5.15 \text{ nF}$$

$$C_3 = 0.492 \text{ nF} = 492 \text{ pF}$$

$$C_4 = 5.15 \text{ nF}$$

The second second-order section also can be realized using the circuit in Fig. 13.34(c). Here,

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C_4 = C$$

$$C_3 = \frac{C}{4Q^2}$$

where $Q = Q_2 = 0.618$. Thus

$$C_3 = \frac{C}{4 \times 0.618^2} = 0.655C$$

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 0.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 0.618}{2\pi \times 10^4 \times 10^4} = 1.97 \text{ nF}$$

$$C_3 = 1.29 \text{ nF}$$

$$C_4 = 1.97 \text{ nF}$$

The first-order section can be realized using the circuit in Fig. 13.13(a) with

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C = \frac{1}{\omega_0 R}$$

$$= \frac{1}{2\pi \times 10^4 \times 10^4} = 1.59 \text{ nF}$$

The complete circuit is shown in Fig. 2 below.

**13.66** Refer to Fig. 13.31 and let the network $n$ have a transfer function

$$\frac{V_a}{V_b} = \frac{s\dfrac{\omega_0}{Q}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

This figure belongs to Problem 13.65.

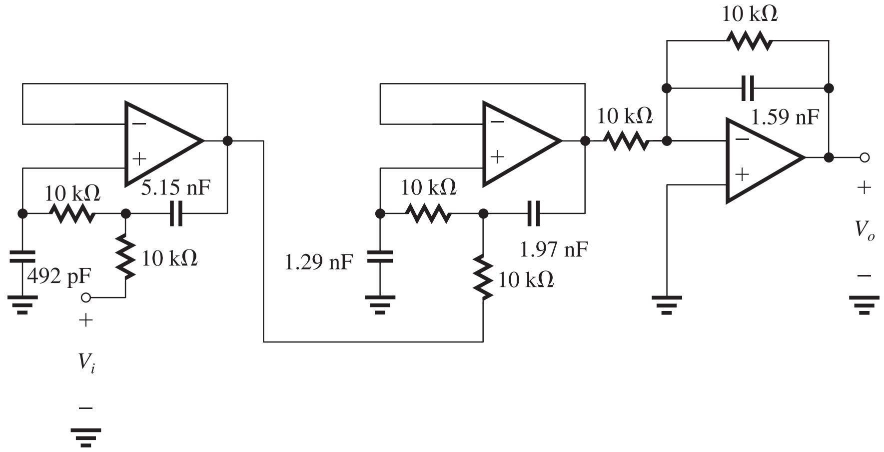



Figure 2

which is a bandpass with a unity center-frequency gain. The complementary network in (b) will have a transfer function

$$\frac{V_a}{V_c} = 1 - \frac{V_a}{V_b}$$

$$= 1 - \frac{s\dfrac{\omega_0}{Q}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

which is a notch function.

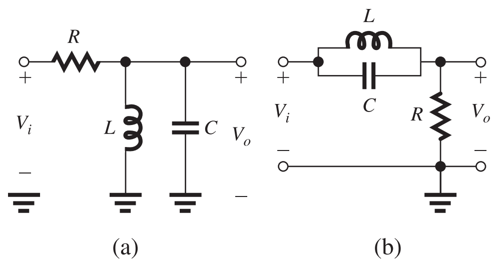



Figure 1

As an example, consider the RLC bandpass circuit shown in Fig. 1(a). It has the transfer function

$$T_1(s) = \frac{V_o}{V_i} = \frac{s\dfrac{1}{CR}}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{LC}}$$

Interchanging the input terminal with ground, we obtain the circuit shown in Fig. 1(b).

Straightforward analysis shows that this circuit has the transfer function

$$T_2(s) = \frac{s^2 + \dfrac{1}{LC}}{s^2 + s\dfrac{1}{CR} + \dfrac{1}{LC}}$$

which is a notch function. Observe that

$$T_2(s) = 1 - T_1(s)$$

that is, the circuits in (a) and (b) are complementary.

**13.67** For the circuit in Fig. 13.18(b) we have

$$T(s) = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad Q = R\sqrt{\frac{C}{L}}$$

For $\omega_0$ we have

$$\frac{\partial\omega_0}{\partial L} = \frac{\partial(LC)^{-1/2}}{\partial L} = -\frac{1}{2}L^{-3/2}C^{-1/2} = \frac{\omega_0}{2L}$$

$$\frac{\partial\omega_0}{\partial C} = -\frac{\omega_0}{2C}$$

$$\frac{\partial\omega_0}{\partial R} = 0$$

$$\therefore\ S_L^{\omega_0} = \frac{\partial\omega_0}{\partial L}\frac{L}{\omega_0} = -1/2$$

$$S_C^{\omega_0} = \frac{\partial\omega_0}{\partial C} \times \frac{C}{\omega_0} = -1/2$$

$$S_R^{\omega_0} = \frac{\partial\omega_0}{\partial R}\frac{R}{\omega_0} = 0$$

For $Q$ we have

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}}\left(\frac{-1}{2}\right) = \frac{-Q}{2L}$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2}\frac{R}{\sqrt{LC}} = \frac{1}{2}\frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R}\sqrt{C/L} = Q/R$$

$$S_L^Q = -\frac{Q}{2L} \times \frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C} \times \frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = \frac{Q}{R} \cdot \frac{R}{Q} = 1$$

**13.68** (a) $y = uv$

$$S_x^y = \frac{\partial(uv)}{\partial x}\frac{x}{uv}$$

$$= v\frac{\partial u}{\partial x}\frac{x}{uv} + u\frac{\partial v}{\partial x}\frac{x}{uv}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u} + \frac{\partial v}{\partial x}\frac{x}{v}$$

$$= S_x^u + S_x^v$$

(b) $y = u/v$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = \frac{\partial(u/v)}{\partial x}\frac{x}{u/v}$$

$$= \frac{1}{v}\frac{\partial u}{\partial x}\frac{xv}{u} - \frac{u}{v^2}\frac{\partial v}{\partial x}\frac{xv}{u}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u} - \frac{\partial v}{\partial x}\frac{x}{v}$$

$$= S_x^u - S_x^v$$

(c) $y = ku$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y} = k\frac{\partial u}{\partial x}\frac{x}{ku}$$

$$= \frac{\partial u}{\partial x}\frac{x}{u}$$

$$= S_x^u$$

(d) $y = u^n$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$$

$$= nu^{n-1}\frac{\partial u}{\partial x}\frac{x}{u^n}$$

$$= n\frac{\partial u}{\partial n}\frac{x}{u}$$

$$= nS_x^u$$

(e) $y = f_1(u),\ u = f_2(x)$

$$S_x^y = \frac{\partial y}{\partial x}\frac{x}{y}$$

$$= \frac{\partial f_1(u)}{\partial u}\frac{\partial u}{\partial x}\frac{x}{f_1(u)}\frac{u}{u}$$

$$= \left[\frac{\partial f_1(u)}{\partial u}\frac{u}{f_1(u)}\right]\left[\frac{\partial f_2(x)}{\partial x}\frac{x}{u}\right]$$

$$= \left[\frac{\partial f_1(u)}{\partial u}\frac{u}{f_1(u)}\right]\left[\frac{\partial f_2(x)}{\partial x}\frac{x}{f_2(x)}\right]$$

$$= S_u^{f_1}S_x^{f_2}$$

$$= S_u^y S_x^u$$

**13.69** From Table 13.1 we have

$$\omega_0 = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5/\mathrm{R}_2}}$$

$$Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$$

$$\frac{\partial\omega_0}{\partial C_4} = \frac{-\omega_0}{2C_4}$$

$$\therefore\ S_{C_4}^{\omega_0} = \frac{-\omega_0}{2C_4}\times\frac{C_4}{\omega_0} = -\frac{1}{2}$$

Similarly, $S_{C_6}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_5}^{\omega_o} = -\frac{1}{2}$

$$\frac{\partial\omega_0}{\partial R_2} = \frac{\omega_O}{2R_2} \Rightarrow S_{R_2}^{\omega_o} = \frac{1}{2}$$

Now for $Q$:

$$\frac{\partial Q}{\partial R_6} = \frac{Q}{R_6} \Rightarrow S_{R_6}^{Q} = \frac{\partial Q}{\partial R_6}\frac{R_6}{Q} = +1$$

$$\frac{\partial Q}{\partial C_6} = \frac{Q}{2C_6} \Rightarrow S_{C_6}^{Q} = S_{R_2}^{Q} = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = -\frac{Q}{2C_4} \Rightarrow S_{C_4}^{Q} = S_{R_1,R_2,R_3}^{Q} = -\frac{1}{2}$$

**13.70** Using Eqs. (13.78) and (13.79), we have

$$\omega_0 = \frac{1}{\sqrt{C_3C_4R_1R_2}}$$

$$Q = \frac{1}{\sqrt{C_3C_4R_1R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

$$\frac{\partial\omega_0}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}}$$

$$= \frac{-\omega_0}{2C_3}$$

$$S_{C_3}^{\omega_0} = \frac{\partial\omega_0}{\partial C_3}\frac{C_3}{\omega_O} = -\frac{1}{2}$$

Clearly, $S_{C_3}^{\omega_0} = S_{C_4}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = -\frac{1}{2}$

$$\frac{\partial Q}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

$$= \frac{-Q}{2C_3}$$

$$\therefore S_{C_3}^{Q} = -\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = \frac{Q}{2C_4} \Rightarrow S_{C_4}^{Q} = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial R_1} = \frac{1/\sqrt{R_1} - \sqrt{R_1}/\mathrm{R}_2}{R_1\left(\frac{1}{\sqrt{R_1}}+\frac{\sqrt{R_1}}{R_2}\right)}\cdot\frac{Q}{2}$$

$$= \frac{\sqrt{R_2/\mathrm{R}_1} - \sqrt{R_1/\mathrm{R}_2}}{R_1\left(\sqrt{\frac{R_2}{R_1}}+\sqrt{\frac{R_1}{R_2}}\right)}\cdot\frac{Q}{2}$$

$$\therefore S_{R_1}^{Q} = \frac{1}{2}\frac{\sqrt{R_2/\mathrm{R}_1} - \sqrt{R_1/\mathrm{R}_2}}{\sqrt{R_2/\mathrm{R}_1} + \sqrt{R_1/\mathrm{R}_2}}$$

If $R_1 = R_2 \Rightarrow S_{R_1}^{Q} = 0$.

Similarly,

$$S_{R_2}^{Q} = 0$$

**13.71** Refer to the circuit in Fig. 13.35(f).

$$I_o = g_{m1,2}\left(\frac{V_i}{2}\right)$$

Thus,

$$G_m = \frac{1}{2}g_{m1,2}$$

But,

$$g_{m1,2} = \sqrt{2k_nI_{D1,2}}$$

$$= \sqrt{2k_n(I/2)}$$

$$= \sqrt{k_nI}$$

Thus,

$$G_m = \frac{1}{2}\sqrt{k_nI}$$

For $G_m = 0.25$ mA/V and $k_n = 0.5$ mA/V$^2$, we have

$$0.25 = \frac{1}{2}\sqrt{0.5I}$$

$$\Rightarrow I = 0.5 \text{ mA}$$

Since $G_m$ is proportional to $\sqrt{I}$, tuning $G_m$ in the range ±5% requires tuning $I$ in the range $0.95^2 I_{\text{nominal}}$ to $1.05^2 I_{\text{nominal}}$, which is approximately ±10% of the nominal value.

**13.72** $R = \frac{1}{G_m}$

$$\Rightarrow G_m = \frac{1}{R} = \frac{1}{10^3} = 10^{-3}\ \text{A/V} = 1\ \text{mA/V}$$

Since the output terminal is connected back to the input, the output resistance appears in effect in parallel with the resistance $1/G_m$, thus the actual resistance realized is

$$\frac{1}{G_m} \parallel R_o = 1 \parallel 100 = 0.99\ \text{k}\Omega$$

**13.73**

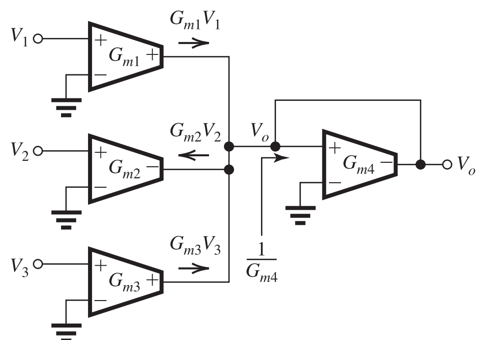



Figure 1

The circuit is shown in Fig. 1, for which we can write

$$V_o = \frac{1}{G_{m4}}(G_{m1}V_1 - G_{m2}V_2 + G_{m3}V_3)$$

$$= \frac{G_{m1}}{G_{m4}}V_1 - \frac{G_{m2}}{G_{m4}}V_2 + \frac{G_{m3}}{G_{m4}}V_3$$

To obtain

$$V_o = V_1 - 2V_2 + 3V_3$$

we select

$$G_{m1} = G_{m4}$$

$$G_{m2} = 2\ G_{m4}$$

$$G_{m3} = 3\ G_{m4}$$

**13.74** For the integrator in Fig. 13.36(b), we have

$$\frac{V_o}{V_1} = \frac{G_m}{sC}$$

$$\text{Unity-gain frequency} = \frac{G_m}{2\pi C}$$

Thus,

$$10 \times 10^6 = \frac{G_m}{2\pi \times 5 \times 10^{-12}}$$

$$\Rightarrow G_m = 0.314\ \text{mA/V}$$

**13.75** Both $R_o$ and $C_o$ will appear in parallel with $C$, thus

$$V_o = G_m V_i \frac{1}{\frac{1}{R_o} + s(C + C_o)}$$

The transfer function realized will be

$$\frac{V_o}{V_1} = \frac{G_m}{\frac{1}{R_o} + s(C + C_o)}$$

The integrator time constant is

$$\tau = (C + C_o)/G_m$$

$$= \frac{C}{G_m}\left(1 + \frac{C_o}{C}\right)$$

The quantity $\left(1 + \frac{C_o}{C}\right)$ represents the error factor. For the error to be less than 1%, we must have

$$\frac{C_o}{C} \leq 0.01$$

$$\Rightarrow C \geq 100C_o$$

Thus, the smallest value of $C$ is $100C_o$.

Frequency of the low-frequency pole

$$= \frac{1}{(C + C_o)R_o}$$

$$\simeq \frac{1}{CR_o}$$

If this frequency is to be at least two decades lower than the unity gain frequency $\frac{G_m}{C}$, then

$$\frac{1}{CR_o} \leq 0.01\frac{G_m}{C}$$

$$\Rightarrow G_m \geq \frac{100}{R_o}$$

Thus, the smallest value of $G_m$ must be $100/R_o$.

**13.76** Refer to the circuit in Fig. 13.36(c) and its transfer function in Eq. (13.91), namely

$$\frac{V_o}{V_i} = -\frac{G_{m1}}{sC + G_{m2}}$$

$$\text{Pole frequency} = \frac{G_{m2}}{2\pi C}$$

$$20 \times 10^6 = \frac{G_{m2}}{2\pi \times 2 \times 10^{-12}}$$

$$\Rightarrow G_{m2} = 0.251\ \text{mA/V}$$

$$|\text{DC gain}| = \frac{G_{m1}}{G_{m2}}$$

$$10 = \frac{G_{m1}}{G_{m2}}$$

$$\Rightarrow G_{m1} = 2.51 \text{ mA/V}$$

**13.77**

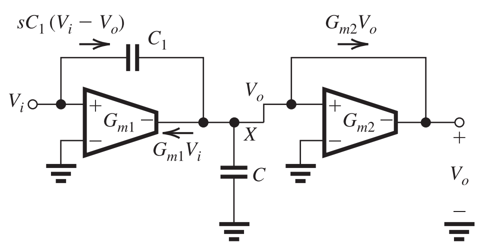



Figure 1

Figure 1 shows the circuit. A node equation at $X$ yields

$$sC_1(V_i - V_o) = G_{m1}V_i + G_{m2}V_o + sCV_o$$

$$(sC_1V_i - G_{m1})V_i = V_o(G_{m2} + sC + sC_1)$$

$$\Rightarrow \frac{V_o}{V_i} = -\frac{G_{m1} - sC_1}{G_{m2} + s(C + C_1)}$$

**13.78** (a) Refer to the circuit in Fig. P13.78. The output current of the $G_{m2}$ transconductor is $G_{m2}V_1$. Thus,

$$V_2 = \frac{G_{m2}}{sC}V_1$$

This is the input voltage to the negative transconductor $G_{m1}$. Thus the output current of $G_{m1}$, which is equal to $I_1$, will be

$$I_1 = G_{m1}V_2$$

$$= \frac{G_{m1}G_{m2}}{sC}V_1$$

Thus,

$$Z_{\text{in}} \equiv \frac{V_1}{I_1} = s\frac{C}{G_{m1}G_{m2}}$$

which is that of an inductance $L$,

$$L = \frac{C}{G_{m1}G_{m2}} \qquad \text{Q.E.D.}$$

(b) To obtain an LCR resonator, we connected a capacitor $C$ from node 1 to ground and a resistance $R$ realized by the transconductor $G_{m3}$, as shown in Fig. 1 below.

(c) A fourth transconductor $G_{m4}$ is used to feed a current $G_{m4}V_i$ to node 1, as shown in Fig. 1. The resulting circuit is identical to that in Fig. 13.37(b) except here $C_1 = C_2 = C$.

(d) With analogy to the identical circuit in Fig. 13.37(b), $V_1/V_i$ will be a second-order bandpass filter with a transfer function given by Eq. (13.93), and $V_2/V_i$ will be a second-order low-pass filter with a transfer function given by Eq. (13.94). Thus,

$$\frac{V_1}{V_i} = -\frac{s(G_{m4}/C)}{s^2 + s\dfrac{G_{m3}}{C_1} + \dfrac{G_{m1}G_{m2}}{C^2}}$$

and,

$$\frac{V_2}{V_i} = -\frac{G_{m2}G_{m4}/C^2}{s^2 + s\dfrac{G_{m2}}{C} + \dfrac{G_{m1}G_{m2}}{C^2}}$$

**13.79** Using Eqs. (13.95) and (13.96), we have

$$\omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}} \tag{1}$$

$$Q = \frac{\sqrt{G_{m1}G_{m2}}}{G_{m3}}\sqrt{\frac{C_1}{C_2}} \tag{2}$$

This figure belongs to Problem 13.78, part (b).

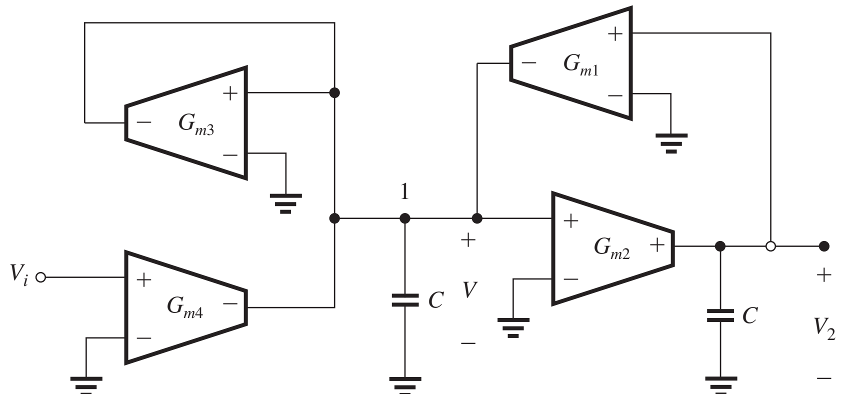



Figure 1

Selecting $G_{m1} = G_{m2} = G_{m3} = G_m$, we obtain

$$\omega_0 = \frac{G_m}{\sqrt{C_1 C_2}} \qquad (3)$$

$$Q = \sqrt{\frac{C_1}{C_2}} \qquad (4)$$

Selecting $C_2 = C$, then from Eq. (4) we have

$C_1 = Q^2 C$

and from Eq. (3) we have

$G_m = \omega_0 Q C$

**13.80** $f_0 = 25$ MHz, $Q = 5$,

Center-frequency gain $= 5$

The design equations are given by (13.99), (13.100), and (13.101). Thus

$G_m = \omega_0 C$

where

$C = C_1 = C_2 = 5$ pF

$G_m = 2\pi \times 25 \times 10^6 \times 5 \times 10^{-12}$

$= 0.785$ mA/V

Thus,

$G_{m2} = G_m = 0.785$ mA/V

$$G_{m3} = \frac{G_m}{Q} = \frac{0.785}{5} = 0.157 \text{ mA/V}$$

$$G_{m4} = \frac{G_m}{Q}|\text{Gain}|$$

$$= \frac{0.785}{5} \times 5 = 0.785 \text{ mA/V}$$

**13.81** The resulting circuit is shown in Fig. 1. For $V_2$ we can write

$$V_2 = \frac{1}{sC_2}(G_{m2}V_1 - G_{m5}V_i) \qquad (1)$$

A node equation at $X$ can be written as

$$G_{m1}V_2 + sC_3(V_1 - V_i) + G_{m4}V_i + G_{m3}V_1 + sC_1V_1$$
$$= 0 \qquad (2)$$

Substituting for $V_2$ from Eq. (1) into Eq. (2) and collecting terms results in the transfer function

$$\frac{V_1}{V_i} = \frac{s^2\left(\dfrac{C_3}{C_1 + C_3}\right) - s\dfrac{G_{m4}}{C_1 + C_3} + \dfrac{G_{m1}G_{m5}}{(C_1 + C_3)C_2}}{s^2 + s\dfrac{G_{m3}}{C_1 + C_3} + \dfrac{G_{m1}G_{m2}}{(C_1 + C_3)C_2}}$$

**13.82** $R_{\text{eq}} = \dfrac{T_c}{C_1} = \dfrac{1}{f_c C_1} = \dfrac{1}{200 \times 10^3 C_1}$

For $C_1 = 1$ pF,

$$R_{\text{eq}} = \frac{1}{200 \times 10^3 \times 1 \times 10^{-12}} = 5\ \text{M}\Omega$$

This figure belongs to Problem 13.81.

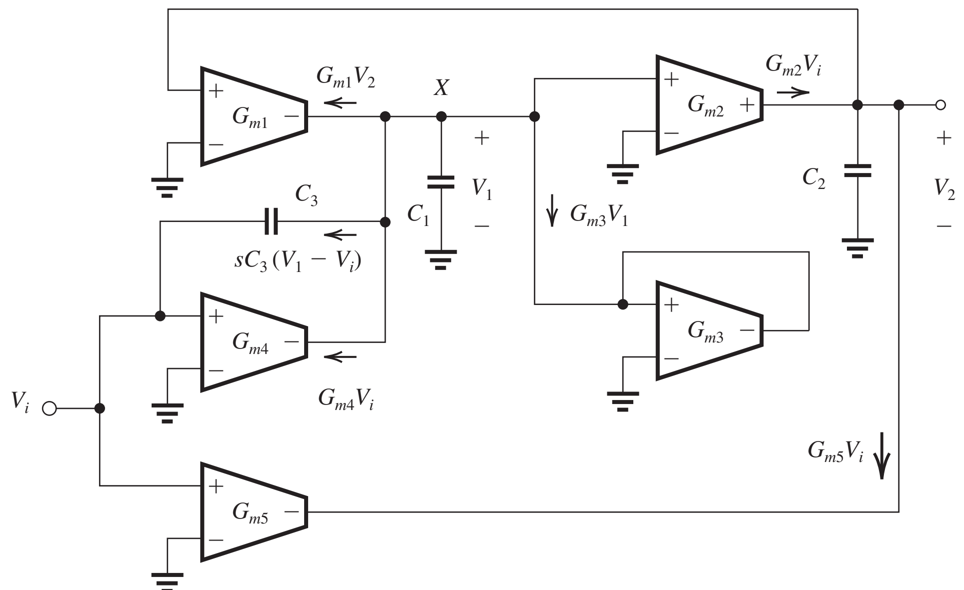



Figure 1

For $C_1 = 5$ pF,

$$R_{\text{eq}} = \frac{1}{200 \times 10^3 \times 5 \times 10^{-12}} = 1\ \text{M}\Omega$$

For $C_1 = 10$ pF,

$$R_{\text{eq}} = \frac{1}{200 \times 10^3 \times 10 \times 10^{-12}} = 500\ \text{k}\Omega$$

**13.83** Change transferred $\Rightarrow Q = CV$

$= 10^{-12}\,(1)$

$= 1$ pC

For $f_0 = 100$ kHz, average current is given by

$$I_{\text{AV}} = \frac{Q}{T} = 1\ \text{pC} \times 100 \times 10^3$$

$= 0.1\ \mu$A

For each clock cycle, the output will change by the same amount as the change in voltage across $C_2$.

$$\therefore \Delta V = Q/C_2 = \frac{1\ \text{pC}}{10\ \text{pF}} = 0.1\ \text{V}$$

For $\Delta V = 0.1$ V for each Clock cycle, the amplifier will saturate in

$$= \frac{10\ \text{V}}{0.1\ \text{V}} = 100\ \text{cycles}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10\text{V}}{(100\ \text{cycles})\left(1/100 \times 10^3\right)}$$

$= 10^4$ V/s

**13.84** $\omega_0 = \omega_{3dB} = 10^3$ rad/s

$Q = 1/\sqrt{2}$ and DC gain $= 1$

$f_c = 100$ kHz, $C_1 = C_2 = C = 5$ pF

From Eqs. (13.109) and (13.110),

$C_3 = C_4 = \omega_0 T_c C$

$$= 10^3 \times \frac{1}{100 \times 10^3} \times 5 \times 10^{-12}$$

$= 0.05$ pF

From Eq. (13.112),

$$C_5 = \frac{C_4}{Q} = \frac{0.05}{1/\sqrt{2}} = 0.071\ \text{pF}$$

The dc gain of the low-pass circuit is

$$\text{DC gain} = \frac{C_6}{C_4}$$

For DC gain $= 1$,

$C_6 = C_4 = 0.05$ pF

**13.85** Refer Fig. 1 below and Fig. 2 on next page. For the BJT we have

$$g_m = \frac{I_C}{V_T} \simeq \frac{1\ \text{mA}}{0.025\ \text{V}} = 40\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{40} = 5\ \text{k}\Omega$$

$C_\pi = 10$ pF

This figure belongs to Problem 13.85, part (a).

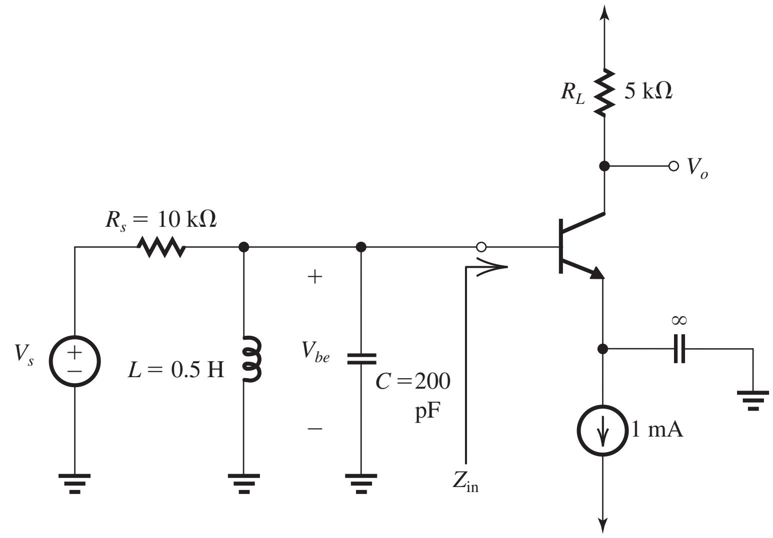



Figure 1

Miller capacitance $= C_\mu(1 + g_m R_L)$

$= 0.5(1 + 40 \times 5)$

$= 100.5$ pF

Total capacitance $= C + C_\pi + C_{\text{Miller}}$

$= 200 + 10 + 100.5$

$= 310.5$ pF

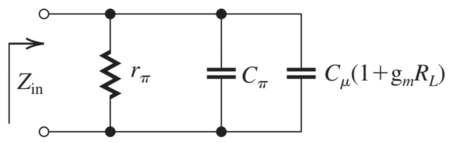



Figure 2

$$\omega_0 = \frac{1}{\sqrt{LC_{\text{total}}}}$$

$$= \frac{1}{\sqrt{0.5 \times 10^{-6} \times 310.5 \times 10^{-12}}} = 80.3 \text{ Mrad/s}$$

Total effective parallel resistance $=$
$10 \text{ k}\Omega \parallel 5 \text{ k}\Omega = 3.33 \text{ k}\Omega$

$$Q = \frac{R}{\omega_0 L} = \frac{3.33 \times 10^3}{80.3 \times 10^6 \times 0.5 \times 10^{-6}} = 83$$

$$\text{3-dB } BW = \frac{\omega_0}{Q} = \frac{80.3 \times 10^6}{83} = 967 \text{ kHz}$$

$$V_{be}(\omega_0) = V_s \frac{r_\pi}{r_\pi + R_s}$$

$$V_o(\omega_0) = -g_m R_L V_{be}$$

$$= -40 \times 5 \times \frac{1}{3} V_s$$

$$= -66.7 V_s$$

$$\left| \frac{V_o(\omega_0)}{V_s(\omega_0)} \right| = 66.7 \text{ V/V}$$

**13.86** $R_p = \omega_0 L Q$

$= 2\pi \times 10^6 \times 10 \times 10^{-6} \times 250$

$= 15.71$ k$\Omega$

$$C = \frac{1}{\omega_0^2 L}$$

$$= \frac{1}{(2\pi \times 10^6)^2 \times 10 \times 10^{-6}} = 2.53 \text{ nF}$$

For a 3-dB bandwidth of 12 kHz, we have

$$Q = \frac{1 \times 10^6}{12 \times 10^3} = 83.3$$

which requires a parallel resistance of

$R = \omega_0 L Q$

$= 2\pi \times 10^6 \times 10 \times 10^{-6} \times 83.3$

$= 5.23$ k$\Omega$

Thus, the additional parallel resistance required, $R_a$, can be determined from

$R_a \parallel R_p = R$

$R_a \parallel 15.71 = 5.23$

$\Rightarrow R_a = 7.84$ k$\Omega$

**13.87** $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{36 \times 10^{-6} \times 10^3 \times 10^{-12}}}$$

$= 838.8$ kHz

Equivalent parallel resistance $= 3^2 \times 1 = 9$ k$\Omega$

$$Q = \frac{R_p}{\omega_0 L}$$

$$= \frac{9 \times 10^3}{2\pi \times 838.8 \times 10^3 \times 36 \times 10^{-6}}$$

$= 47.4$

**13.88**

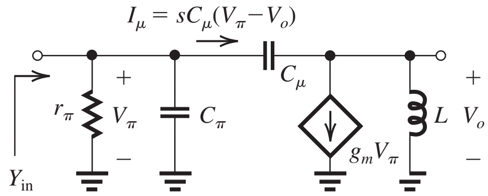



Figure 1

Refer to Fig. 1. A node equation at the output node yields

$$sC_\mu(V_\pi - V_o) = g_m V_\pi + \frac{V_o}{sL}$$

$$\Rightarrow V_o = V_\pi \frac{sC_\mu - g_m}{\dfrac{1}{sL} + sC_\mu}$$

Now, we can find $I_\mu$ as

$$I_\mu = sC_\mu(V_\pi - V_o)$$

$$= sC_\mu V_\pi - sC_\mu V_o$$

Thus,

$$\frac{I_\mu}{V_\pi} = sC_\mu - sC_\mu \frac{sC_\mu - g_m}{\dfrac{1}{sL} + sC_\mu}$$

$$= sC_\mu \left[ \frac{\dfrac{1}{sL} + sC_\mu - sC_\mu + g_m}{\dfrac{1}{sL} + sC_\mu} \right]$$

$$= sC_\mu \frac{\dfrac{1}{sL} + g_m}{\dfrac{1}{sL} + sC_\mu}$$

For $s = j\omega$, we obtain

$$\frac{I_\mu}{V_\pi} = j\omega C_\mu \frac{g_m + \dfrac{1}{j\omega L}}{j\omega C_\mu + \dfrac{1}{j\omega L}}$$

$$= j\omega C_\mu \frac{1 + j\omega L g_m}{1 - \omega^2 L C_\mu}$$

since $\omega C_\mu \ll \dfrac{1}{\omega L}$

$\Rightarrow \omega^2 L C_\mu \ll 1$

Thus,

$$\frac{I_\mu}{V_\pi} \simeq j\omega C_\mu (1 + j\omega L g_m)$$

$$= j\omega C_\mu - \omega^2 L C_\mu g_m$$

Returning to Fig. 1, we can write

$$Y_{\text{in}} = \frac{1}{r_\pi} + j\omega C_\mu + \frac{I_\mu}{V_\pi}$$

$$= \frac{1}{r_\pi} + j\omega C_\pi + j\omega C_\mu - \omega^2 C_\mu L g_m$$

$$= \left(\frac{1}{r_\pi} - \omega^2 C_\mu L g_m\right) + j\omega(C_\pi + C_\mu) \qquad \text{Q.E.D.}$$

**13.89** (a) $T(s) = \dfrac{sK\left(\dfrac{\omega_0}{Q}\right)}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$

where

$K = |T(j\omega_0)|$

Thus,

$$|T(j\omega)| = \frac{|T(j\omega_0)|}{\sqrt{1 + Q^2\left(\dfrac{\omega_0^2 - \omega^2}{\omega\omega_0}\right)^2}} \qquad (1)$$

Now, consider the quantity $\dfrac{\omega_0^2 - \omega^2}{\omega\omega_0}$. For $\omega = \omega_0 + \delta\omega$ where $(\delta\omega/\omega_0) \ll 1$,

$$\omega = \omega_0\left(1 + \frac{\delta\omega}{\omega_0}\right)$$

$$\omega^2 \simeq \omega_0^2\left(1 + \frac{2\delta\omega}{\omega_0}\right)$$

Thus,

$$\frac{\omega_0^2 - \omega^2}{\omega\omega_0} = \frac{\omega_0^2 - \omega_0^2(1 + 2\delta\omega/\omega_0)}{\omega_0^2(1 + \delta\omega/\omega_0)}$$

$$= \frac{-2\delta\omega/\omega_0}{1 + \dfrac{\delta\omega}{\omega_0}} \simeq -\frac{2\delta\omega}{\omega_0}$$

Substituting in Eq. (1), we obtain

$$|T(j\omega)| = \frac{|T(j\omega_0)|}{\sqrt{1 + 4Q^2\left(\dfrac{\delta\omega}{\omega_0}\right)^2}}$$

(b) For $N$ syncronously tuned sections connected in cascade, we obtain

$$|T(j\omega)|^N = \frac{|T(j\omega_0)|^N}{[1 + 4Q^2(\delta\omega/\omega_0)^2]^{N/2}}$$

The 3-dB bandwidth is the value of $(2\delta\omega)$ at which

$$|T(j\omega)|^N = \frac{1}{\sqrt{2}}|T(j\omega_0)|^N$$

Denoting, $2\delta\omega = B$, we obtain

$$[1 + Q^2(B/\omega_0)^2]^{N/2} = \sqrt{2}$$

$$B = \left(\frac{\omega_0}{Q}\right)\sqrt{2^{1/N} - 1} \qquad \text{Q.E.D.}$$