# HW5

## 13.53

**13.53** Bandpass with $f_0 = 2$ kHz, 3-dB bandwidth of 50 Hz, thus

$$Q = \frac{f_0}{BW} = \frac{2 \text{ kHz}}{50 \text{ Hz}} = 40$$

Refer to the circuit in Fig. 13.24(a). Using

$$C = 10 \text{ nF}$$

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi \times 2 \times 10^3 \times 10 \times 10^{-9}}$$

$$= 7.96 \text{ k}\Omega$$

Select

$$R_1 = 10 \text{ k}\Omega$$

then

$$R_f = R_1 = 10 \text{ k}\Omega$$

Select

$$R_2 = 1 \text{ k}\Omega$$

then

$$R_3 = (2Q - 1)R_2 = (80 - 1) \times 1 = 79 \text{ k}\Omega$$

$$K = 2 - \frac{1}{Q} = 2 - \frac{1}{40} = 1.975$$

Center-frequency gain $= KQ = 1.975 \times 40 = 79$

13.56

**13.56** Refer to Fig. 13.26 and Table 13.2. Using

$C = 10$ nF,

then

$$R = \frac{1}{\omega_0 C} = \frac{1}{10^5 \times 10 \times 10^{-9}}$$

$= 1$ k$\Omega$

$R_d = QR = 10 \times 1 = 10$ k$\Omega$

Select

$r = 20$ k$\Omega$

$R_1 = R_3 = \infty$

If the dc gain is unity, then

$$1 = \text{HF gain} \times \frac{\omega_n^2}{\omega_0^2}$$

$$\Rightarrow \text{HF gain} = \left(\frac{10^5}{1.3 \times 10^5}\right)^2$$

$= 0.5917$

$C_1 = C \times$ high-frequency gain

$= 10 \times 0.5917 = 5.92$ nF

$$R_2 = R\frac{(\omega_0/\omega_n)^2}{\text{HF gain}} = 1 \text{ k}\Omega$$

13.63

**13.63**

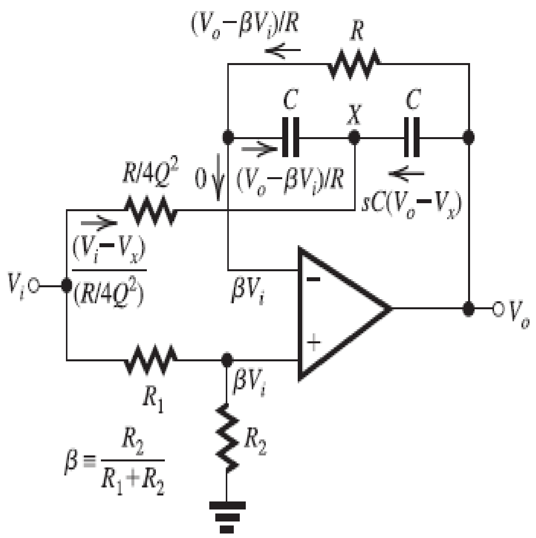



Figure 1

The circuit is shown in Fig. 1. The voltage at node $X$ is given by

$$V_x = \beta V_i - \frac{1}{sC}\,\frac{V_o - \beta V_i}{R}$$

$$= \beta V_i\left(1 + \frac{1}{sCR}\right) - \frac{V_o}{sCR} \qquad (1)$$

Writing a node equation at $X$ gives

$$\frac{V_o - \beta V_i}{R} + \frac{V_i - V_x}{R/4Q^2} + sC(V_o - V_x) = 0$$

Substituting for $V_x$ from Eq. (1) and collecting terms gives

$$\frac{V_i}{V_x} = \beta\,\frac{s^2 + s\dfrac{2}{CR}\left[1 + 2Q^2\left(1 - \dfrac{1}{\beta}\right)\right] + \dfrac{4Q^2}{C^2R^2}}{s^2 + s\dfrac{2}{CR} + \dfrac{4Q^2}{C^2R^2}}$$

We observe that, as expected,

$$\omega_0 = \frac{2Q}{CR}$$

(a) To obtain an all-pass function, we set

$$\frac{2}{CR}\left[1 + 2Q^2\left(1 - \frac{1}{\beta}\right)\right] = -\frac{2}{CR}$$

$$\Rightarrow \frac{1}{\beta} = 1 + \frac{1}{Q^2}$$

But,

$$\beta = \frac{R_2}{R_1 + R_2}$$

Thus,

$$\frac{R_2}{R_1} = Q^2$$

(b) To obtain a notch function, we set

$$\frac{2}{CR}\left[1 + 2Q^2\left(1 - \frac{1}{\beta}\right)\right] = 0$$

$$\Rightarrow \frac{1}{\beta} = 1 + \frac{1}{2Q^2}$$

or, equivalently,

$$\frac{R_2}{R_1} = 2Q^2$$

13.64

**13.64**

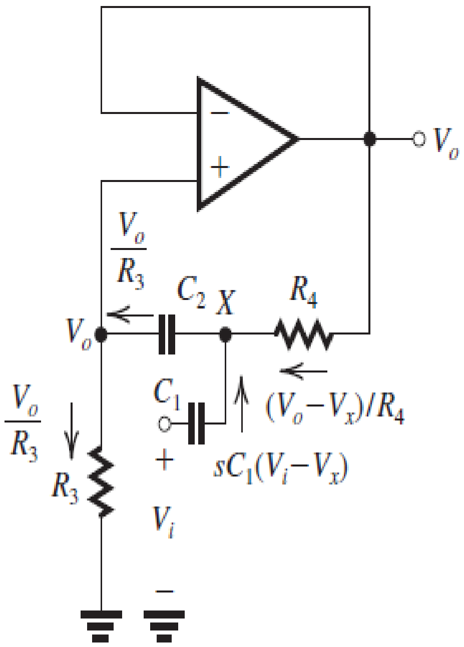



Figure 1

The analysis is shown in Fig. 1, below, left. The voltage at node $X$ is given by

$$V_x = V_o + \frac{V_o}{R_3}\frac{1}{sC_2}$$

$$= V_o\left(1 + \frac{1}{sC_2R_3}\right) \qquad (1)$$

A node equation at $X$ provides

$$\frac{V_o - V_x}{R_4} + sC_1(V_i - V_x) = \frac{V_o}{R_3}$$

$$V_o\left(\frac{1}{R_4} - \frac{1}{R_3}\right) + sCV_i - V_x\left(\frac{1}{R_4} + sC_1\right) = 0$$

Substituting for $V_x$ from Eq. (1) and collecting terms, we obtain

$$\frac{V_o}{V_i} = \frac{s^2}{s^2 + s\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{1}{C_1C_2R_3R_4}}$$

This is a high-pass function with a high-frequency gain of unity. To obtain a maximally flat response with $\omega_{3dB} = 10^4$ rad/s and using

$$C_1 = C_2 = C = 10 \text{ nF}$$

then

$$\omega_0 = \omega_{3dB} = 10^4 \text{ rad/s}$$

$$Q = \frac{1}{\sqrt{2}} = \omega_0 R_3 \Big/ \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{1}{\sqrt{2}} = 10^4 R_3 \Big/ \left(\frac{2}{10 \times 10^{-9}}\right)$$

$$\Rightarrow R_3 = \frac{1}{\sqrt{2}} \times \frac{2}{10^{-8}} \times 10^{-4} = 14.14 \text{ k}\Omega$$

$$\omega_0^2 = \frac{1}{C_1C_2R_3R_4}$$

$$10^8 = \frac{1}{10^{-8} \times 10^{-8} \times 14.4 \times 10^3 \times R_4}$$

$$\Rightarrow R_4 = \frac{100}{14.14} \text{ k}\Omega = 7.07 \text{ k}\Omega$$

13.65

**13.65**

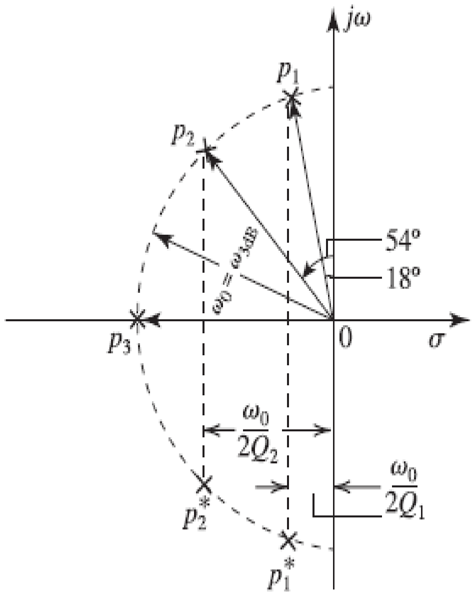



Figure 1

Figure 1 (see previous page) shows a graphical construct to determine the poles of the fifth order Butterworth filter. The pair of complex conjugate poles $p_1$ and $p_1^*$ have a frequency

$$\omega_{01} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a $Q$ factor

$$Q_1 = \frac{1}{2 \sin 18^\circ} = 1.618$$

The pair of complex conjugate poles $p_2$ and $p_2^*$ have

$$\omega_{02} = \omega_{3dB} = 2\pi \times 10^4 \text{ rad/s}$$

and a $Q$ factor,

$$Q_2 = \frac{1}{2 \sin 54^\circ} = 0.618$$

The real-axis pole $p_3$ is at

$$s = -\omega_0 = -2\pi \times 10^4 \text{ rad/s}$$

The first second-order section can be realized using the circuit in Fig. 13.34(c). The design equations are (13.77)–(13.80).

$$R_1 = R_2 = R = 10 \text{ k}\Omega$$

$$C_4 = C$$

$$C_3 = C/4Q^2$$

Here, $Q = Q_1 = 1.618$, thus

$$C_3 = \frac{C}{4 \times 1.618^2} = 0.095C$$

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 1.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 1.618}{2\pi \times 10^4 \times 10 \times 10^3} = 5.15 \text{ nF}$$

$$C_3 = 0.492 \text{ nF} = 492 \text{ pF}$$

$$C_4 = 5.15 \text{ nF}$$

The second second-order section also can be realized using the circuit in Fig. 13.34(c). Here,

$$R_1 = R_2 = R = 10\ \text{k}\Omega$$

$$C_4 = C$$

$$C_3 = \frac{C}{4Q^2}$$

where $Q = Q_2 = 0.618$. Thus

$$C_3 = \frac{C}{4 \times 0.618^2} = 0.655C$$

$$CR = \frac{2Q}{\omega_0} = \frac{2 \times 0.618}{2\pi \times 10^4}$$

$$\Rightarrow C = \frac{2 \times 0.618}{2\pi \times 10^4 \times 10^4} = 1.97\ \text{nF}$$

$$C_3 = 1.29\ \text{nF}$$

$$C_4 = 1.97\ \text{nF}$$

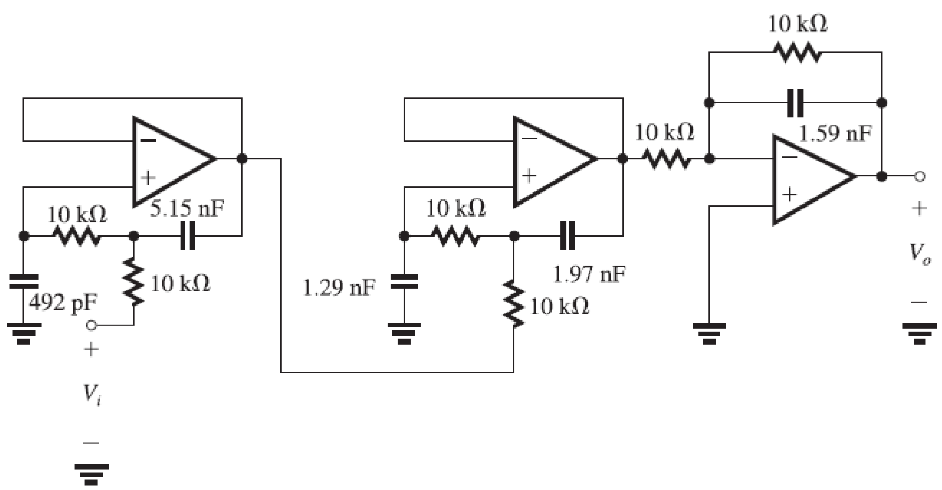
The first-order section can be realized using the circuit in Fig. 13.13(a) with

$$R_1 = R_2 = R = 10\ \text{k}\Omega$$

$$C = \frac{1}{\omega_0 R}$$

$$= \frac{1}{2\pi \times 10^4 \times 10^4} = 1.59\ \text{nF}$$

This figure belongs to Problem 13.65.



# HW6

## 13.74

**13.74** For the integrator in Fig. 13.36(b), we have

$$\frac{V_o}{V_1} = \frac{G_m}{sC}$$

$$\text{Unity-gain frequency} = \frac{G_m}{2\pi C}$$

Thus,

$$10 \times 10^6 = \frac{G_m}{2\pi \times 5 \times 10^{-12}}$$

$$\Rightarrow G_m = 0.314 \text{ mA/V}$$

## 13.76

**13.76** Refer to the circuit in Fig. 13.36(c) and its transfer function in Eq. (13.91), namely

$$\frac{V_o}{V_i} = -\frac{G_{m1}}{sC + G_{m2}}$$

$$\text{Pole frequency} = \frac{G_{m2}}{2\pi C}$$

$$20 \times 10^6 = \frac{G_{m2}}{2\pi \times 2 \times 10^{-12}}$$

$$\Rightarrow G_{m2} = 0.251 \text{ mA/V}$$

$$|\text{DC gain}| = \frac{G_{m1}}{G_{m2}}$$

$$10 = \frac{G_{m1}}{G_{m2}}$$

$$\Rightarrow G_{m1} = 2.51 \text{ mA/V}$$

13.78

**13.78** (a) Refer to the circuit in Fig. P13.78. The output current of the $G_{m2}$ transconductor is $G_{m2}V_1$. Thus,

$$V_2 = \frac{G_{m2}}{sC} V_1$$

This is the input voltage to the negative transconductor $G_{m1}$. Thus the output current of $G_{m1}$, which is equal to $I_1$, will be

$$I_1 = G_{m1} V_2$$
$$= \frac{G_{m1} G_{m2}}{sC} V_1$$

$$Z_{\text{in}} \equiv \frac{V_1}{I_1} = s\frac{C}{G_{m1} G_{m2}}$$

which is that of an inductance $L$,

$$L = \frac{C}{G_{m1} G_{m2}} \qquad \text{Q.E.D.}$$

(b) To obtain an LCR resonator, we connected a capacitor $C$ from node 1 to ground and a resistance $R$ realized by the transconductor $G_{m3}$, as shown in Fig. 1 below.

(c) A fourth transconductor $G_{m4}$ is used to feed a current $G_{m4} V_i$ to node 1, as shown in Fig. 1. The resulting circuit is identical to that in Fig. 13.37(b) except here $C_1 = C_2 = C$.

(d) With analogy to the identical circuit in Fig. 13.37(b), $V_1/V_i$ will be a second-order bandpass filter with a transfer function given by Eq. (13.93), and $V_2/V_i$ will be a second-order low-pass filter with a transfer function given by Eq. (13.94). Thus,

$$\frac{V_1}{V_i} = -\frac{s(G_{m4}/C)}{s^2 + s\dfrac{G_{m3}}{C_1} + \dfrac{G_{m1} G_{m2}}{C^2}}$$

and,

$$\frac{V_2}{V_i} = -\frac{G_{m2} G_{m4}/C^2}{s^2 + s\dfrac{G_{m2}}{C} + \dfrac{G_{m1} G_{m2}}{C^2}}$$

This figure belongs to Problem 13.78, part (b).

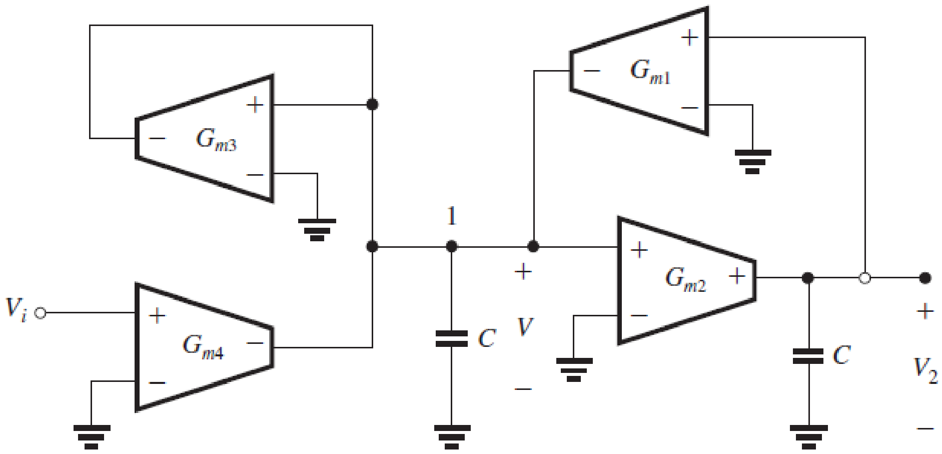



Figure 1

13.80

**13.80** $f_0 = 25$ MHz, $Q = 5$,

Center-frequency gain $= 5$

The design equations are given by (13.99), (13.100), and (13.101). Thus

$G_m = \omega_0 C$

where

$C = C_1 = C_2 = 5$ pF

$G_m = 2\pi \times 25 \times 10^6 \times 5 \times 10^{-12}$

$= 0.785$ mA/V

Thus,

$G_{m2} = G_m = 0.785$ mA/V

$$G_{m3} = \frac{G_m}{Q} = \frac{0.785}{5} = 0.157 \text{ mA/V}$$

$$G_{m4} = \frac{G_m}{Q}|\text{Gain}|$$

$$= \frac{0.785}{5} \times 5 = 0.785 \text{ mA/V}$$

13.82

$$\textbf{13.82}\ R_{eq} = \frac{T_c}{C_1} = \frac{1}{f_c C_1} = \frac{1}{200 \times 10^3 C_1}$$

For $C_1 = 1$ pF,

$$R_{eq} = \frac{1}{200 \times 10^3 \times 1 \times 10^{-12}} = 5\ \text{M}\Omega$$

For $C_1 = 5$ pF,

$$R_{eq} = \frac{1}{200 \times 10^3 \times 5 \times 10^{-12}} = 1\ \text{M}\Omega$$

For $C_1 = 10$ pF,

$$R_{eq} = \frac{1}{200 \times 10^3 \times 10 \times 10^{-12}} = 500\ \text{k}\Omega$$

13.84

**13.84** $\omega_0 = \omega_{3dB} = 10^3$ rad/s

$Q = 1/\sqrt{2}$ and DC gain $= 1$

$f_c = 100$ kHz, $C_1 = C_2 = C = 5$ pF

From Eqs. (13.109) and (13.110),

$$C_3 = C_4 = \omega_0 T_c C$$

$$= 10^3 \times \frac{1}{100 \times 10^3} \times 5 \times 10^{-12}$$

$$= 0.05 \text{ pF}$$

From Eq. (13.112),

$$C_5 = \frac{C_4}{Q} = \frac{0.05}{1/\sqrt{2}} = 0.071 \text{ pF}$$

The dc gain of the low-pass circuit is

$$\text{DC gain} = \frac{C_6}{C_4}$$

For DC gain $= 1$,

$$C_6 = C_4 = 0.05 \text{ pF}$$