HW7_ch14_12.15.18.22.27.28

HW8_ch14_30.36.40

HW7

14.12

14.12 If the closed-loop amplifier in Fig. 14.5 exhibits a phase shift of $-3°$ for ω around ω_0, then the loop-gain expression in Eq. (14.11) becomes

$$L(j\omega) = \frac{(1 + R_2/R_1)e^{-j\phi}}{3 + j(\omega CR - 1/\omega CR)}$$

where

$$\phi = \frac{3\pi}{180} = \pi/60$$

Oscillation will occur at the frequency ω_0 for which the phase angle of $L(j\omega)$ is $0°$:

$$-\phi = \tan^{-1}\frac{1}{3}\left(\omega_0 CR - \frac{1}{\omega_0 CR}\right)$$

$$\omega_0 CR - \frac{1}{\omega_0 CR} = -3\tan 3° = -0.157$$

$$\Rightarrow \omega_0^2 + \frac{0.157}{CR}\omega_0 - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_0 = \frac{0.925}{CR}$$

14.15

14.15 First we design the circuit to operate at 10 kHz.

$$\omega_0 = \frac{1}{CR}$$

$$2\pi \times 10 \times 10^3 = \frac{1}{CR}$$

$$\Rightarrow CR = 0.159 \times 10^{-4}\ \text{s}$$

For $R = 10\ \text{k}\Omega$, we have

$$C = \frac{0.159 \times 10^{-4}}{10 \times 10^3} = 1.59\ \text{nF}$$

Now, refer to Eq. (14.11). If the closed-loop amplifier has an excess phase lag of 5.7°, then the gain will be $\left(1 + \frac{R_2}{R_1}\right)e^{-j5.7°}$. Oscillations will occur at the frequency ω_{01} at which the phase angle of the denominator is −5.7°, that is,

$$\tan^{-1}\frac{1}{3}\left(\omega_{01}CR - \frac{1}{\omega_{01}CR}\right) = -5.7°$$

$$\omega_{01}CR - \frac{1}{\omega_{01}CR} = 3\tan(-5.7°) = -0.3$$

$$\Rightarrow \omega_{01}^2 + \frac{0.3}{CR}\omega_{01} - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_{01} = \frac{0.86}{CR}$$

That is, the frequency of oscillation is reduced by 14% to

$$f_{01} = 0.86 f_0 = 8.6\ \text{kHz}$$

To restore operation to $f_0 = 10$ kHz, we modify the shunt resistor R to R_x, as indicated in Fig. 1.

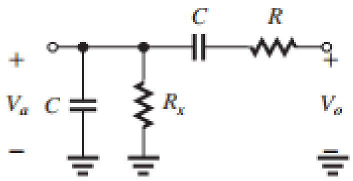


Figure 1

We now require the feedback RC circuit to have a phase shift of −(−5.7°) = +5.7° at f = 10 kHz. The transfer function of the RC circuit can be found as follows:

$$\frac{V_a}{V_o} = \frac{Z_p}{Z_p + Z_s}$$

$$= \frac{1}{1 + Z_s Y_p}$$

$$= \frac{1}{1 + \left(R + \frac{1}{sC}\right)\left(\frac{1}{R_x} + sC\right)}$$

$$= \frac{1}{\left(2 + \frac{R}{R_x}\right) + sCR + \frac{1}{sCR_x}}$$

For $s = j\omega$, we have

$$\frac{V_a}{V_o} = \frac{1}{\left(2 + \frac{R}{R_x}\right) + j\left(\omega CR - \frac{1}{\omega CR_x}\right)}$$

At $\omega = \omega_0$, the phase angle of $\frac{V_a}{V_o}$ must be +5.7° or equivalently, the phase angle of the denominator must be −5.7°. Thus,

$$\tan^{-1}\frac{\omega_0 CR - \frac{1}{\omega_0 CR_x}}{2 + \frac{R}{R_x}} = -5.7°$$

$$\omega_0 CR - \frac{1}{\omega_0 CR_x} = \left(2 + \frac{R}{R_x}\right)\tan(-5.7°)$$

$$= \left(2 + \frac{R}{R_x}\right) \times -0.0998$$

Now, $\omega_0 CR = 1$, thus

$$1 - \frac{R}{R_x} = -0.0998\left(2 + \frac{R}{R_x}\right)$$

$$1 + 2 \times 0.0998 = \frac{R}{R_x}(1 - 0.0998) \Rightarrow \frac{R}{R_x} = 1.33$$

$$\Rightarrow R_x = 0.75\ R = 7.5\ \text{k}\Omega$$

At $\omega = \omega_0$ and for $R_x = 7.5\ \text{k}\Omega$

$$\frac{V_a}{V_o}(\omega_0) = \frac{1}{\left(2 + \frac{10}{7.5}\right) + j\left(1 - \frac{10}{7.5}\right)}$$

$$= \frac{1}{3.33 - j0.33}$$

$$\left|\frac{V_a}{V_o}(\omega_0)\right| = \frac{1}{\sqrt{(3.33)^2 + (0.33)^2}} = \frac{1}{3.35}$$

Thus the magnitude of the gain of the amplifier must be 3.35 V/V. Thus, R_2/R_1 must be changed to

$$\frac{R_2}{R_1} = 2.35$$

14.18

14.18 Figure 1 shows the circuit with the additional resistance R included. The loop has been broken at the output of the op amp. The analysis will determine V_o/V_x and equate it to unity, which is the condition for sustained oscillations.

To begin, observe that the voltage V_1 is related to V_o by

$$\frac{V_o}{V_1} = -\frac{R_f}{R} \qquad (1)$$

Also, the current I_1 is given by

$$I_1 = \frac{V_1}{R} \qquad (2)$$

We now proceed to determine the various currents and voltages of the RC network as follows:

$$V_2 = V_1 + \frac{1}{sC} I_1$$

$$= V_1 + \frac{1}{sC}\frac{V_1}{R} = V_1\left(1 + \frac{1}{sCR}\right)$$

$$I_2 = \frac{V_2}{R} = \frac{V_1}{R}\left(1 + \frac{1}{sCR}\right)$$

$$I_3 = I_1 + I_2 = \frac{V_1}{R} + \frac{V_1}{R}\left(1 + \frac{1}{sCR}\right)$$

$$= \frac{V_1}{R}\left(2 + \frac{1}{sCR}\right)$$

$$V_3 = V_2 + \frac{I_3}{sC}$$

$$= V_1\left(1 + \frac{1}{sCR}\right) + \frac{V_1}{sCR}\left(2 + \frac{1}{sCR}\right)$$

$$= V_1\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$I_4 = \frac{V_3}{R} = \frac{V_1}{R}\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

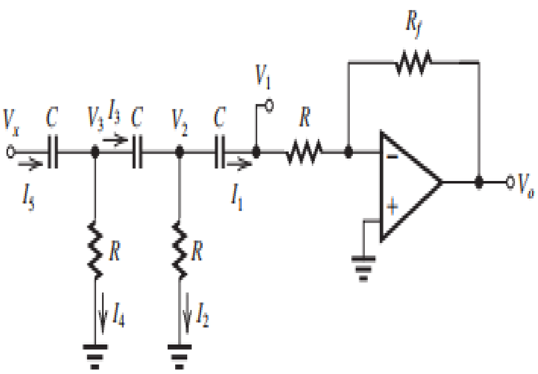


Figure 1

$$I_5 = I_3 + I_4$$

$$= \frac{V_1}{R}\left(2 + \frac{1}{sCR}\right) + \frac{V_1}{R}\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$= \frac{V_1}{R}\left(3 + \frac{4}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$V_x = V_3 + \frac{I_5}{sC}$$

$$= V_1\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$+ \frac{V_1}{sCR}\left(3 + \frac{4}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$= V_1\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)$$

Now, by replacing V_1 by the value from Eq. (1), we obtain

$$V_x = -V_o\frac{R}{R_f}\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)$$

For sustained oscillations $V_o = V_x$, thus

$$-\frac{R_f}{R} = 1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}$$

For $s = j\omega$, we have

$$-\frac{R_f}{R} = 1 + \frac{6}{j\omega CR} - \frac{5}{\omega^2C^2R^2} - \frac{1}{j\omega^3C^3R^3}$$

$$= \left(1 - \frac{5}{\omega^2C^2R^2}\right) - j\left(\frac{6}{\omega CR} - \frac{1}{\omega^3C^3R^3}\right)$$

Thus, oscillation will occur at the frequency that renders the imaginary part of the RHS zero:

$$\frac{6}{\omega_0 CR} = \frac{1}{\omega_0^3C^3R^3} :$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{6}CR}$$

At this frequency, the real part of the RHS must be equal to $(-R_f/R)$:

$$-\frac{R_f}{R} = 1 - \frac{5}{1/6} = -29$$

Thus,

$$R_f = 29R$$

which is the minimum required value for R_f to obtain sustained oscillations. Numerical values:

$$f_0 = \frac{1}{2\pi\sqrt{6} \times 16 \times 10^{-9} \times 10 \times 10^3}$$

$$= 406 \text{ Hz}$$

$$R_f = 290 \text{ k}\Omega$$

14.22

14.22

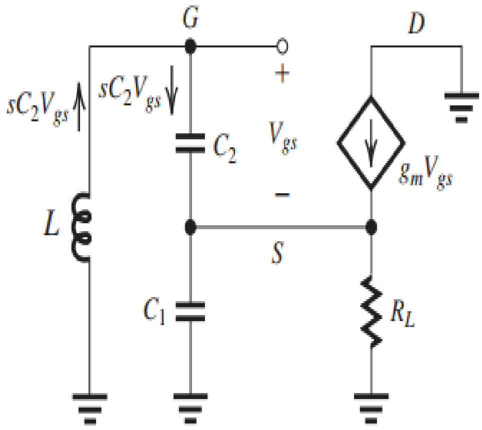


Figure 1

Figure 1 shows the equivalent circuit together with some of the analysis. The voltage at the gate, V_g, can be expressed as

$$V_g = -s^2LC_2V_{gs} \tag{1}$$

The voltage at the source, V_s, can be expressed as

$$V_s = V_g - V_{gs}$$

Thus,

$$V_s = -s^2LC_2V_{gs} - V_{gs} \tag{2}$$

A node equation at S provides

$$sC_2V_{gs} + g_mV_{gs} = \left(\frac{1}{R_L} + sC_1\right)V_s$$

Substituting for V_s from Eq. (2), we obtain

$$sC_2V_{gs} + g_mV_{gs} = -\left(\frac{1}{R_L} + sC_1\right)(s^2LC_2 + 1)V_{gs}$$

Dividing by V_{gs} and collecting terms, we obtain

$$s^3LC_1C_2 + s^2\frac{LC_2}{R_L} + s(C_1 + C_2) + \left(g_m + \frac{1}{R_L}\right) = 0$$

For $s = j\omega$, we have

$$j\omega[-\omega^2LC_1C_2 + (C_1 + C_2)] + \left(g_m + \frac{1}{R_L} - \omega^2\frac{LC_2}{R_L}\right) = 0 \tag{3}$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω at which the imaginary part is zero, thus

$$\omega_0^2 = 1\bigg/\left[L\left(\frac{C_1C_2}{C_1 + C_2}\right)\right] \tag{4}$$

$$\Rightarrow \omega_0 = 1\bigg/\sqrt{L\left(\frac{C_1C_2}{C_1 + C_2}\right)}$$

The condition for sustained oscillations can be found by equating the real part of Eq. (3) to zero and making use of (4), thus

$$g_m + \frac{1}{R_L} = \left(\frac{C_1 + C_2}{C_1}\right)\left(\frac{1}{R_L}\right)$$

$$\Rightarrow g_mR_L = \frac{C_2}{C_1}$$

To ensure that oscillations start, we use

$$g_mR_L > \frac{C_2}{C_1}$$

14.27

14.27 $\omega_0 = 20$ Grad/s $= 20 \times 10^9$ rad/s

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$20 \times 10^9 = \frac{1}{\sqrt{5 \times 10^{-9} \times C}}$$

$$\Rightarrow C = 0.5 \text{ pF}$$

$$R_p = \omega_0 LQ$$

$$= 20 \times 10^9 \times 5 \times 10^{-9} \times 10$$

$$= 1000\ \Omega = 1\ \text{k}\Omega$$

$$r_o \parallel R_p = 5 \parallel 1 = \frac{5}{6}\ \text{k}\Omega$$

$$g_m|_{\min} = \frac{1}{\frac{5}{6} \times 10^3} = 1.2 \text{ mA/V}$$

14.28

14.28 From Exercise 14.13, we have

$$L = 0.52 \text{ H}$$

$$C_s = 0.012 \text{ pF}$$

$$C_p = 4 \text{ pF}$$

$$C_{eq} = \frac{C_s\left(C_p + \dfrac{C_1 C_2}{C_1 + C_2}\right)}{C_s + C_p + \dfrac{C_1 C_2}{C_1 + C_2}}$$

$C_2 = 10$ pF $\quad C_1 = 1$ to 10 pF

$$C_L = \frac{0.012\left(4 + \dfrac{10 \times 1}{10 + 1}\right)}{\left(0.012 + 4 + \dfrac{10}{11}\right)} = 0.01197 \text{ pF}$$

$$C_H = \frac{0.012\left(4 + \dfrac{10 \times 10}{10 + 10}\right)}{\left(0.012 + 4 + \dfrac{100}{20}\right)} = 0.01198 \text{ pF}$$

$$\therefore f_{0H} = \frac{1}{2\pi\left[0.52 \times 0.01197 \times 10^{-12}\right]^{1/2}}$$

$$= 2.0173 \text{ MHz}$$

$$f_{0L} = \left[2\pi\left(0.52 \times 0.01198 \times 10^{-12}\right)^{1/2}\right]^{-1}$$

$$= 2.0165 \text{ MHz}$$

Difference $= 800$ Hz

HW8

14.30

14.30

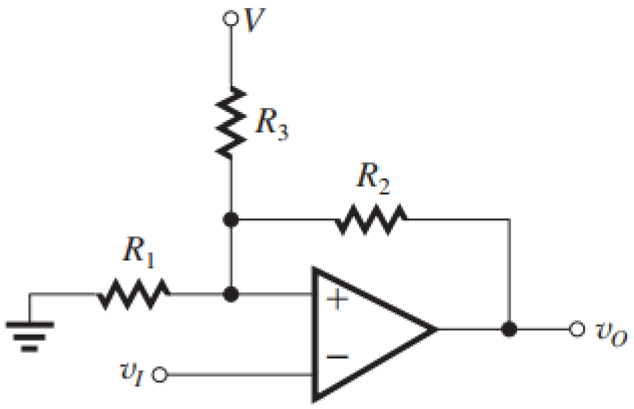


Figure 1

(a) Refer to Fig. 1. With $v_O = L_+$, the voltage at the op amp positive input terminal will be V_{TH}. Now, writing a node equation at the op amp positive input terminal, we have

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L_+ - V_{TH}}{R_2}$$

$$V_{TH}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{L_+}{R_2}+\frac{V}{R_3}$$

$$\Rightarrow V_{TH}=\left(\frac{L_+}{R_2}+\frac{V}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

Similarly, we can obtain

$$V_{TL}=\left(\frac{L_-}{R_2}+\frac{V}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

(b) $L_+ = -L_- = 10\text{ V},\; V = 15\text{ V},\; R_1 = 10\text{ k}\Omega$

$$V_{TH}=5.1=\left(\frac{10}{R_2}+\frac{15}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

$$\frac{5.1}{R_1}+\frac{5.1}{R_2}+\frac{5.1}{R_3}=\frac{10}{R_2}+\frac{15}{R_3}$$

$$0.51=\frac{4.9}{R_2}+\frac{9.9}{R_3} \qquad (1)$$

$$V_{TL}=4.9=\left(\frac{-10}{R_2}+\frac{15}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

$$\frac{4.9}{R_1}+\frac{4.9}{R_2}+\frac{4.9}{R_3}=-\frac{10}{R_2}+\frac{15}{R_3}$$

$$0.49=\frac{-14.9}{R_2}+\frac{10.1}{R_3} \qquad (2)$$

Multiplying Eq. (1) by $\left(\frac{14.9}{4.9}\right)$, we obtain

$$1.55=\frac{-14.9}{R_2}+\frac{30.1}{R_3} \qquad (3)$$

Adding (2) and (3) gives

$$2.04=\frac{40.2}{R_3}$$

$$\Rightarrow R_3 = 19.7\text{ k}\Omega$$

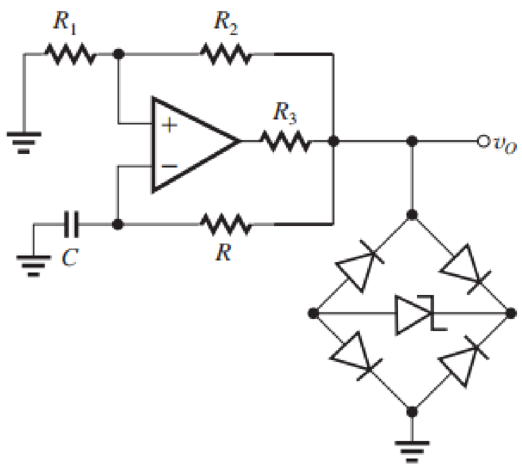
Substituting in Eq. (1), we obtain

$$0.51=\frac{4.9}{R_2}+\frac{9.9}{19.7}$$

$$\Rightarrow R_2=\frac{4.9}{0.0076}=656.7\text{ k}\Omega$$

14.36

14.36



$\beta = 0.462$

For $V_D = 0.7$ V and $V_O = \pm 5$ V, we have

$V_Z = 5 - 2V_D$

$V_Z = 3.6$ V

$$T = 2\tau \ln\left(\frac{1+\beta}{1-\beta}\right)$$

$$10^{-3} = 2\tau \ln\left(\frac{1.462}{1-0.462}\right) \Rightarrow \tau = 0.5 \text{ ms}$$

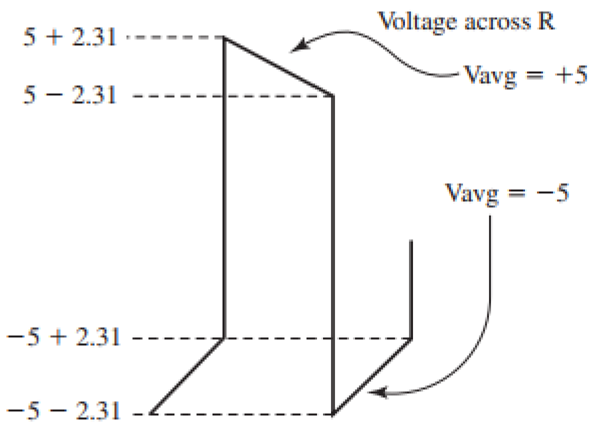
$\tau = RC \Rightarrow R = \tau/C = 50\ \text{k}\Omega$

Thresholds $= \pm 0.462 \times 5 = \pm 2.31$ V

Average current in R in $\frac{1}{2}$ cycle:

$$I \cong \frac{1}{R}\left(\frac{5 - 2.31 + 2.31 + 5}{2}\right)$$

$$= \frac{5}{R} = \frac{5}{50\ \text{k}\Omega} = 0.1 \text{ mA}$$



$$R_1 + R_2 = \frac{5 \text{ V}}{0.1 \text{ mA}} = 50\ \text{k}\Omega$$

$$\frac{R_1}{R_1 + R_2} = 0.462 \rightarrow R_1 = 50\,(0.462)$$

$= 23.1\ \text{k}\Omega$

$\therefore\ R_2 = 26.9\ \text{k}\Omega$

$$1 = \frac{13-5}{R_3} - 0.1 - 0.1$$

$$R_3 = \frac{8}{1.2}$$

$= 6.67\ \text{k}\Omega$

14.40

14.40 Choose $C_1 = 1$ nF and $C_2 = 0.1$ nF:

$$R_1 = R_2 = 100\ \text{k}\Omega \Rightarrow \beta \equiv \frac{1}{2}$$

$$T = C_1 R_3 \ln\left(\frac{0.7 + 13}{0.5 \times (-13) + 13}\right)$$

$$10^{-4} = 10^{-9} R_3 \ln\left(\frac{13.7}{13\,(0.5)}\right)$$

$$R_3 = 134.1\ \text{k}\Omega$$

Need $R_4 \gg R_1 \Rightarrow$ choose $R_4 = 470\ \text{k}\Omega$

The trigger pulse must be sufficiently large to lower the voltage at node C from βL_+ to V_D, that is, from +6.5 V to +0.7 V; thus it must be at least 5.8 V.

For recovery we have

$$v_B = 13 - \left(13 - \beta L_-\right) e^{-t/\tau}$$

$$= 13 - 19.5 e^{-t/\tau} = 0.7$$

$$\therefore\ t_{\text{recovery}} = -\tau \ln\left(\frac{12.3}{19.5}\right)$$

$$= -\left(134.1 \times 10^3\right)\left(10^{-9}\right)(-0.4608)$$

$$= 61.8\ \mu\text{s}$$