14.1 Since the second-order bandpass filter will exhibit zero phase at $\omega = \omega_0$, the circuit if it oscillates will do so at

$\omega = \omega_0$

For the circuit to oscillate, the magnitude of the gain at $\omega = \omega_0$ must be at least unity, thus

$AK \geq 1$

For sustained oscillations, we have

$AK = 1$

14.2 (a) The bandpass function can be written as

$$T(s) = \frac{Ks\dfrac{\omega_0}{Q}}{s^2 + s\left(\dfrac{\omega_0}{Q}\right) + \omega_0^2}$$

$$T(j\omega) = \frac{j\dfrac{\omega\omega_0}{Q}K}{(\omega_0^2 - \omega^2) + j\dfrac{\omega\omega_0}{Q}}$$

Thus,

$$\phi(\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2}\right)$$

$$\frac{d\phi}{d\omega} = \frac{1}{1 + \dfrac{1}{Q^2}\left[\dfrac{\omega\omega_0}{\omega_0^2 - \omega^2}\right]^2} \times \frac{1}{Q}$$

$$\times \frac{(\omega_0^2 - \omega^2)\omega_0 - \omega\omega_0(-2\omega)}{(\omega_0^2 - \omega^2)^2}$$

$$= -\frac{(\omega_0/Q)(\omega_0^2 + \omega^2)}{(\omega_0^2 - \omega^2)^2 + \dfrac{1}{Q^2}\omega^2\omega_0^2}$$

$$\frac{d\phi}{d\omega}(\omega = \omega_0) = -\frac{2Q}{\omega_0}$$

(b) For a change in phase $\triangle\phi$, the corresponding change in ω_0 will be

$$\triangle\omega_0 = \frac{\triangle\phi}{d\phi/d\omega}$$

$$= -\frac{\triangle\phi}{2Q/\omega_0}$$

$$\Rightarrow \frac{\triangle\omega_0}{\omega_0} = -\frac{\triangle\phi}{2Q}$$

14.3 The characteristic equation is obtained as follows:

$1 - L(s) = 0$

$$1 - A\,\frac{Ks\left(\dfrac{\omega_0}{Q}\right)}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} = 0$$

$$\Rightarrow s^2 + s\frac{\omega_0}{Q}(1 - AK) + \omega_0^2 = 0$$

Thus, the poles will be in the left half of the s-plane at a radial frequency ω_0 and a horizontal distance from the $j\omega$ axis of

$$\sigma = -\frac{\omega_0}{2Q}(1 - AK)$$

(a) For the poles to be on the $j\omega$ axis, we need

$\sigma = 0$

$\Rightarrow AK = 1$

(b) For the poles in the right half of the s-plane at a horizontal distance from the $j\omega$ axis of $\dfrac{\omega_0}{2Q}$, we have

$$\sigma = +\frac{\omega_0}{2Q}$$

which is achieved by making

$AK = 2$

14.4

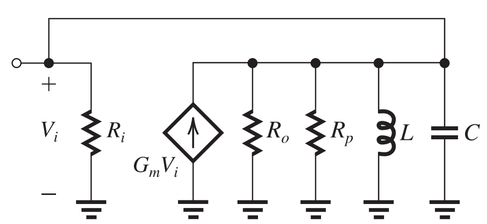


Figure 1

Figure 1 shows the resulting circuit. Here R_p can be found from

$R_p = \omega_0 LQ$

where

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-6} \times 100 \times 10^{-12}}}$$

$= 10^8$ rad/s

Thus,

$R_p = 10^8 \times 1 \times 10^{-6} \times 50$

$= 5\ \text{k}\Omega$

The total parallel resistance can now be found as

$R = R_i \parallel R_o \parallel R_p$

$= 5 \parallel 5 \parallel 5 = 1.67\ \text{k}\Omega$

The circuit will oscillate when the loop gain is unity, that is,

$G_m R = 1$

$$\Rightarrow G_m = \frac{1}{R} = 0.6\ \text{mA/V}$$

and the frequency of oscillation will be

$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^8}{2\pi} = 15.92 \text{ MHz}$$

14.5 (a) A change of $+1\%$ in the value of L causes a change of -0.5% in the value of ω_0.

(b) A change of $+1\%$ in the value C causes a change of -0.5% in the value of ω_0.

(c) Since ω_0 does not depend on the value of R, there will be no change in ω_0 as R changes by $+1\%$.

14.6 For the circuit to oscillate, two conditions must be satisfied:

(1) The total phase shift around the loop should be 0 or 360°, and

(2) The loop gain must be at least unity.

Here we have three amplifier stages (Fig. P14.6). Thus the phase angle of each amplifier at the oscillation frequency ω_0, must be 120°. Now, for each amplifier stage we have

$$V_o = -g_m V_i \frac{1}{\dfrac{1}{R} + sC}$$

$$\frac{V_o}{V_i} = -\frac{g_m R}{1 + sCR}$$

At $s = j\omega_0$, we have

$$\frac{V_o}{V_i}(j\omega_0) = -\frac{g_m R}{1 + j\omega_0 CR}$$

$$= -\frac{g_m R(1 - j\omega_0 CR)}{1 + (\omega_0 CR)^2}$$

$$= \frac{g_m R}{1 + (\omega_0 CR)^2}(-1 + j\omega_0 CR)$$

For the phase angle to be 120°, we need

$$\tan 60 = \omega_0 CR$$

$$\Rightarrow \omega_0 = \frac{\sqrt{3}}{CR}$$

$$\left|\frac{V_o}{V_i}(j\omega_0)\right| = \frac{g_m R}{\sqrt{1 + (\omega_0 CR)^2}} = \frac{g_m R}{\sqrt{1+3}}$$

$$= 0.5 g_m R$$

For a loop gain of unity, we have

$$(0.5 g_m R)^3 = 1$$

$$g_m R = 2$$

$$g_m|_{\min} = \frac{2}{R}$$

14.7 For oscillations to begin, the total phase shift around the loop at ω_0 must be 0 or 360°. Since the phase shift of the frequency-selective network is 180°, the amplifier must have a phase shift of 180°. Also, the loop gain at ω_0 must be at least unity. Since the frequency-selective network has 12-dB attenuation, the amplifier gain must be at least 12 dB or 4 V/V.

14.8 Refer to Fig. 14.4(e).

$$L_+ = -L_- = 3 \text{ V}$$

$$\frac{R_3}{R_1} = \frac{R_4}{R_1} = 0.05 \qquad (1)$$

Now,

$$L_+ = V\frac{R_4}{R_5} + 0.7\left(1 + \frac{R_4}{R_5}\right)$$

thus,

$$3 = 5\frac{R_4}{R_5} + 0.7\left(1 + \frac{R_4}{R_5}\right)$$

$$\Rightarrow \frac{R_4}{R_5} = \frac{2.3}{5.7} \qquad (2)$$

Similarly, from the equation for L_- we obtain

$$\frac{R_3}{R_2} = \frac{2.3}{5.7} \qquad (3)$$

Selecting $R_1 = 100$ kΩ, Eq. (1) gives

$$R_3 = R_4 = 5 \text{ k}\Omega$$

Then using Eqs. (2) and (3), we obtain

$$R_2 = R_5 = \frac{5 \times 5.7}{2.3} = 12.4 \text{ k}\Omega$$

14.9 Refer to Fig. 1 on the next page. By connecting V_B and R_B, we are injecting a current into the virtual ground node of V_B/R_B. To neutralize this current, v_I has to be at a negative value that pulls an equal current through R_1. Thus, the comparator threshold shifts from $v_I = 0$ to

$$v_I = -V_B\frac{R_1}{R_B}$$

To obtain a −2-V threshold with $V_B = 5$ V, we have

$$-2 = -5\frac{R_1}{R_B}$$

$$\Rightarrow \frac{R_1}{R_B} = \frac{2}{5} \qquad (1)$$

For a comparator input resistance of 100 kΩ, we have

$$R_1 = 100 \text{ k}\Omega$$

Using Eq. (1), we obtain

$$R_B = 250 \text{ k}\Omega$$

This figure belongs to Problem 14.9.

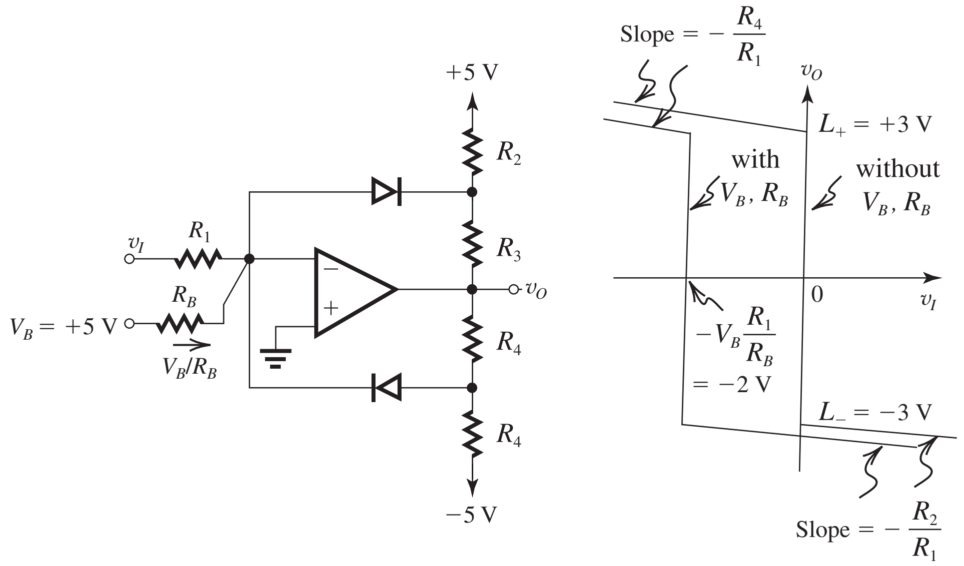


Figure 1

To obtain a slope of 0.05 in the limiting regions, we use

$$\frac{R_3}{R_1} = \frac{R_4}{R_1} = 0.05$$

thus,

$$R_3 = R_4 = 5\ \text{k}\Omega$$

To obtain $L_+ = -L_- = 3$ V, we use Eqs. (14.8) and (14.9) with $V_D = 0$, thus

$$\frac{R_3}{R_2} = \frac{R_4}{R_5} = \frac{3}{5}$$

$$\Rightarrow R_2 = R_5 = \frac{5 \times 5}{3} = 8.33\ \text{k}\Omega$$

For standard 5% resistors, use: $R_1 = 100\ \text{k}\Omega$, $R_B = 240\ \text{k}\Omega$, $R_3 = R_4 = 5.1\ \text{k}\Omega$, $R_2 = R_4 = 8.2\ \text{k}\Omega$.

14.10 Refer to Fig. 14.5.

$$\frac{V_a}{V_b} = \frac{Z_p}{Z_p + Z_s}$$

$$= \frac{1}{1 + Z_s Y_p}$$

$$= \frac{1}{1 + \left(R + \frac{1}{sC}\right)\left(\frac{1}{R} + sC\right)}$$

$$= \frac{1}{1 + 1 + 1 + sCR + \frac{1}{sCR}}$$

$$= \frac{s/CR}{s^2 + s(3/CR) + \frac{1}{(CR)^2}}$$

which is a bandpass function with a center frequency ω_0 given by

$$\omega_0 = \frac{1}{CR}$$

and a pole-Q of

$$Q = \frac{1}{3}$$

and a center-frequency gain of

$$\text{Gain} = \frac{1}{3}$$

14.11 The characteristic equation can be written using the expression for the loop gain in Eq. (14.10) as follows:

$$1 - L(s) = 0$$

$$1 - \frac{1 + R_2/R_1}{3 + sCR + \frac{1}{sCR}} = 0$$

$$3 + sCR + \frac{1}{sCR} - 1 - \frac{R_2}{R_1} = 0$$

$$s^2 + s\left(2 - \frac{R_2}{R_1}\right) \Big/ CR + \frac{1}{(CR)^2} = 0$$

Thus the poles have

$$\omega_0 = \frac{1}{CR}$$

and

$$Q = \frac{1}{2 - \frac{R_2}{R_1}}$$

The poles will be on the $j\omega$ axis for $R_2/R_1 = 2$ and will be in the right half of the s-plane for $R_2/R_1 > 2$. Q.E.D.

14.12 If the closed-loop amplifier in Fig. 14.5 exhibits a phase shift of $-3°$ for ω around ω_0, then the loop-gain expression in Eq. (14.11) becomes

$$L(j\omega) = \frac{(1 + R_2/R_1)e^{-j\phi}}{3 + j(\omega CR - 1/\omega CR)}$$

where

$$\phi = \frac{3\pi}{180} = \pi/60$$

Oscillation will occur at the frequency ω_0 for which the phase angle of $L(j\omega)$ is $0°$:

$$-\phi = \tan^{-1}\frac{1}{3}\left(\omega_0 CR - \frac{1}{\omega_0 CR}\right)$$

$$\omega_0 CR - \frac{1}{\omega_0 CR} = -3\tan 3° = -0.157$$

$$\Rightarrow \omega_0^2 + \frac{0.157}{CR}\omega_0 - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_0 = \frac{0.925}{CR}$$

14.13 Refer to Fig. 14.6. Assume that $R_3 = R_6$ and $R_4 = R_5$ and consider the magnitude of the positive peak. When $v_O = \hat{V}_o$, D_2 just conducts and clamps node b to a voltage

$V_b = V_1 + V_D \simeq \frac{\hat{V}_o}{3} + V_D$. Neglecting the current through D_2, we can write

$$\frac{\hat{V}_o - V_b}{R_5} = \frac{V_b - (-15)}{R_6}$$

Thus,

$$\hat{V}_o = \left(1 + \frac{R_5}{R_6}\right)V_b + 15\frac{R_5}{R_6}$$

Substituting

$$V_b = \frac{1}{3}\hat{V}_o + V_D$$

we obtain

$$\hat{V}_o = \frac{15\left(\frac{R_5}{R_6}\right) + \left(1 + \frac{R_5}{R_6}\right)V_D}{\frac{2}{3} - \frac{1}{3}\frac{R_5}{R_6}} \qquad (1)$$

To obtain an 8-V peak-to-peak output, we use

$$\hat{V}_o = 4 = \frac{15\left(\frac{R_5}{R_6}\right) + \left(1 + \frac{R_5}{R_6}\right) \times 0.7}{\frac{2}{3} - \frac{1}{3}\frac{R_5}{R_6}}$$

$$\Rightarrow \frac{R_5}{R_6} = 0.115$$

Since $R_5 = 1\ \text{k}\Omega$, then

$$R_6 = 8.66\ \text{k}\Omega$$

and

$$R_3 = 8.66\ \text{k}\Omega$$

If R_3 and R_6 are open circuited, substituting $R_6 = \infty$ in Eq. (1) yields

$$\hat{V}_o = 1.5V_D = 1.05\ \text{V}$$

This result can be obtained directly from the circuit: If R_3 and R_6 are open circuited, the positive peak of the output will be the voltage at which D_2 conducts. At this point, $v_O = \hat{V}_o$ and $v_1 = \frac{1}{3}\hat{V}_o$, thus the voltage across V_D is $\frac{2}{3}\hat{V}_o$ or, equivalently,

$$\hat{V}_o = 1.5V_D$$

A similar situation occurs at the negative peak at which diode D_1 conducts and the voltage across it will be $\frac{2}{3}\hat{V}_o$, etc.

14.14

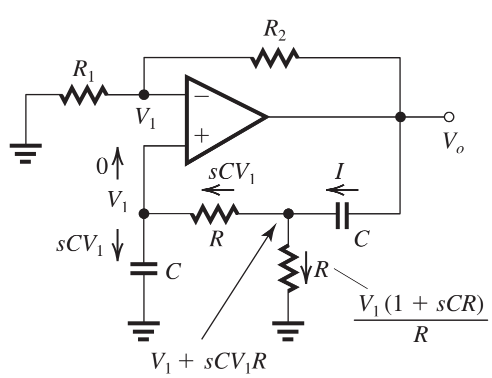


Figure 1

Figure 1 shows the circuit and some of the analysis for the purpose of determining the transfer function of the RC circuit. The current I is given by

$$I = \frac{V_1}{R}(1 + sCR) + sCV_1$$

For V_o we now can write

$$V_o = V_1(1 + sCR) + \frac{I}{sC}$$

$$= V_1(1 + sCR) + \frac{V_1}{sCR}(1 + sCR) + V_1$$

$$\Rightarrow \frac{V_1}{V_o} = \frac{s/CR}{s^2 + s\left(\dfrac{3}{CR}\right) + \dfrac{1}{(CR)^2}}$$

The loop gain $L(s)$ can now be found as

$$L(s) = \frac{s\left[\left(1 + \dfrac{R_2}{R_1}\right)\Big/ CR\right]}{s^2 + s\dfrac{3}{CR} + \dfrac{1}{(CR)^2}}$$

$$L(j\omega) = \frac{j\omega\left[\left(1 + \dfrac{R_2}{R_1}\right)\Big/ CR\right]}{\left[\dfrac{1}{(CR)^2} - \omega^2\right] + j\dfrac{3\omega}{CR}}$$

Zero phase shift will occur at $\omega = \omega_0$:

$$\omega_0 = \frac{1}{CR}$$

At $\omega = \omega_0$, we have

$$|L(j\omega)| = \frac{1}{3}\left(1 + \frac{R_2}{R_1}\right)$$

For oscillations to begin, we need

$$\frac{1}{3}\left(1 + \frac{R_2}{R_1}\right) \geq 1$$

$$\frac{R_2}{R_1} \geq 2$$

14.15 First we design the circuit to operate at 10 kHz.

$$\omega_0 = \frac{1}{CR}$$

$$2\pi \times 10 \times 10^3 = \frac{1}{CR}$$

$$\Rightarrow CR = 0.159 \times 10^{-4}\ \text{s}$$

For $R = 10\ \text{k}\Omega$, we have

$$C = \frac{0.159 \times 10^{-4}}{10 \times 10^3} = 1.59\ \text{nF}$$

Now, refer to Eq. (14.11). If the closed-loop amplifier has an excess phase lag of 5.7°, then the gain will be $\left(1 + \dfrac{R_2}{R_1}\right)e^{-j5.7^\circ}$. Oscillations will occur at the frequency ω_{01} at which the phase angle of the denominator is −5.7°, that is,

$$\tan^{-1}\frac{1}{3}\left(\omega_{01}CR - \frac{1}{\omega_{01}CR}\right) = -5.7^\circ$$

$$\omega_{01}CR - \frac{1}{\omega_{01}CR} = 3\tan(-5.7^\circ) = -0.3$$

$$\Rightarrow \omega_{01}^2 + \frac{0.3}{CR}\omega_{01} - \frac{1}{(CR)^2} = 0$$

$$\Rightarrow \omega_{01} = \frac{0.86}{CR}$$

That is, the frequency of oscillation is reduced by 14% to

$$f_{01} = 0.86 f_0 = 8.6\ \text{kHz}$$

To restore operation to $f_0 = 10$ kHz, we modify the shunt resistor R to R_x, as indicated in Fig. 1.

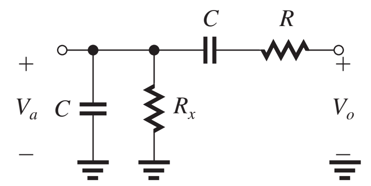


Figure 1

We now require the feedback RC circuit to have a phase shift of $-(-5.7^\circ) = +5.7^\circ$ at $f = 10$ kHz. The transfer function of the RC circuit can be found as follows:

$$\frac{V_a}{V_o} = \frac{Z_p}{Z_p + Z_s}$$

$$= \frac{1}{1 + Z_s Y_p}$$

$$= \frac{1}{1 + \left(R + \dfrac{1}{sC}\right)\left(\dfrac{1}{R_x} + sC\right)}$$

$$= \frac{1}{\left(2 + \dfrac{R}{R_x}\right) + sCR + \dfrac{1}{sCR_x}}$$

For $s = j\omega$, we have

$$\frac{V_a}{V_o} = \frac{1}{\left(2 + \dfrac{R}{R_x}\right) + j\left(\omega CR - \dfrac{1}{\omega CR_x}\right)}$$

At $\omega = \omega_0$, the phase angle of $\dfrac{V_a}{V_o}$ must be +5.7° or equivalently, the phase angle of the denominator must be −5.7°. Thus,

$$\tan^{-1}\frac{\omega_0 CR - \dfrac{1}{\omega_0 CR_x}}{2 + \dfrac{R}{R_x}} = -5.7^\circ$$

$$\omega_0 CR - \frac{1}{\omega_0 CR_x} = \left(2 + \frac{R}{R_x}\right)\tan(-5.7^\circ)$$

$$= \left(2 + \frac{R}{R_x}\right) \times -0.0998$$

Now, $\omega_0 CR = 1$, thus

$$1 - \frac{R}{R_x} = -0.0998\left(2 + \frac{R}{R_x}\right)$$

$$1 + 2 \times 0.0998 = \frac{R}{R_x}(1 - 0.0998) \Rightarrow \frac{R}{R_x} = 1.33$$

$$\Rightarrow R_x = 0.75\ R = 7.5\ \text{k}\Omega$$

At $\omega = \omega_0$ and for $R_x = 7.5\ \text{k}\Omega$

$$\frac{V_a}{V_o}(\omega_0) = \frac{1}{\left(2 + \frac{10}{7.5}\right) + j\left(1 - \frac{10}{7.5}\right)}$$

$$= \frac{1}{3.33 - j0.33}$$

$$\left|\frac{V_a}{V_o}(\omega_0)\right| = \frac{1}{\sqrt{(3.33)^2 + (0.33)^2}} = \frac{1}{3.35}$$

Thus the magnitude of the gain of the amplifier must be 3.35 V/V. Thus, R_2/R_1 must be changed to

$$\frac{R_2}{R_1} = 2.35$$

14.16

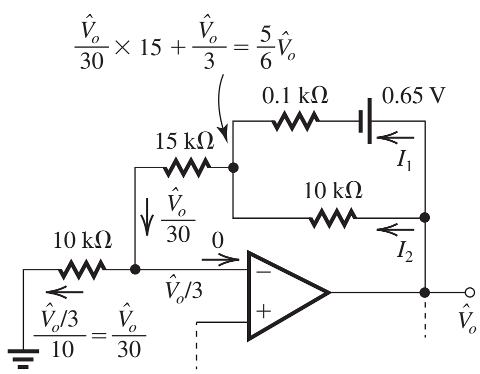


Figure 1

The circuit in Fig. 1 depicts the situation when $v_O = \hat{V}_o$. We made use of the fact that for sustained oscillations the closed-loop gain of the amplifier must be 3. Thus the voltage at the inverting input terminal will be $(\hat{V}_o/3)$. Some of the analysis is shown in the figure. We complete the analysis as follows:

$$I_1 = \frac{\hat{V}_o - \frac{5}{6}\hat{V}_o - 0.65}{0.1} = 10\hat{V}_o - \frac{50}{6}\hat{V}_o - 6.5$$

$$I_2 = \frac{\hat{V}_o - \frac{5}{6}\hat{V}_o}{10} = 0.1\hat{V}_o - \frac{5}{60}\hat{V}_o$$

$$I_1 + I_2 = \frac{\hat{V}_o}{30}$$

Thus,

$$\hat{V}_o\left(10 - \frac{50}{6} + 0.1 - \frac{5}{60}\right) - 6.5 = \frac{\hat{V}_o}{30}$$

$$\Rightarrow \hat{V}_o = 3.94\ \text{V}$$

Thus the peak-to-peak of the output sinusoid will be $2 \times 3.94 = 7.88$ V.

14.17 Refer to Fig. 1, below.

$$I_1 = \frac{V_1}{R}$$

$$V_o = -R_f I_1 = -\frac{R_f}{R}V_1$$

$$\Rightarrow V_1 = -\left(\frac{R}{R_f}\right)V_o \qquad (1)$$

$$I_2 = sCV_1$$

$$I_3 = I_1 + I_2 = \frac{V_1}{R} + sCV_1$$

$$I_3 = \frac{V_1}{R}(1 + sCR)$$

$$V_2 = V_1 + I_3 R = V_1(2 + sCR)$$

$$I_4 = sCV_2 = sCV_1(2 + sCR)$$

$$I_5 = I_3 + I_4 = \frac{V_1}{R}(1 + sCR) + sCV_1(2 + sCR)$$

This figure belongs to Problem 14.17.

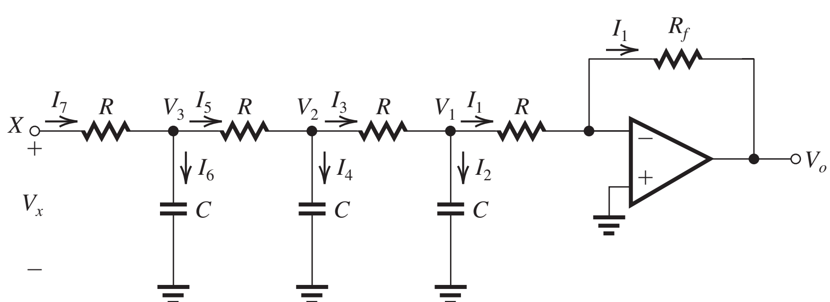


Figure 1

$V_3 = V_2 + I_5R$

$= V_1(2 + sCR) + V_1(1 + sCR) + sCRV_1(2 + sCR)$

$= V_1(3 + s4CR + s^2C^2R^2)$

$I_6 = sCV_3$

$= sCV_1(3 + s4CR + s^2C^2R^2)$

$I_7 = I_5 + I_6$

$= \frac{V_1}{R}(1 + sCR) + sCV_1(5 + s5CR + s^2C^2R^2)$

$V_x = V_3 + I_7R$

$= V_1(3 + s4CR + s^2C^2R^2)$

$+V_1(1 + sCR) + V_1(s5CR + s^2 5C^2R^2 + s^3C^3R^3)$

$= V_1(4 + s\ 10CR + s^2 6C^2R^2 + s^3C^3R^3)$

Substituting for V_1 from Eq. (1) and equating V_x with V_o, we obtain

$-\frac{R_f}{R} = 4 + s\ 10CR + s^2 6\ C^2R^2 + s^3C^3R^3$

For $s = j\omega$

$-\frac{R_f}{R} = (4 - 6\ \omega^2C^2R^2) + j\omega\ (10CR - \omega^2C^3R^3)$

Equating the imaginary part on the RHS to zero, we obtain for the frequency of oscillation ω_0

$\omega_0 = \frac{\sqrt{10}}{CR}$

Equating the real parts on both sides gives the condition for sustained oscillations as

$\frac{R_f}{R} = 6\frac{10}{C^2R^2}C^2R^2 - 4 = 56$

That is,

$R_f = 56R$

Numerical values:

$f_0 = 15$ kHz

$R = 10$ kΩ

$CR = \frac{\sqrt{10}}{2\pi \times 15 \times 10^3}$

$C = \frac{\sqrt{10}}{2\pi \times 15 \times 10^3 \times 10 \times 10^3} = 3.36$ nF

$R_f = 56 \times 10 = 560$ kΩ

14.18 Figure 1 shows the circuit with the additional resistance R included. The loop has been broken at the output of the op amp. The analysis will determine V_o/V_x and equate it to unity, which is the condition for sustained oscillations.

To begin, observe that the voltage V_1 is related to V_o by

$$\frac{V_o}{V_1} = -\frac{R_f}{R} \quad (1)$$

Also, the current I_1 is given by

$$I_1 = \frac{V_1}{R} \quad (2)$$

We now proceed to determine the various currents and voltages of the RC network as follows:

$V_2 = V_1 + \frac{1}{sC}I_1$

$= V_1 + \frac{1}{sC}\ \frac{V_1}{R} = V_1\left(1 + \frac{1}{sCR}\right)$

$I_2 = \frac{V_2}{R} = \frac{V_1}{R}\left(1 + \frac{1}{sCR}\right)$

$I_3 = I_1 + I_2 = \frac{V_1}{R} + \frac{V_1}{R}\left(1 + \frac{1}{sCR}\right)$

$= \frac{V_1}{R}\left(2 + \frac{1}{sCR}\right)$

$V_3 = V_2 + \frac{I_3}{sC}$

$= V_1\left(1 + \frac{1}{sCR}\right) + \frac{V_1}{sCR}\left(2 + \frac{1}{sCR}\right)$

$= V_1\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$

$I_4 = \frac{V_3}{R} = \frac{V_1}{R}\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$

This figure belongs to Problem 14.18.

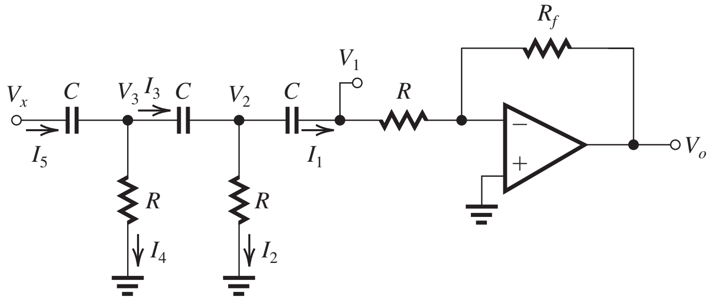


Figure 1

$$I_5 = I_3 + I_4$$

$$= \frac{V_1}{R}\left(2 + \frac{1}{sCR}\right) + \frac{V_1}{R}\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$= \frac{V_1}{R}\left(3 + \frac{4}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$V_x = V_3 + \frac{I_5}{sC}$$

$$= V_1\left(1 + \frac{3}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$+ \frac{V_1}{sCR}\left(3 + \frac{4}{sCR} + \frac{1}{s^2C^2R^2}\right)$$

$$= V_1\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)$$

Now, by replacing V_1 by the value from Eq. (1), we obtain

$$V_x = -V_o\frac{R}{R_f}\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)$$

For sustained oscillations $V_o = V_x$, thus

$$-\frac{R_f}{R} = 1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}$$

For $s = j\omega$, we have

$$-\frac{R_f}{R} = 1 + \frac{6}{j\omega CR} - \frac{5}{\omega^2C^2R^2} - \frac{1}{j\omega^3C^3R^3}$$

$$= \left(1 - \frac{5}{\omega^2C^2R^2}\right) - j\left(\frac{6}{\omega CR} - \frac{1}{\omega^3C^3R^3}\right)$$

Thus, oscillation will occur at the frequency that renders the imaginary part of the RHS zero:

$$\frac{6}{\omega_0 CR} = \frac{1}{\omega_0^3C^3R^3}:$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{6}CR}$$

At this frequency, the real part of the RHS must be equal to $(-R_f/R)$:

$$-\frac{R_f}{R} = 1 - \frac{5}{1/6} = -29$$

Thus,

$$R_f = 29R$$

which is the minimum required value for R_f to obtain sustained oscillations. Numerical values:

$$f_0 = \frac{1}{2\pi\sqrt{6} \times 16 \times 10^{-9} \times 10 \times 10^3}$$

$$= 406 \text{ Hz}$$

$$R_f = 290 \text{ k}\Omega$$

14.19 Refer to the circuit in Fig. 14.10 with the limiter eliminated and with the loop broken at X to determine the loop gain

$$L(s) = \frac{V_{o2}}{V_x}$$

To determine $L(s)$, we note that it is the product of the transfer functions of the inverting integrator formed around op amp 1:

$$\frac{V_{o1}}{V_x} = -\frac{1}{sCR} \qquad (1)$$

and the noninverting integrator formed around op amp 2. To obtain the transfer function of the latter, refer to Fig. 1.

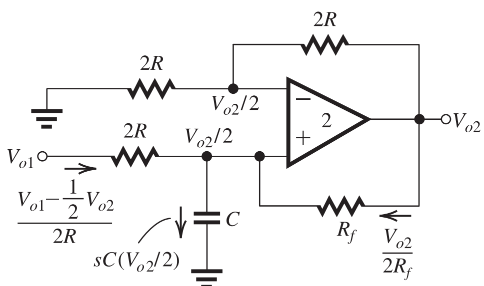


Figure 1

Writing a node equation at the positive input terminal of op amp 2, we obtain

$$\frac{V_{o1} - \frac{1}{2}V_{o2}}{2R} + \frac{V_{o2}}{2R_f} = sC\frac{V_{o2}}{2}$$

$$\Rightarrow \frac{V_{o2}}{V_{o1}} = \frac{1}{sCR + \frac{1}{2} - \frac{R}{R_f}}$$

Substituting for $R_f = \dfrac{2R}{1 + \triangle}$, we obtain

$$\frac{V_{o2}}{V_{o1}} = \frac{1}{sCR - \frac{\triangle}{2}} \qquad (2)$$

Using Eqs. (1) and (2), we obtain the loop gain as

$$L(s) = -\frac{1}{sCR}\,\frac{1}{sCR - \frac{\triangle}{2}}$$

The characteristic equation can now be written as

$$1 - L(s) = 0$$

$$1 + \frac{1}{sCR}\,\frac{1}{sCR - \frac{\triangle}{2}} = 0$$

$$sCR\left(sCR - \frac{\triangle}{2}\right) + 1 = 0$$

$$s^2 - s\frac{1}{CR}\,\frac{\triangle}{2} + \frac{1}{C^2R^2} = 0$$

Thus, the poles are

$$s = \frac{1}{2CR}\left(\frac{\triangle}{2}\right) \pm \frac{1}{2}\sqrt{\frac{1}{C^2R^2}\left(\frac{\triangle}{2}\right)^2 - \frac{4}{C^2R^2}}$$

For $\triangle \ll 4$, we have

$$s \simeq \frac{1}{CR}\left[\left(\frac{\triangle}{4}\right) \pm j\right]$$

from which it is obvious that the poles are in the right half of the s-plane. Q.E.D.

14.20 $\omega_0 = \frac{1}{CR}$

$$R = \frac{1}{2\pi \times 10 \times 10^3 \times 1.6 \times 10^{-9}}$$

$= 9.95\ \text{k}\Omega$

The square wave v_2 will have a peak-to-peak amplitude

$V = 1.4$ V

The component at the fundamental frequency ω_0 will have a peak-to-peak amplitude of $4\ \text{V}/\pi = 1.78$ V. The filter has a center-frequency gain of 2, thus at v_1 the sine wave will have a peak-to-peak amplitude of approximately 3.6 V.

The output amplitude can be doubled by adding a diode in series with each of the diodes in the limiter.

14.21 The different harmonic components will be attenuated relative to the fundamental by the selective response of the bandpass circuit. Let us first determine the magnitude of the transmission of the bandpass filter at a frequency ω relative to that at the fundamental frequency ω_0,

$$T(s) = \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$|T(j\omega)| = \frac{\frac{\omega\omega_0}{Q}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}}$$

$$= 1\Big/\sqrt{1 + Q^2\left(\frac{\omega_0^2 - \omega^2}{\omega\omega_0}\right)^2}$$

$$= 1\Big/\sqrt{1 + Q^2\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2}$$

For the nth harmonic, we have $\omega = n\omega_0$, where $n = 3, 5, 7, ...$ Since n is large and $Q = 20$ is large, we obtain

$$|T(j\omega)| \simeq 1/Q\left(n - \frac{1}{n}\right)$$

Thus, relative to the fundamental, we have:

(a) The second harmonic = 0.

(b) The third harmonic

$$= \frac{1/3}{20\left(3 - \frac{1}{3}\right)} = 6.25 \times 10^{-3}$$

(c) The fifth harmonic

$$\frac{1/5}{20\left(5 - \frac{1}{5}\right)} = 2.08 \times 10^{-3}$$

(d) The rms of harmonics to the tenth:

The seventh harmonic $= \dfrac{1/7}{20\left(7 - \frac{1}{7}\right)}$

$= 1.04 \times 10^{-3}$

The ninth harmonic $= \dfrac{1/9}{20\left(9 - \frac{1}{9}\right)}$

$= 0.625 \times 10^{-3}$

Thus, the rms of harmonics to the tenth

$= \sqrt{6.25^2 + 2.08^2 + 1.04^2 + 0.625^2} \times 10^{-3}$

$= 6.70 \times 10^{-3}$

14.22

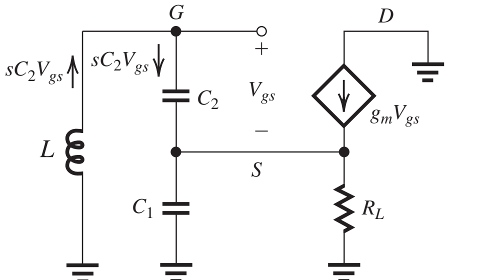


Figure 1

Figure 1 shows the equivalent circuit together with some of the analysis. The voltage at the gate, V_g, can be expressed as

$$V_g = -s^2LC_2V_{gs} \quad (1)$$

The voltage at the source, V_s, can be expressed as

$$V_s = V_g - V_{gs}$$

Thus,

$$V_s = -s^2 LC_2 V_{gs} - V_{gs} \quad (2)$$

A node equation at S provides

$$sC_2V_{gs} + g_mV_{gs} = \left(\frac{1}{R_L} + sC_1\right)V_s$$

Substituting for V_s from Eq. (2), we obtain

$$sC_2V_{gs} + g_mV_{gs} = -\left(\frac{1}{R_L} + sC_1\right)(s^2LC_2 + 1)V_{gs}$$

Dividing by V_{gs} and collecting terms, we obtain

$$s^3LC_1C_2 + s^2\frac{LC_2}{R_L} + s(C_1 + C_2) + \left(g_m + \frac{1}{R_L}\right) = 0$$

For $s = j\omega$, we have

$$j\omega[-\omega^2LC_1C_2 + (C_1 + C_2)] + \left(g_m + \frac{1}{R_L} - \omega^2\frac{LC_2}{R_L}\right) = 0 \quad (3)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω at which the imaginary part is zero, thus

$$\omega_0^2 = 1\Big/\left[L\left(\frac{C_1C_2}{C_1 + C_2}\right)\right] \quad (4)$$

$$\Rightarrow \omega_0 = 1\Big/\sqrt{L\left(\frac{C_1C_2}{C_1 + C_2}\right)}$$

The condition for sustained oscillations can be found by equating the real part of Eq. (3) to zero and making use of (4), thus

$$g_m + \frac{1}{R_L} = \left(\frac{C_1 + C_2}{C_1}\right)\left(\frac{1}{R_L}\right)$$

$$\Rightarrow g_mR_L = \frac{C_2}{C_1}$$

To ensure that oscillations start, we use

$$g_mR_L > \frac{C_2}{C_1}$$

14.23

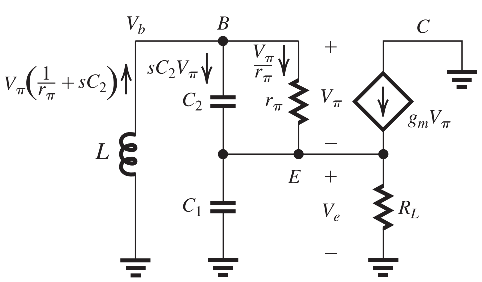


Figure 1

Figure 1 shows the equivalent circuit together with some of the analysis. The voltage V_b can be expressed as

$$V_b = -sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi \quad (1)$$

The voltage V_e can be expressed as

$$V_e = V_b - V_\pi$$

Thus,

$$V_e = -sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi - V_\pi \quad (2)$$

Writing a node equation at E, we obtain

$$\left(sC_1 + \frac{1}{R_L}\right)V_e = \left(g_m + \frac{1}{r_\pi}\right)V_\pi + sC_2V_\pi$$

Substituting for V_e from Eq. (2), we obtain

$$-\left(sC_1 + \frac{1}{R_L}\right)sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi - \left(sC_1 + \frac{1}{R_L}\right)V_\pi$$

$$= \left(g_m + \frac{1}{r_\pi}\right)V_\pi + sC_2V_\pi$$

Dividing by V_π and collecting terms, we obtain

$$s^3LC_1C_2 + s^2L\left(\frac{C_1}{r_\pi} + \frac{C_2}{R_L}\right) + s\left[\frac{L}{R_Lr_\pi} + (C_1 + C_2)\right] + \left(g_m + \frac{1}{r_\pi} + \frac{1}{R_L}\right) = 0$$

For $s = j\omega$, we have

$$j\omega\left[-\omega^2LC_1C_2 + \frac{L}{R_Lr_\pi} + (C_1 + C_2)\right] + \left[\left(g_m + \frac{1}{r_\pi} + \frac{1}{R_L}\right) - \omega^2L\left(\frac{C_1}{r_\pi} + \frac{C_2}{R_L}\right)\right] = 0 \quad (3)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω at which the imaginary part becomes zero, thus

$$\omega_0^2 = \frac{C_1 + C_2}{LC_1C_2} + \frac{1}{R_Lr_\pi C_1C_2} \quad (4)$$

Note that for r_π large so that the second term on the RHS can be neglected, we have

$$\omega_0^2 \simeq 1\Big/\left[L\left(\frac{C_1C_2}{C_1 + C_2}\right)\right]$$

$$\omega_0 = 1\Big/\sqrt{L\left(\frac{C_1C_2}{C_1 + C_2}\right)} \quad (5)$$

which is the expected value. Taking r_π into account will shift the oscillation frequency slightly from this value.

The condition for sustained oscillations can be obtained by equating the real part of Eq. (3) to zero and making use of (4), thus

$$g_m + \frac{1}{r_\pi} + \frac{1}{R_L} =$$

$$L\left[\frac{C_1 + C_2}{LC_1C_2} + \frac{1}{R_L r_\pi C_1 C_2}\right]\left[\frac{C_1}{r_\pi} + \frac{C_2}{R_L}\right] \qquad (6)$$

For r_π large, we have

$$g_m + \frac{1}{R_L} \simeq \left(\frac{C_1 + C_2}{C_1C_2}\right)\left(\frac{C_2}{R_L}\right)$$

$$g_m R_L + 1 = 1 + \frac{C_2}{C_1}$$

$$\Rightarrow g_m R_L = \frac{C_2}{C_1} \qquad (7)$$

To ensure that oscillations start, we use

$$g_m R_L > \frac{C_2}{C_1}$$

14.24

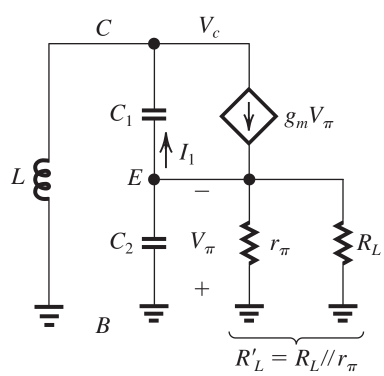


Figure 1

Figure 1 shows the equivalent circuit where we have neglected r_o. A node equation at E provides the following expression for I_1:

$$I_1 = V_\pi\left(g_m + \frac{1}{R'_L} + sC_2\right)$$

The collector voltage V_c can now be found as

$$V_c = -V_\pi - \frac{1}{sC_1} I_1$$

$$V_c = -V_\pi - \frac{1}{sC_1}\left(g_m + \frac{1}{R'_L} + sC_2\right)V_\pi \qquad (1)$$

A node equation at C provides

$$\frac{V_c}{sL} = V_\pi\left(g_m + \frac{1}{R'_L} + sC_2\right) - g_m V_\pi$$

$$= V_\pi\left(\frac{1}{R'_L} + sC_2\right)$$

Substituting for V_c from (1), we obtain

$$-V_\pi - \frac{1}{sC_1}\left(g_m + \frac{1}{R'_L} + sC_2\right)V_\pi =$$

$$V_\pi sL\left(\frac{1}{R'_L} + sC_2\right)$$

Dividing by V_π and collecting terms results in

$$s^3 LC_1C_2 + s^2\frac{LC_1}{R'_L} + s(C_1 + C_2) + g_m + \frac{1}{R'_L} = 0$$

For $s = j\omega$, we have

$$j\omega[-\omega^2 LC_1C_2 + (C_1 + C_2)] +$$

$$\left(-\omega^2\frac{LC_1}{R'_L} + g_m + \frac{1}{R'_L}\right) = 0 \qquad (2)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω at which the imaginary part is zero, thus

$$\omega_0^2 = \frac{1}{L\left(\dfrac{C_1C_2}{C_1 + C_2}\right)} \qquad (3)$$

or

$$\omega_0 = 1\Big/\sqrt{L\frac{C_1C_2}{C_1 + C_2}}$$

The condition for sustained oscillation can be found by equating the real part in Eq. (2) to zero at $\omega = \omega_0$ and by making use of Eq. (3). Thus,

$$-\frac{C_1 + C_2}{C_2}\frac{1}{R'_L} + g_m + \frac{1}{R'_L} = 0$$

$$\Rightarrow g_m R'_L = \frac{C_1}{C_2}$$

While to ensure that oscillations start, we make

$$g_m R'_L > \frac{C_1}{C_2}$$

Observe then in this circuit, we did not have to resort to assuming r_π to be large in order to obtain simplified expressions. Here r_π is simply included with R_L to obtain R'_L and thus can be easily taken into account. A drawback of our analysis, however, is that r_o was not taken into account.

14.25

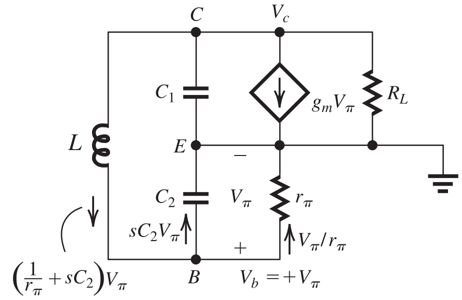


Figure 1

Figure 1 shows the equivalent circuit where r_o has been absorbed into R_L. We have neglected R_f with the assumption that $R_f \gg \omega_0 L$. Some of the analysis is shown on Fig. 1.

Note that,

$$V_b = V_\pi$$

$$V_c = V_b + sL\left(\frac{1}{r_\pi} + sC_2\right)V_\pi$$

Thus,

$$V_c = V_\pi + sL\left(sC_2 + \frac{1}{r_\pi}\right)V_\pi \qquad (1)$$

A node equation at E yields

$$V_\pi\left(\frac{1}{r_\pi} + sC_2\right) + g_m V_\pi + V_c\left(\frac{1}{R_L} + sC_1\right) = 0$$

Substituting for V_c from Eq. (1), we obtain

$$V_\pi\left(\frac{1}{r_\pi} + sC_2\right) + g_m V_\pi + \left(\frac{1}{R_L} + sC_1\right)V_\pi$$

$$+\left(\frac{1}{R_L} + sC_1\right)sL\left(sC_2 + \frac{1}{r_\pi}\right) = 0$$

Dividing by V_π and collecting terms gives

$$s^3 LC_1C_2 + s^2\left(\frac{LC_2}{R_L} + \frac{LC_1}{r_\pi}\right) +$$

$$s\left(C_1 + C_2 + \frac{L}{R_L r_\pi}\right) + g_m + \frac{1}{r_\pi} + \frac{1}{R_L} = 0$$

For $s = j\omega$, we have

$$j\omega\left(-\omega^2 LC_1C_2 + C_1 + C_2 + \frac{1}{R_L r_\pi}\right)$$

$$+\left[g_m + \frac{1}{r_\pi} + \frac{1}{R_L} - \omega^2 L\left(\frac{C_2}{R_L} + \frac{C_1}{r_\pi}\right)\right] = 0 \quad (2)$$

This is the equation that governs the operation of the oscillator circuit. The frequency of oscillation ω_0 is the value of ω that makes the imaginary part zero, thus

$$\omega_0^2 = 1\Bigg/\left[L\frac{C_1C_2}{(C_1+C_2) + \dfrac{L}{R_L r_\pi}}\right] \qquad (3)$$

Observe that including r_π changes the frequency of oscillation slightly from that of the frequency of the resonance circuit. If we neglect the term containing r_π in Eq. (3), we obtain

$$\omega_0 \simeq 1\Bigg/\sqrt{L\frac{C_1C_2}{C_1+C_2}} \qquad (4)$$

The condition for sustained oscillations can be found from Eq. (2) by equating the real part to zero at $\omega = \omega_0$ and making use of (3), thus

$$g_m + \frac{1}{R_L} + \frac{1}{r_\pi} = \frac{1}{\dfrac{C_1C_2}{C_1 + C_2 + \dfrac{L}{R_L r_\pi}}}\left(\frac{C_2}{R_L} + \frac{C_1}{r_\pi}\right)$$

$$g_m + \frac{1}{R_L} + \frac{1}{r_\pi} =$$

$$\left[\frac{C_1 + C_2 + (L/R_L r_\pi)}{C_1C_2}\right]\left(\frac{C_2}{R_L} + \frac{C_1}{r_\pi}\right)$$

Neglecting the terms containing r_π, we obtain

$$g_m + \frac{1}{R_L} = \left(1 + \frac{C_2}{C_1}\right)\frac{1}{R_L}$$

$$\Rightarrow g_m R_L = \frac{C_2}{C_1}$$

For oscillations to start, we need

$$g_m R_L > \frac{C_2}{C_1}$$

14.26 (a)

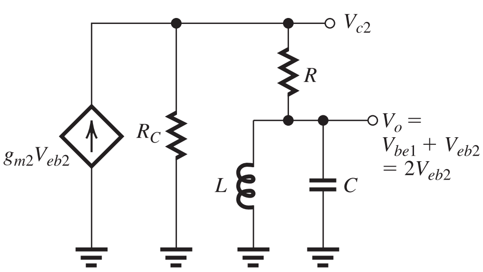


Figure 1

Figure 1 shows the equivalent circuit.

(b) Oscillations will occur at the frequency for which the tuned circuit has infinite impedance. This is because at this frequency the phase shift around the loop will be zero. Thus,

$$\omega_0 = 1/\sqrt{LC}$$

At this frequency, no current flows through R. Thus the voltage V_{c2} will be

$$V_{c2} = (g_{m2}V_{eb2})R_C$$

and the voltage V_o will equal V_{c2}, thus

$$(g_{m2}V_{eb2})R_C = V_o = 2V_{eb2}$$

which yields the condition for sustained oscillation as

$$g_{m2}R_C = 2$$

For oscillations to start, we impose the condition

$$g_{m2}R_C > 2 \qquad (1)$$

But

$$g_{m2} \simeq \frac{I/2}{V_T}$$

Thus, the condition in (1) can be expressed as

$$IR_C > 4V_T$$

that is,

$$IR_C > 0.1 \text{ V}$$

(c) Selecting $IR_C = 1$ V means that oscillations will start and will grow in amplitude until V_o is large enough to cause Q_1 and Q_2 to alternately turn on and off. When this happens the collector current of Q_2 will be 0 in half a cycle and equal to I in the other half cycle. Thus a square wave voltage of amplitude $IR_C = 1$ V peak-to-peak will develop at the collector of Q_2. This square wave voltage is applied through R to the bandpass filter formed by the RLC circuit. Thus, the sinusoid that develops across the LC circuit—that is, V_o—will have a frequency ω_0 and a peak-to-peak amplitude of $\left(\frac{4}{\pi} \times 1\right) = \frac{4}{\pi}$ V . There will be third, fifth, and other odd harmonics, but those will be attenuated because of the selectivity of the bandpass RLC circuit.

14.27 $\omega_0 = 20$ Grad/s $= 20 \times 10^9$ rad/s

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$20 \times 10^9 = \frac{1}{\sqrt{5 \times 10^{-9} \times C}}$$

$$\Rightarrow C = 0.5 \text{ pF}$$

$$R_p = \omega_0 LQ$$

$$= 20 \times 10^9 \times 5 \times 10^{-9} \times 10$$

$$= 1000\ \Omega = 1 \text{ k}\Omega$$

$$r_o \parallel R_p = 5 \parallel 1 = \frac{5}{6} \text{ k}\Omega$$

$$g_m|_{\min} = \frac{1}{\frac{5}{6} \times 10^3} = 1.2 \text{ mA/V}$$

14.28 From Exercise 14.13, we have

$$L = 0.52 \text{ H}$$

$$C_s = 0.012 \text{ pF}$$

$$C_p = 4 \text{ pF}$$

$$C_{\text{eq}} = \frac{C_s\left(C_p + \frac{C_1C_2}{C_1+C_2}\right)}{C_s + C_p + \frac{C_1C_2}{C_1+C_2}}$$

$C_2 = 10$ pF $\quad C_1 = 1$ to 10 pF

$$C_L = \frac{0.012\left(4 + \frac{10 \times 1}{10+1}\right)}{\left(0.012 + 4 + \frac{10}{11}\right)} = 0.01197 \text{ pF}$$

$$C_H = \frac{0.012\left(4 + \frac{10 \times 10}{10+10}\right)}{\left(0.012 + 4 + \frac{100}{20}\right)} = 0.01198 \text{ pF}$$

$$\therefore f_{0H} = \frac{1}{2\pi\left[0.52 \times 0.01197 \times 10^{-12}\right]^{1/2}}$$

$$= 2.0173 \text{ MHz}$$

$$f_{0L} = \left[2\pi\left(0.52 \times 0.01198 \times 10^{-12}\right)^{1/2}\right]^{-1}$$

$$= 2.0165 \text{ MHz}$$

Difference $= 800$ Hz

14.29 $V_{TH} = -V_{TL} = 1$ V

$$L_+ = -L_- = 5 \text{ V}$$

$$V_{TH} = \beta L_+$$

$$1 = \frac{R_1}{R_1 + R_2} \times 5$$

$$\Rightarrow \frac{R_2}{R_1} = 4$$

For $R_1 = 10$ kΩ, we have

$$R_2 = 40 \text{ k}\Omega$$

14.30

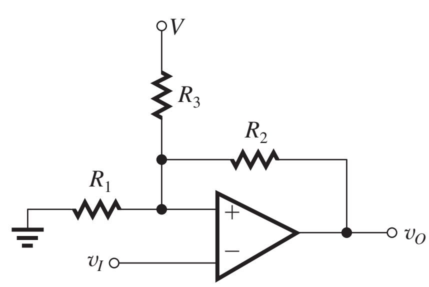


Figure 1

(a) Refer to Fig. 1. With $v_O = L_+$, the voltage at the op amp positive input terminal will be V_{TH}. Now, writing a node equation at the op amp positive input terminal, we have

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L_+ - V_{TH}}{R_2}$$

$$V_{TH}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{L_+}{R_2}+\frac{V}{R_3}$$

$$\Rightarrow V_{TH}=\left(\frac{L_+}{R_2}+\frac{V}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

Similarly, we can obtain

$$V_{TL}=\left(\frac{L_-}{R_2}+\frac{V}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

(b) $L_+ = -L_- = 10\text{ V},\ V = 15\text{ V},\ R_1 = 10\text{ k}\Omega$

$$V_{TH}=5.1=\left(\frac{10}{R_2}+\frac{15}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

$$\frac{5.1}{R_1}+\frac{5.1}{R_2}+\frac{5.1}{R_3}=\frac{10}{R_2}+\frac{15}{R_3}$$

$$0.51=\frac{4.9}{R_2}+\frac{9.9}{R_3} \quad (1)$$

$$V_{TL}=4.9=\left(\frac{-10}{R_2}+\frac{15}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

$$\frac{4.9}{R_1}+\frac{4.9}{R_2}+\frac{4.9}{R_3}=-\frac{10}{R_2}+\frac{15}{R_3}$$

$$0.49=\frac{-14.9}{R_2}+\frac{10.1}{R_3} \quad (2)$$

Multiplying Eq. (1) by $\left(\frac{14.9}{4.9}\right)$, we obtain

$$1.55=\frac{-14.9}{R_2}+\frac{30.1}{R_3} \quad (3)$$

Adding (2) and (3) gives

$$2.04=\frac{40.2}{R_3}$$

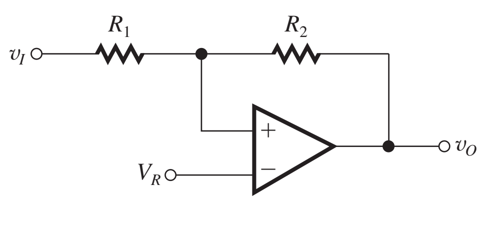
$$\Rightarrow R_3 = 19.7\text{ k}\Omega$$

Substituting in Eq. (1), we obtain

$$0.51=\frac{4.9}{R_2}+\frac{9.9}{19.7}$$

$$\Rightarrow R_2=\frac{4.9}{0.0076}=656.7\text{ k}\Omega$$

14.31



(a) For $v_I = V_{TL}$ and $v_O = L_+$ initially

$$\frac{L_+ - V_R}{R_2}=\frac{V_R - V_{TL}}{R_1}$$

$$V_{TL}=V_R+\frac{R_1}{R_2}V_R-\frac{R_1}{R_2}L_+$$

$$\therefore\ V_{TL}=V_R\left(1+\frac{R_1}{R_2}\right)-\frac{R_1}{R_2}L_+$$

Similarly,

$$\frac{L_- - V_R}{R_2}=\frac{V_R - V_{TH}}{R_1}$$

$$V_{TH}=V_R(1+R_1/R_2)-\frac{R_1}{R_2}L_-$$

(b) Given

$L_+ = -L_- = V$

$R_1 = 10\text{ k}\Omega$

$V_{TL} = 0$

$V_{TH} = V/10$

Substituting these values, we get

$$0 = V_R(1+10/R_2) - (10/R_2)V \quad (1)$$

$$\frac{V}{10} = V_R(1+10/R_2) + (10/R_2)V \quad (2)$$

Subtracting Eq. (2) from Eq. (1), we obtain

$$-\frac{V}{10}=-\frac{20}{R_2}\times V$$

$$\Rightarrow R_2 = 200\text{ k}\Omega$$

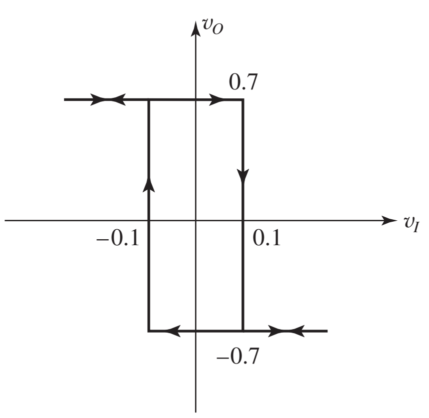
$$0 = V_R\,(1+10/200)-\frac{10}{200}V$$

$$V_R=\frac{10/200\text{ V}}{1+10/200}=\frac{V}{21}$$

14.32 Output levels $= \pm 0.7$ V

$$\text{Threshold levels} = \pm\frac{10}{10+60}\times 0.7 = 0.1\text{ V}$$

$$i_{D,\,\max}=\frac{12-0.7}{10}-\frac{0.7}{10+60}=1.12\text{ mA}$$

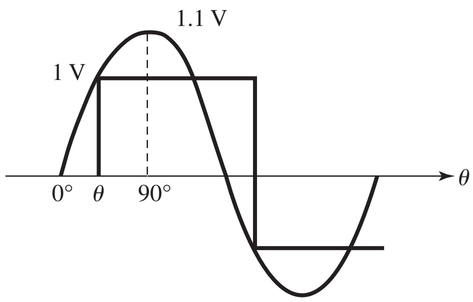


14.33 (a) A 0.5-V peak sine wave is not large enough to change the state of the circuit. Hence, the output will be either +12 V or –12 V.

(b) The 1.1-V peak sine wave will change the state when

$1.1 \sin \theta = 1$

$\theta = 65.40$



$\therefore$ The output is a symmetric square wave of frequency f, and it lags the sine wave by an angle of 65.4°. The square wave has a swing of ±12 V.

Since $V_{TH} = -V_{TL} = 1$ V, if the average shifts by an amount so either the +ve or -ve swing is < 1 V, then no change of state will occur. Clearly, if the shift is 0.1 V, the output will be a DC voltage.

14.34 For $L_+ = -L_- = 7.5$ V, we have

$V_Z = 6.8$ V with $V_D = 0.7$ V.

For $V_{TH} = -V_{TL} = 7.5,$ we have V $\Rightarrow R_1 = R_2$.

For $v_i = 0,$ we have $I_{R_2} = 0.5\,\text{mA} = \dfrac{7.5}{R_1 + R_2}$

$\Rightarrow R_1 = R_2 = 7.5\ \text{k}\Omega$

$I_D = 1\ \text{mA} = \dfrac{12 - 7.5}{R} - \dfrac{7.5}{2R_1}$

$1 = \dfrac{4.5}{R} - 0.5$

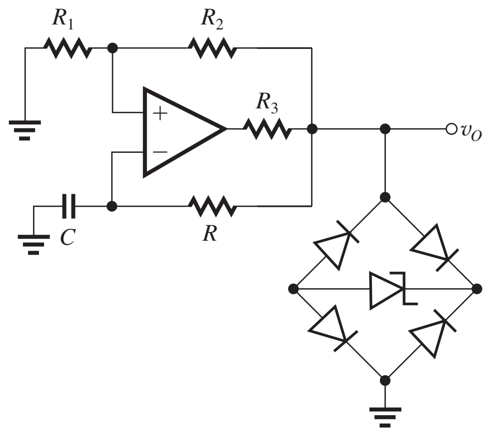
$R = 3\ \text{k}\Omega$

14.35 $T = 2\tau \ln \dfrac{1 + \beta}{1 - \beta}, \quad \beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{10}{26}$

$T = 2(5 \times 10^{-9})(62 \times 10^3) \ln\left(\dfrac{1 + 10/26}{1 - 10/26}\right)$

$T = 0.503\ \text{ms} \Rightarrow f = 1989\ \text{Hz}$

14.36



$\beta = 0.462$

For $V_D = 0.7$ V and $V_O = \pm 5$ V, we have

$V_Z = 5 - 2V_D$

$V_Z = 3.6$ V

$T = 2\tau \ln\left(\dfrac{1 + \beta}{1 - \beta}\right)$

$10^{-3} = 2\tau \ln\left(\dfrac{1.462}{1 - 0.462}\right) \Rightarrow \tau = 0.5\ \text{ms}$

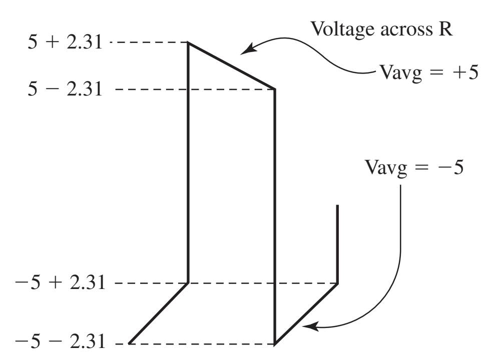
$\tau = RC \Rightarrow R = \tau/C = 50\ \text{k}\Omega$

Thresholds $= \pm 0.462 \times 5 = \pm 2.31$ V

Average current in R in $\dfrac{1}{2}$ cycle:

$I \cong \dfrac{1}{R}\left(\dfrac{5 - 2.31 + 2.31 + 5}{2}\right)$

$= \dfrac{5}{R} = \dfrac{5}{50\ \text{k}\Omega} = 0.1\ \text{mA}$



$R_1 + R_2 = \dfrac{5\ \text{V}}{0.1\ \text{mA}} = 50\ \text{k}\Omega$

$\dfrac{R_1}{R_1 + R_2} = 0.462 \rightarrow R_1 = 50\,(0.462)$

$= 23.1\ \text{k}\Omega$

$\therefore\ R_2 = 26.9\ \text{k}\Omega$

$$1 = \frac{13-5}{R_3} - 0.1 - 0.1$$

$$R_3 = \frac{8}{1.2}$$

$$= 6.67\ \text{k}\Omega$$

14.37 From Fig. 14.25(b), for ±5-V output, we have

$$V_Z = 5 - 2V_{\text{DIODE}} = 5 - 1.4 = 3.6\ \text{V}$$

For ±5-V output:

$$R_1 = R_2,\ L_+ = -L_- = 5\ \text{V}$$

$$V_{TH} = -V_{TL} = 5\ \text{V}$$

Max current in feedback network = 0.2 mA

$$\therefore 0.2 = \frac{10}{R_1 + R_2} \Rightarrow R_1 = R_2 = 25\ \text{k}\Omega$$

Minimum zener current = 1 mA

$$\therefore \frac{12-5}{R_3} = (0.2 + 1)\ \text{mA}$$

$$R_3 = \frac{7}{1.2} = 5.83\ \text{k}\Omega$$

Now from Fig. 14.27(c) we have

$$\text{Slope} = \frac{-L_-}{RC} = \frac{V_{TH} - V_{TL}}{T/2}$$

for $f = 1\ \text{kHz}$

$$T = 10^{-3}\ \text{s}$$

$$C = 0.01\ \mu\text{F}$$

$$\frac{5}{RC} = \frac{10}{10^{-3}/2} \Rightarrow R = 25\ \text{k}\Omega$$

14.38 Refer to the circuit of Fig. P14.38. To obtain a square-wave voltage of ±7.5 V levels across the zeners, we need

$$V_Z + V_D = 7.5\ \text{V}$$

For $V_D = 0.7$ V, we have

$$V_Z = 6.8\ \text{V}$$

Now, $R_1 = R_2$, thus for the bistable we obtain

$$L_+ = -L_- = 7.5\ \text{V}$$

$$\beta = \frac{R_1}{R_1 + R_2} = 0.5$$

$$V_{TH} = -V_{TL} = \frac{7.5}{2}\ \text{V}$$

As shown in Fig. 1, the voltage across the capacitor will be triangular, ranging between $-V_{TL}$ and $+V_{TH}$. The slope of the triangular waveform edges is

$$\text{Slope} = \frac{7.5}{R_5} \times \frac{1}{C} = \frac{7.5}{T/2}$$

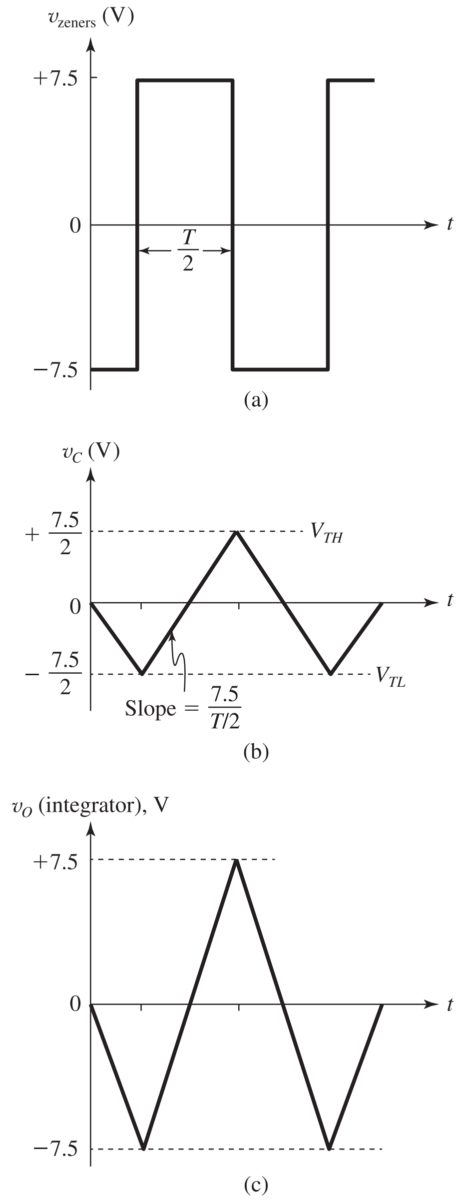


Figure 1

For $R_5 = R$, we have

$$CR = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2 \times 10 \times 10^3}$$

$$R = \frac{1}{2 \times 10 \times 10^3 \times 0.5 \times 10^{-9}} = 100\ \text{k}\Omega$$

Since all resistors, except R_7, are equal, we have

$$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 100\ \text{k}\Omega$$

To determine R_7, we note that the minimum zener current occurs when the current through R_5 is at its maximum. The latter condition occurs when the output of the bistable is at +7.5 V while v_C is $(-7.5/2)$ V at which time

$$I_{R5} = \frac{7.5 - (-7.5/2)}{R_s}$$

$$= 0.1125 \text{ mA}$$

Thus, for a minimum zener current of 1 mA, we write

$$\frac{13 - 7.5}{R_7} = 1 + \frac{7.5}{R_1 + R_2} + 0.1125$$

$$= 1 + \frac{7.5}{200} + 0.1125$$

$$\Rightarrow R_7 = 4.8 \text{ k}\Omega$$

14.39 Refer to Fig. 14.28. The recovery time, t_{rec}, is the time for v_B to go from βL_- to V_{D1}:

$$v_B(t) = L_+ - (L_+ - \beta L_-)e^{-t/C_1R_3}$$

Thus,

$$V_D = L_+ - (L_+ - \beta L_-)e^{-t_{\text{rec}}/C_1R_3}$$

$$\Rightarrow t_{\text{rec}} = C_1R_3 \ln \frac{L_+ - \beta L_-}{L_+ - V_D}$$

From Exercise 14.22, we have

$$C_1 = 0.1\ \mu\text{F},\ R_3 = 6171\ \Omega,\ L_+ = -L_- = 12 \text{ V}$$

$$\beta = 0.1,\ \text{and } V_D = 0.7 \text{ V},\ \text{thus}$$

$$t_{\text{rec}} = 0.1 \times 10^{-6} \times 6.171 \times 10^3 \ln\left(\frac{12 + 1.2}{12 - 0.7}\right)$$

$$= 96\ \mu\text{s}$$

14.40 Choose $C_1 = 1$ nF and $C_2 = 0.1$ nF:

$$R_1 = R_2 = 100 \text{ k}\Omega \Rightarrow \beta \equiv \frac{1}{2}$$

$$T = C_1R_3 \ln\left(\frac{0.7 + 13}{0.5 \times (-13) + 13}\right)$$

$$10^{-4} = 10^{-9}R_3 \ln\left(\frac{13.7}{13\,(0.5)}\right)$$

$$R_3 = 134.1 \text{ k}\Omega$$

Need $R_4 \gg R_1 \Rightarrow$ choose $R_4 = 470$ kΩ

The trigger pulse must be sufficiently large to lower the voltage at node C from βL_+ to V_D, that is, from +6.5 V to +0.7 V; thus it must be at least 5.8 V.

For recovery we have

$$v_B = 13 - \left(13 - \beta L_-\right) e^{-t/\tau}$$

$$= 13 - 19.5e^{-t/\tau} = 0.7$$

$$\therefore\ t_{\text{recovery}} = -\tau \ln\left(\frac{12.3}{19.5}\right)$$

$$= -\left(134.1 \times 10^3\right)\left(10^{-9}\right)(-0.4608)$$

$$= 61.8\ \mu\text{s}$$

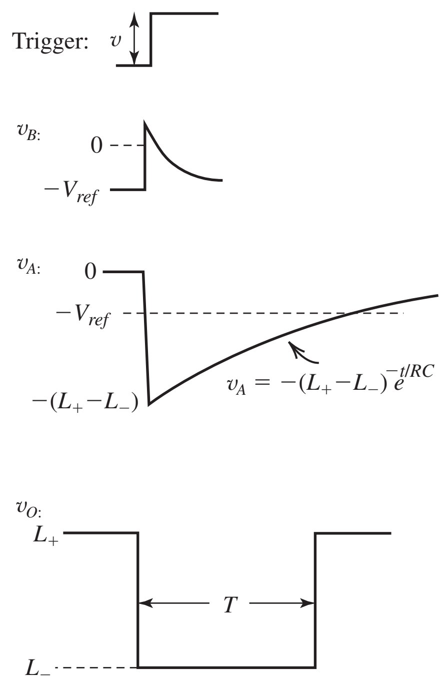
14.41 See sketches that follow:

$$v_A\,(t = T) = -V_{\text{ref}} = -\,(L_+ - L_-)\,e^{-T/RC}$$

$$\frac{V_{\text{ref}}}{L_+ - L_-} = e^{-T/RC}$$

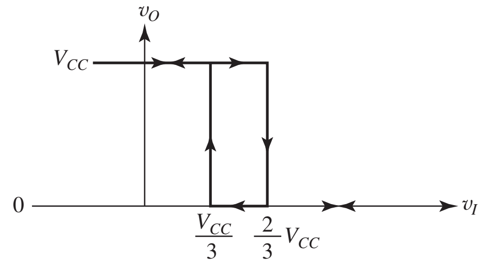
$$T = -RC \ln\left(\frac{V_{\text{ref}}}{L_+ - L_-}\right)$$

$$= RC \ln\left(\frac{L_+ - L_-}{V_{\text{ref}}}\right) \qquad \text{Q.E.D.}$$



14.42 For $v_I > 2/3V_{CC}$, comp –1 = "1" and comp –2 = "0" and flip flop is reset, i.e. $v_O = 0$ V. Now v_O will not change until $v_I = 1/3V_{CC}$, when comp –2 = "1" and comp –1 = "0" and FF is set: i.e. $v_O = V_{CC}$

For $\frac{1}{3}V_{CC} < v_I < \frac{2}{3}V_{CC}$, comp –1 = comp –2 = "0" and no change of state will occur.



i.e. an inverting bistable circuit.

14.43
$C = 680$ pF, $f = 20$ kHz, duty cycle = 80%.

Using Eq. (14.46), we obtain

$T = 0.69C(R_A + 2R_B)$

Thus,

$$\frac{1}{20 \times 10^3} = 0.69 \times 680 \times 10^{-12}\ (R_A + 2R_B)$$

$$\Rightarrow R_A + 2R_B = \frac{1}{20 \times 0.69 \times 0.68 \times 10^{-6}}$$

$$R_A + 2R_B = 106.56\ \text{k}\Omega \qquad (1)$$

Using Eq. (14.47), we have

$$0.8 = \frac{R_A + R_B}{R_A + 2R_B}$$

Thus,

$$R_A + R_B = 0.8 \times 106.6 = 85.25\ \text{k}\Omega \qquad (2)$$

Subtracting Eq. (2) from Eq. (1) gives

$R_B = 21.31\ \text{k}\Omega$

and using Eq. (2), we obtain

$R_A = 63.94\ \text{k}\Omega$

14.44 (a) $C = 0.5$ nF

Using Eq. (14.41), we obtain

$T \cong 1.1CR$

$10 \times 10^{-16} = 1.1 \times 0.5 \times 10^{-9} \times R$

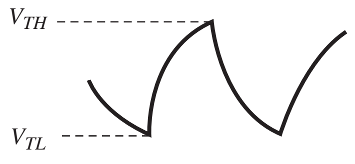
$\Rightarrow R = 18.2\ \text{k}\Omega$

(b) For $T = 20\ \mu\text{s}, R = 18.2\ \text{k}\Omega$,
$C = 0.5$nF, $V_{CC} = 12$ V, and using Eq. (14.40) with $v_C = V_{\text{TH}}$ and $t = T$, we obtain

$$V_{TH} = 12\left(1 - e^{-\frac{20\times10^{-6}}{9.1\times10^{-6}}}\right)$$

$= 10.67$ V

14.45



For the rise:

$$v_C = V_{CC} - (V_{CC} - V_{TL})\, e^{-t/C(R_A+R_B)}$$

$$V_{TH} = V_{CC} - (V_{CC} - V_{TL})\, e^{-T_H/C(R_A+R_B)}$$

$$\frac{V_{CC} - V_{TH}}{V_{CC} - V_{TL}} = e^{-T_H/C(R_A+R_B)}$$

$$T_H = C(R_A + R_B) \ln\left(\frac{V_{CC} - V_{TL}}{V_{CC} - V_{TH}}\right)$$

For exponential fall:

$$v_C = V_{TH} e^{-t/CR_B}$$

$$\therefore\ V_{TL} = V_{TH} e^{-T_L/CR_B}$$

$$T_L = CR_B \ln\left(\frac{V_{TH}}{V_{TL}}\right)$$

for $V_{TH} = 2V_{TL} \Rightarrow T_L = CR_B \ln(2)$

(b) $C = 1$ nF, $R_A = 7.2\ \text{k}\Omega$, $R_B = 3.6\ \text{k}\Omega$

$V_{CC} = 5$ V, no external voltage to V_{TH}.

$\therefore\ T_H + T_L = T = \ln 2 \times (R_A + 2R_B)C$

$T = 9.94\ \mu\text{s} \rightarrow f = 100.6$ kHz

$$\text{Duty cycle} = \frac{T_H}{T_H + T_L} = \frac{R_A + R_B}{R_A + 2R_B} = 0.75$$

$\Rightarrow 75\%$

(c) $V_{CC} = 5$ V,

$$V_{TH} = \frac{2}{3} \times 5 = \frac{10}{3} = 3.33\ \text{V}$$

For 1-V input the high value of V_{TH} will be $V'_{TH} = 4.33$ V

and, $V'_{TL} = \dfrac{1}{2} V'_{TH} = 2.17$ V

$$T'_H = 10^{-9}(3.6 + 7.2) \times 10^3 \ln\left(\frac{5 - 2.17}{5 - 4.33}\right)$$

$= 15.6\ \mu\text{s}$

$T'_L = 10^{-9} \times 3.6 \times 10^3 \ln 2 = 2.5\ \mu\text{s}$

$$\therefore\ f = \frac{1}{(15.6 + 2.5)10^{-6}} = 55.2\ \text{kHz}$$

$$\text{Duty cycle} = \frac{15.6}{2.5 + 15.6} = 86.2\%$$

For 1-V input the low value of V_{TH} will be $V''_{TH} = 2.33$

and $V''_{TL} = 1.17$

$$\therefore\ T''_H = 10^{-9}(3.6 + 7.2)\, 10^3 \ln\left(\frac{5 - 1.17}{5 - 2.33}\right)$$

$= 3.90\ \mu\text{s}$

$T''_L = T'_L = 2.5\ \mu\text{s}$

$$\therefore\ f = \frac{10^6}{(3.90 + 2.5)} = 156\ \text{kHz}$$

$$\text{Duty cycle} = \frac{3.90}{2.5 + 3.90} = 61\%$$

14.46

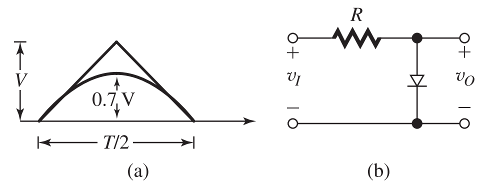


Figure 1

For a sine wave, we have

$v_O = 0.7 \sin \omega t$

Slope at zero crossings

$= 0.7\omega = 0.7 \times 2\pi f$

$= \dfrac{1.4\pi}{T}$

Slope of triangular wave = $\dfrac{V}{T/4}$. Equating slopes, we obtain

$$\frac{1.4\pi}{T} = \frac{V}{T/4}$$

$$\Rightarrow V = \frac{1.4\pi}{4} = 1.1 \text{ V}$$

Refer to Fig. 1(b).

With $v_I = V = 1.1$ V, we have $v_O = 0.7$ V, thus

$v_R = 1.1 - 0.7 = 0.4$ V

At $V_D = 0.7$ V we have $i_D = 1$ mA, thus

$0.4 = 1 \times R$

$\Rightarrow R = 0.4 \text{ k}\Omega = 400\ \Omega$

The angle θ and the ideal value of the output voltage are determined as follows:

Since v_D changes by 0.1 V per decade change in current, we have

$$v_O = 0.7 - 0.1 \log\left(\frac{1 \text{ mA}}{i_D}\right)$$

$$\Rightarrow i_D = 10^{10(v_O - 0.7)}, \text{ mA} \qquad (1)$$

$$v_I = v_O + i_D R \qquad (2)$$

$$\theta = \frac{v_I}{1.1} \times 90^\circ \qquad (3)$$

$$\text{Ideal } v_O = 0.7 \sin\theta \qquad (4)$$

Percentage error in v_O =

$$\frac{v_O - \text{Ideal } v_O}{\text{Ideal } v_O} \times 100 \qquad (5)$$

Equations (1)–(5) can be used for each of the given values of v_O to obtain the results given in the table below.

v_O(V)	θ	0.7 sin θ	% Error
0.70	90°	0.7	0
0.65	63.6°	0.627	3.7
0.60	52.4°	0.554	8.2
0.55	46.1°	0.504	9.1
0.50	41.3°	0.462	8.2
0.40	32.80	0.379	5.5
0.30	24.6°	0.291	3.0
0.20	16.4°	0.198	1.2
0.10	8.2°	0.0998	0.1
0.00	0°	0	0.0

14.47 Slope of triangular wave = $\dfrac{16}{T} = 16f$.
Slope at zero crossings of a sine wave with peak amplitude of $(V_B + 0.7)$ V is

$= (V_B + 0.7)\omega$

Equating slopes, we obtain

$16f = (V_B + 0.7) \times 2\pi f$

$\Rightarrow V_B = 1.85$ V

This figure belongs to Problem 14.47.

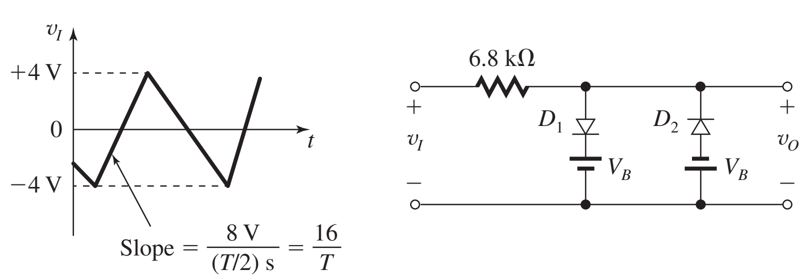
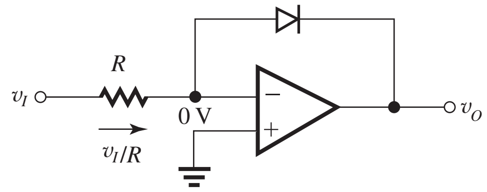


Figure 1

14.48



$v_I > 0$

Voltage across diode is $-v_O$

$i_D = \frac{v_I}{R} = I_S e^{-v_O/V_T}$

$-\frac{v_O}{V_T} = \ln\left(\frac{v_I}{RI_S}\right)$

$v_O = -V_T \ln\left(\frac{v_I}{RI_S}\right), v_I > 0$ Q.E.D.

14.49 From the statement of Problem 14.48, we have

$v_O = -V_T \ln\left(\frac{v_I}{I_S R}\right)$

Now, for the circuit in Fig. P14.49 we can write

$v_A = -V_T \ln\left(\frac{v_1}{I_S}\right)$

$v_B = -V_T \ln\left(\frac{v_2}{I_S}\right)$

$v_C = V_T \ln\left(\frac{1}{I_S}\right)$

$v_D = V_T\left[\ln \frac{v_1}{I_S} + \ln \frac{v_2}{I_S} - \ln \frac{1}{I_S}\right]$

$= V_T \ln^{(v_1 v_2/I_S)}$

$v_O = -I_S e^{\ln\left(\frac{v_1 v_2}{I_S}\right)}$

$v_O = -v_1 v_2$ Q.E.D.

Check:

For $v_1 = 0.5$ V and $v_2 = 1$ V, we have

$i_{D1} = 0.5$ mA

$v_{D1} = 0.7 + 0.025 \ln\left(\frac{0.5}{1}\right)$

$= 0.683$ V

$v_A = -0.683$ V

$i_{D2} = 1$ mA

$v_{D2} = 0.7$ V

$v_B = -0.7$ V

$i_{D3} = 1$ mA

$v_C = 0.7$ V

$v_D = -(-0.683 - 0.7 + 0.7)$

$= 0.683$ V

$i_{D4} = I_S e^{0.683/0.025}$

But,

$1 \text{ mA} = I_S e^{0.7/0.025}$

$\Rightarrow I_S = e^{-0.7/0.025}$, mA

$i_{D4} = e^{(-0.7+0.683)/0.025}$

$= 0.5$ mA

$v_O = -0.5$ V

which is $-v_1 v_2$ Q.E.D.

Other combinations can be used to verify the operation of this analog multiplier.