Microelectronic Circuits

Chapter 13 – Filters and Tuned Amplifiers

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Introduction

- In this chapter, we study the design of an important building block of communications and instrumentation systems, the electronic filter. We shall concentrate on a selection of topics that provide an introduction to the subject as well as a useful arsenal of filter circuits and design methods.
- **Passive LC filters** work well at high frequencies; we shall also study filter realizations that do not require inductors, they are
  - **active-RC filters**, **transconductance-C filters** **(**for high-frequency applications), and
  - **switched-capacitor filters** **(**for audio-frequency applications).

Filter Transmission

- The filter transfer function T (s) is the ratio of the output voltage Vo(s) to the input voltage Vi(s),
  - T(s) ≡ Vo(s)/Vi(s)   (13.1)

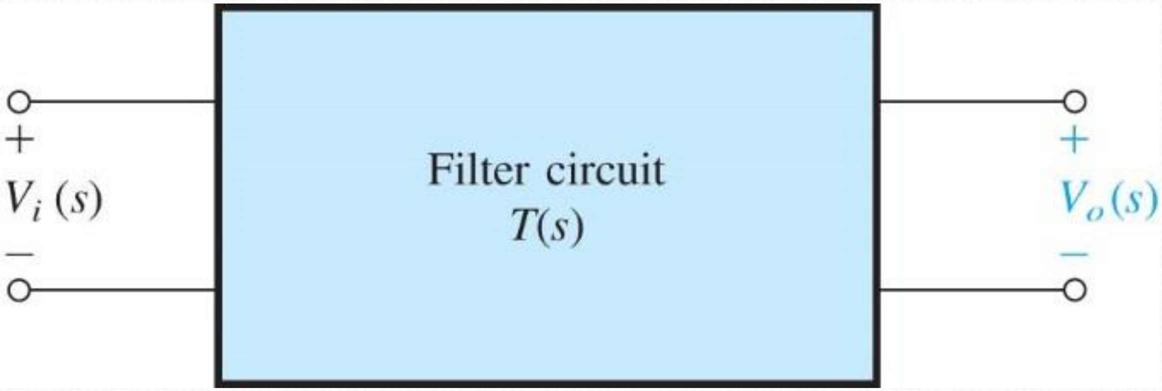


Figure 13.1 The filters studied in this chapter are linear circuits represented by the general two-port network shown. The filter transfer function T(s) ≡ Vo(s)/Vi(s)

Filter Transmission(Cont'd)

- For physical frequencies, $s = j\omega$, $T(s)$ becomes the filter transmission $T(j\omega)$
  - $T(j\omega) = |T(j\omega)|\, e^{j\Phi(\omega)}$ (13.2)
- The magnitude of transmission is often expressed in decibels in terms of the gain function
  - $G(\omega) \equiv 20 \log|T(j\omega)|$, dB (13.3)
- or, alternatively, in terms of the attenuation function
  - $A(\omega) \equiv -20 \log|T(j\omega)|$, dB (13.4)
- Also,
  - $|Vo(j\omega)| = |T(j\omega)|\,|Vi(j\omega)|$ (13.5)

Filter Types

- Figure 13.2 depicts the ideal transmission characteristics of the four major filter types: low-pass (LP) in Fig. 13.2(a), high-pass (HP) in Fig. 13.2(b), bandpass (BP) in Fig. 13.2(c), and bandstop (BS) or band-reject in Fig. 13.2(d).

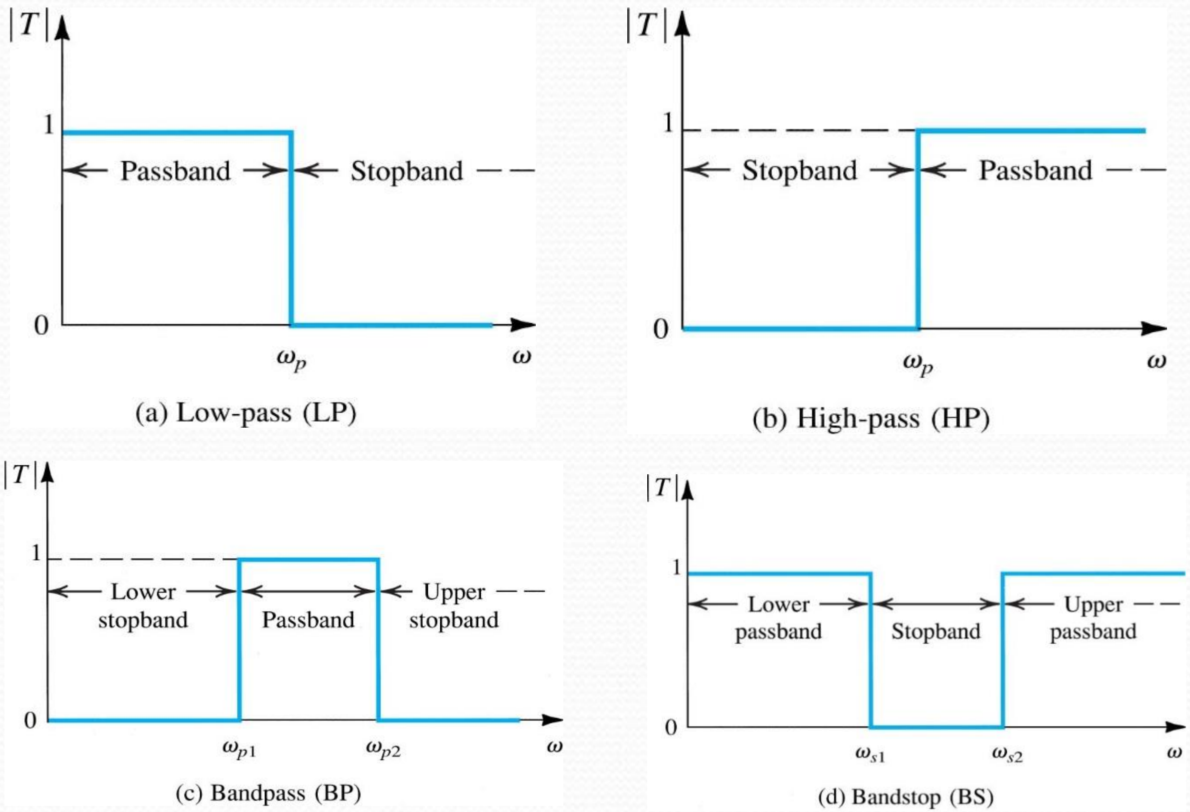


Figure 13.2 Ideal transmission characteristics of the four major filter types: (a) low-pass (LP), (b) high-pass (HP), (c) bandpass (BP), and (d) bandstop (BS).

Filter Specification

- The specifications allow for deviation of the **passband** transmission from the ideal 0 dB, but place an upper bound, A_{max} (dB) (typically ranges from 0.05 dB to 3 dB), on this deviation.
- Also, the specifications require the stopband signals to be attenuated by at least A_{min} (dB)(typically ranges from 20 dB to 100 dB) relative to the passband signals.
- Since the transmission of a physical circuit cannot change abruptly at the edge of the passband, this **transition band** extends from the passband edge ω_p to the stopband edge ω_s. The ratio ω_s / ω_p is called the **selectivity factor**.

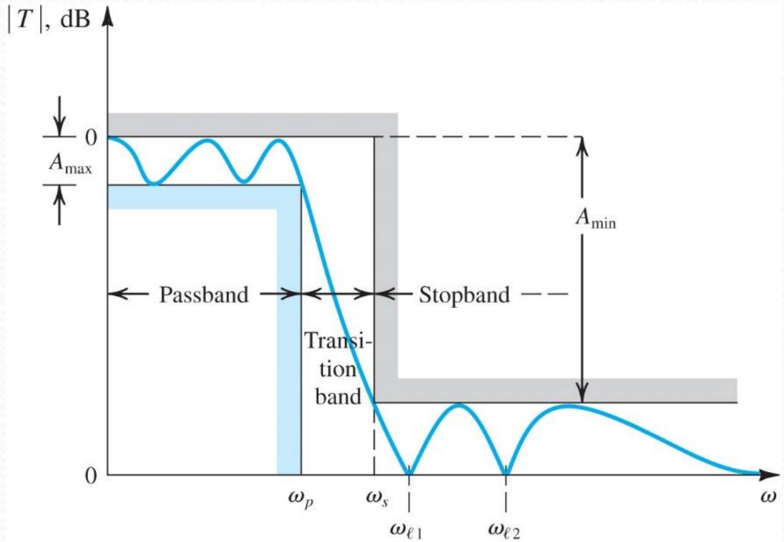
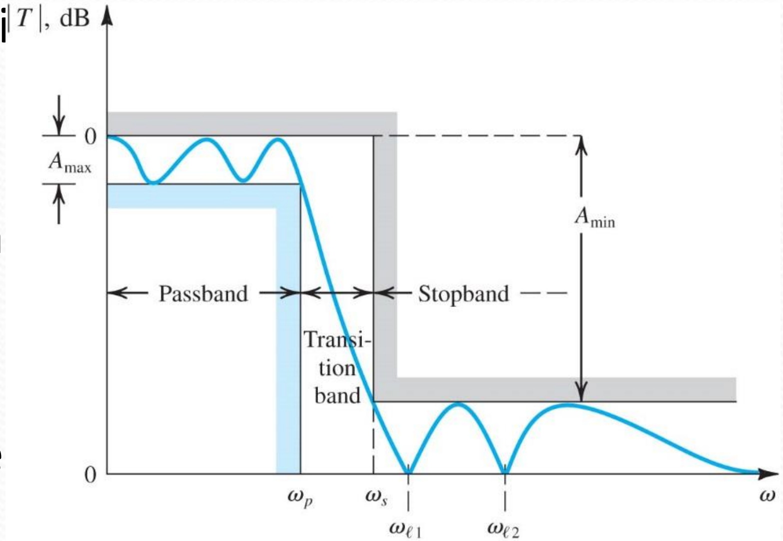


Figure 13.3 Specification of the transmission of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.

Filter Specification(Cont'd)

- The transmission of a low-pass filter is speci
  - **1.** The passband edge ω_p
  - **2.** The maximum allowed variation in pa
  - **3.** The stopband edge ω_s
  - **4.** The minimum required stopband atte



- Since the peak ripple is equal to A_{max}, it is usual to refer to A_{max} as the **passband ripple** and to ω_p as the **ripple bandwidth**.
- The particular filter response shows ripples also in the stopband, and the minimum stopband attenuation achieved is equal to the specified value, A_{min}.
- Thus this particular response is said to be **equiripple** in both the passband and the stopband.
- The process of obtaining a transfer function that meets given specifications is known as **filter approximation**.

Bandpass Filter Specification

- Fig. 13.4 shows transmission specifications for a bandpass filter and the response meets these specifications

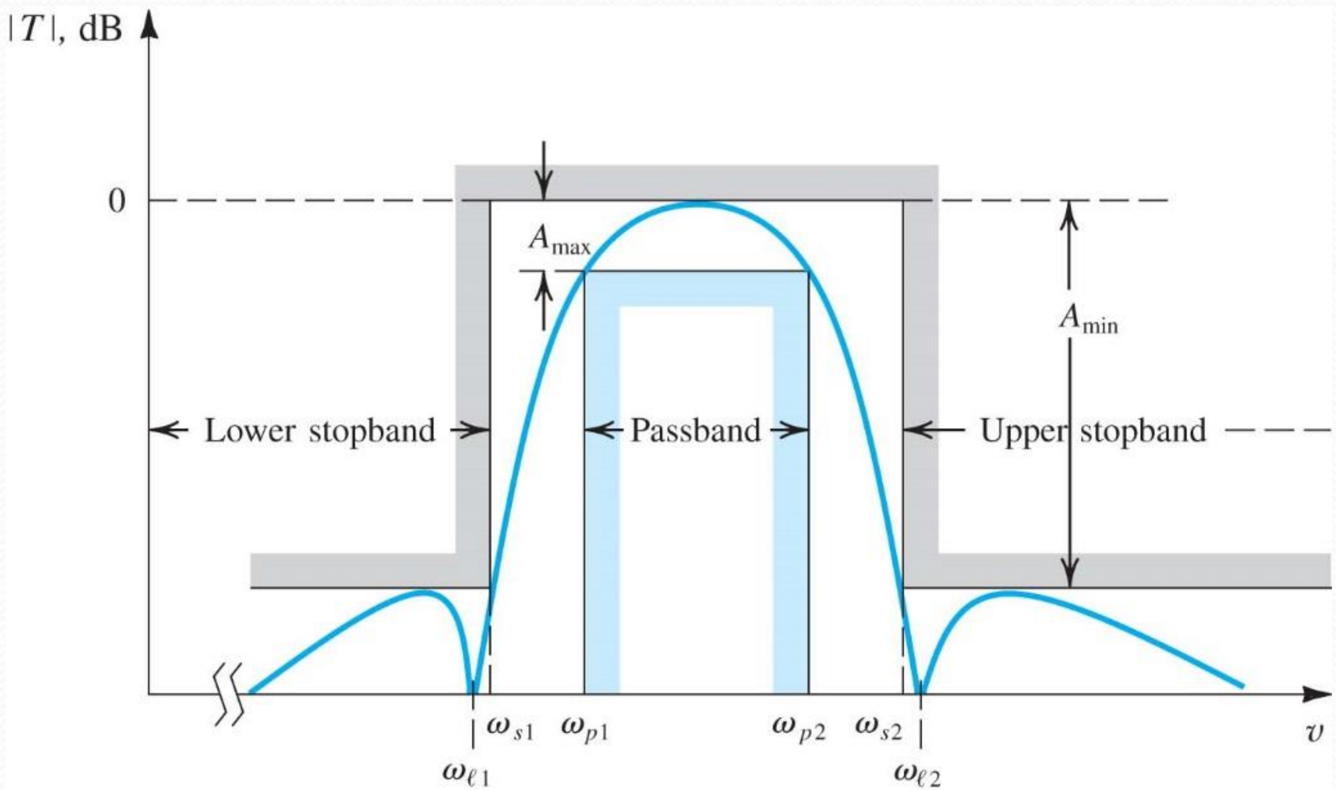


Figure 13.4 Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

The Filter Transfer Function

- The filter transfer function T(s) can be written as

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0} \qquad (13.6)$$

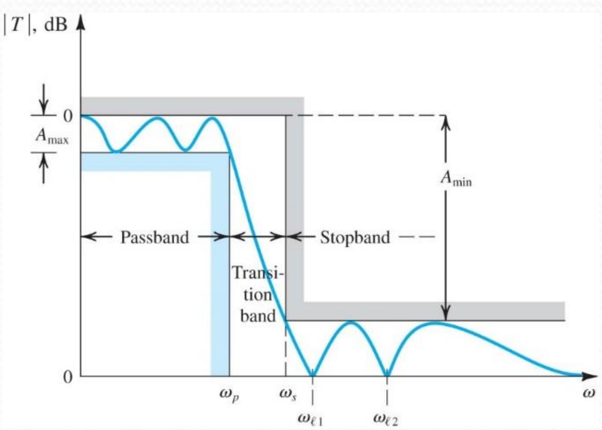
- The degree of the denominator, N, is the **filter order**.
- For the filter circuit to be stable, the degree of the numerator must be less than or equal to that of the denominator; $M \leq N$.
- The numerator and denominator coefficients, $a_0, a_1, \ldots, a_M$ and $b_0, b_1, \ldots, b_{N-1}$, are real numbers.

A general low-pass filter

- $T(s)$ can be expressed in the form

$$T(s) = \frac{a_M(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)} \tag{13.7}$$

- $z_1, z_2, \ldots, z_M$, are **zeros**, or **transmission zeros**; and
- $p_1, p_2, \ldots, p_N$, are **poles**, or the **natural modes**.
- Each transmission zero or pole can be either a real or a complex number.
- The transmission decreases toward zero as ω approaches ∞. Thus the filter must have one or more transmission zeros at $s = \infty$.



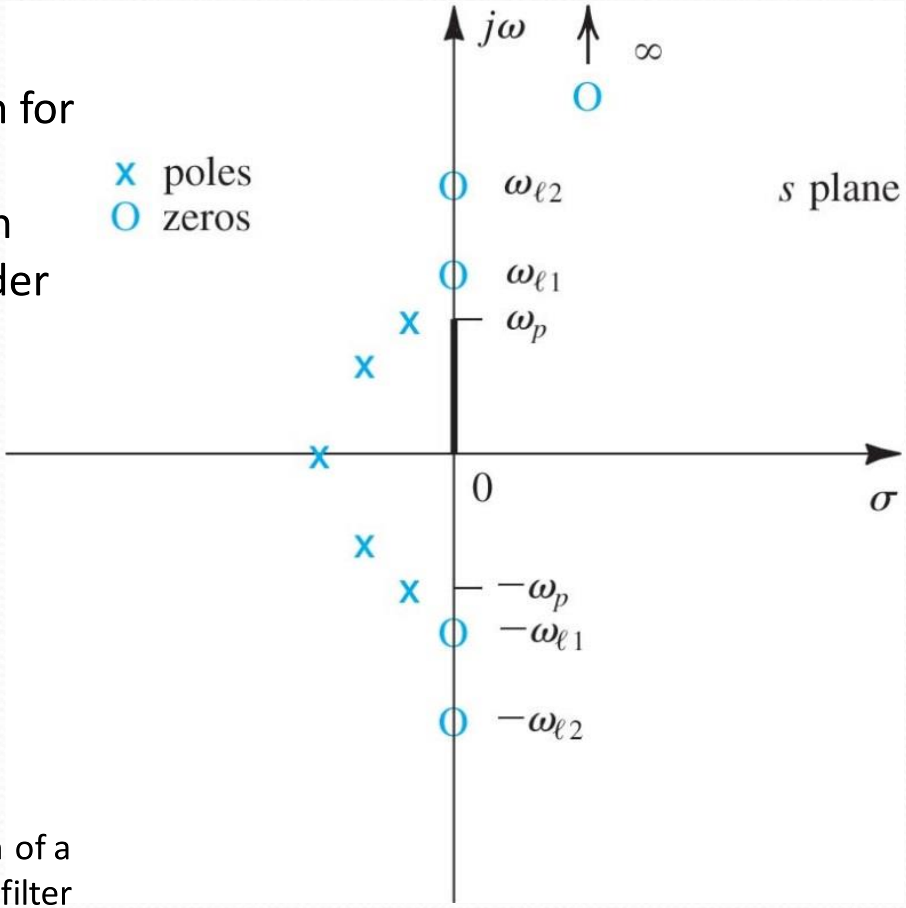
- Eq.(13.6) $$T(s) = \frac{a_M s^M + a_{M-1}s^{M-1} + \cdots + a_0}{s^N + b_{N-1}s^{N-1} + \cdots + b_0}$$

Figure 13.3 Specification of the transmission of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.

A general low-pass filter (Cont'd)

- For a filter circuit to be stable, all its poles must lie in the left half of the *s* plane, and thus $p_1, p_2, \ldots, p_N$ must all have negative real parts..

Fig. 13.5 Pole-zero pattern for the low-pass Filter whose transmission is sketched in Fig. 13.3. This is a fifth-order filter(N=5).



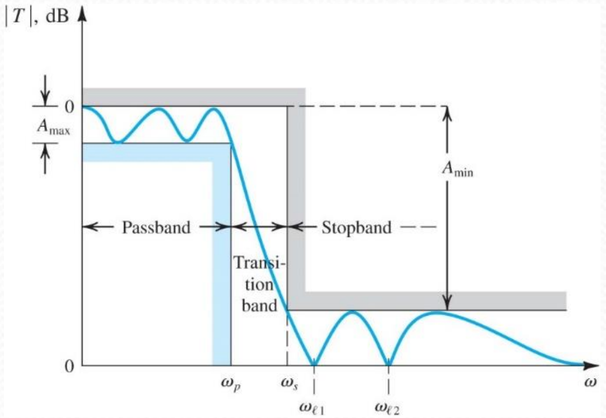


Figure 13.3 Specification of the transmission of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.

A general bandpass

- consider the bandpass filter whose magnitude response is shown in Fig. 13.4. Its transfer function takes the form

$$T(s) = \frac{a_5 s (s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^6 + b_5 s^5 + \ldots + b_0}$$

- A typical pole–zero plot for such a filter is shown in Fig. 13.6.

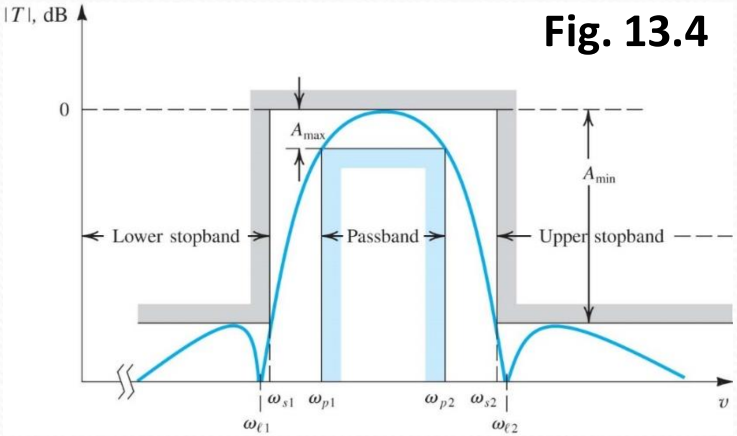


Fig. 13.4

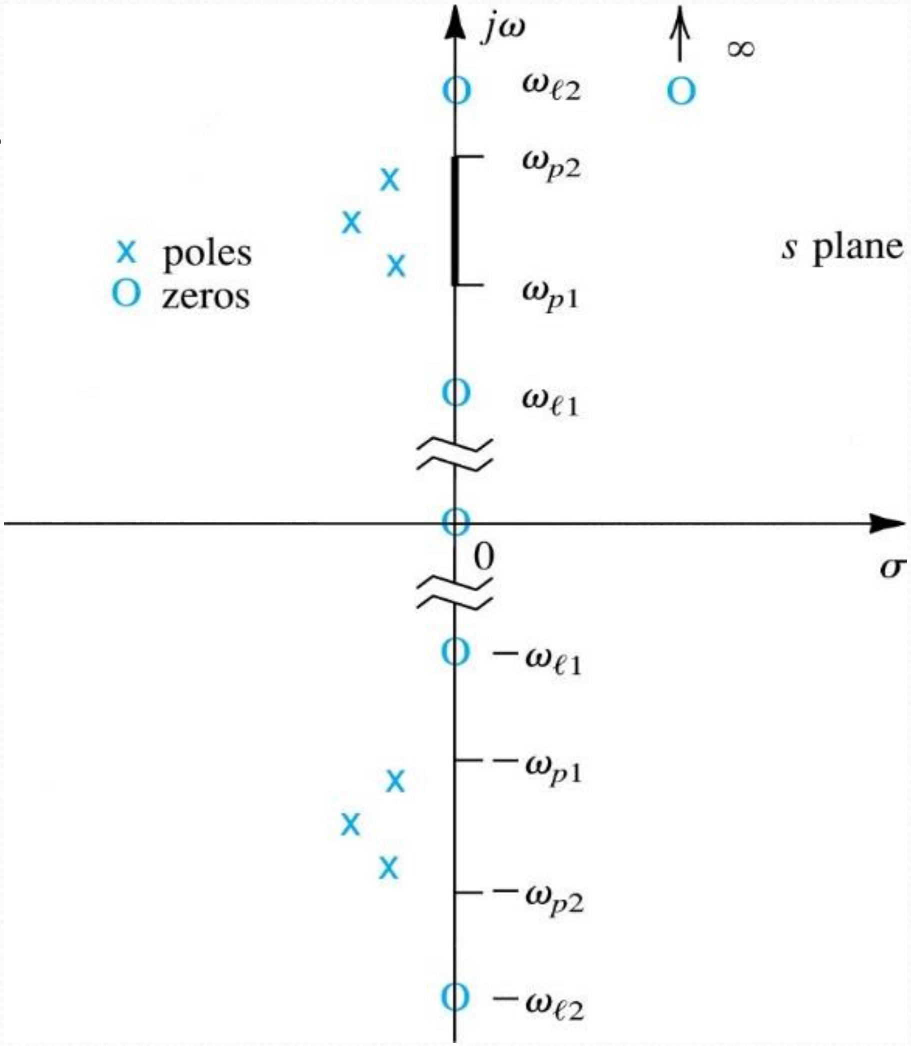


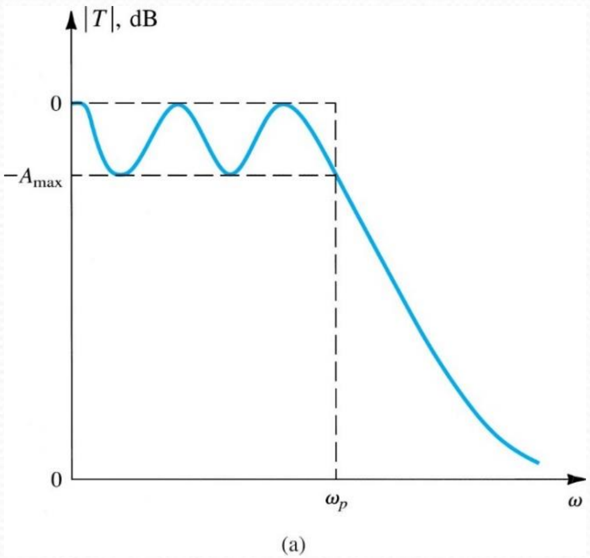
Fig. 13.6 Pole-zero pattern for band-pass filter whose transmission function is shown in Fig. 13.4. This is a sixth-order filter (N=6)

An all-pole filter

- A low-pass filter whose transmission function is depicted in Fig. 13.7(a). We observe that in this case there are no finite values of ω at which attenuation is infinite (zero transmission). Thus it is possible that all the transmission zeros of this filter are at $s = \infty$. If this is the case, the filter transfer function takes the form

$$T(s) = \frac{a_0}{s^N + b_{N-1}s^{N-1} + \ldots + b_0} \qquad (13.10)$$

- Such a filter is known as an **all-pole filter**. Typical pole–zero locations for a fifth-order all-pole low-pass filter are shown in Fig. 13.7(b).



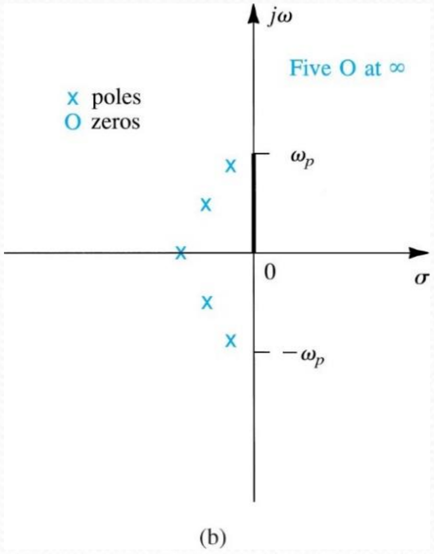


Figure 13.7 (a) Transmission characteristics of a fifth-order low-pass filter having all transmission zeros at infinity. **(b)** Pole–zero pattern for the filter in **(a)**.

Butterworth and Chebyshev Filters

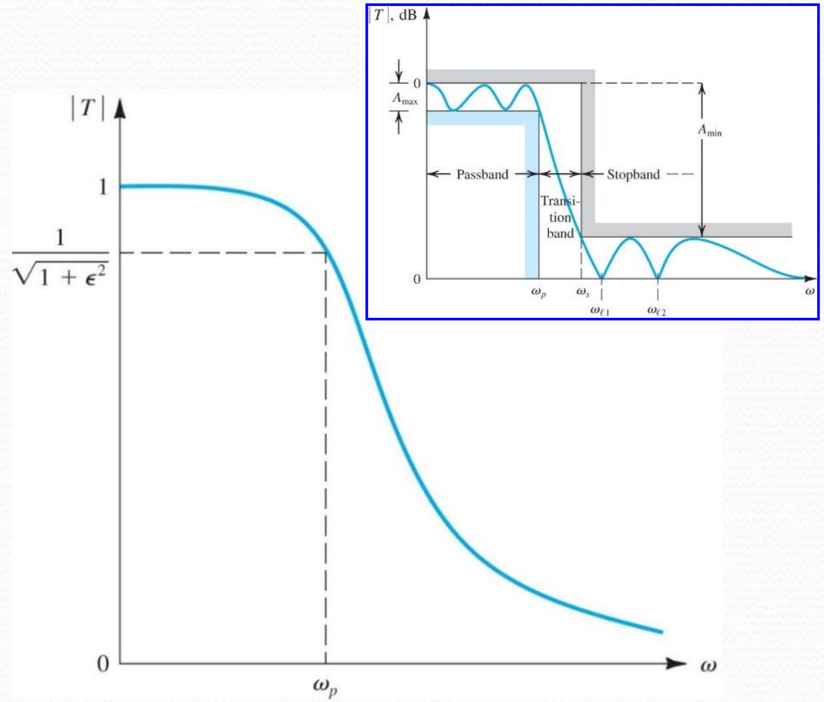
- In this section, we present Two All-Pole Filters that are frequently used in approximating the transmission characteristics of low-pass filters.
- The approximation functions presented can be applied to the design of other filter types through the use of frequency transformations.

The Butterworth Filter

- A Butterworth filter. This filter is with all the transmission zeros at ω = ∞, making it an all-pole filter, $\boxed{T(s)=\frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}}$

- The magnitude function for an *N*th-order Butterworth filter with a passband edge ω_p is given by

  - $$|T(j\omega)|=\frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}} \quad (13.11)$$



- At $\omega = \omega_p$,

  - $$\left|T\left(j\omega_p\right)\right|=\frac{1}{\sqrt{1+\epsilon^2}} \quad (13.12)$$

  - $$A_{\max}=20\log\sqrt{1+\epsilon^2} \quad (13.13)$$

- Conversely, given A_{max}, the value of ε can be determined from

  - $$\boxed{\epsilon=\sqrt{10^{A_{\max}/10}-1}} \quad (13.14)$$

Figure 13.8 The magnitude response of a Butterworth filter.

The Butterworth Filter(Cont'd)

- The Butterworth response is very flat near ω = 0 and results in the name **maximally flat** response. The degree of passband flatness increases as the order *N* is increased, as can be seen from Fig. 13.9. This figure indicates also that as the order *N* is increased, the filter response approaches the ideal brick-wall type of response.

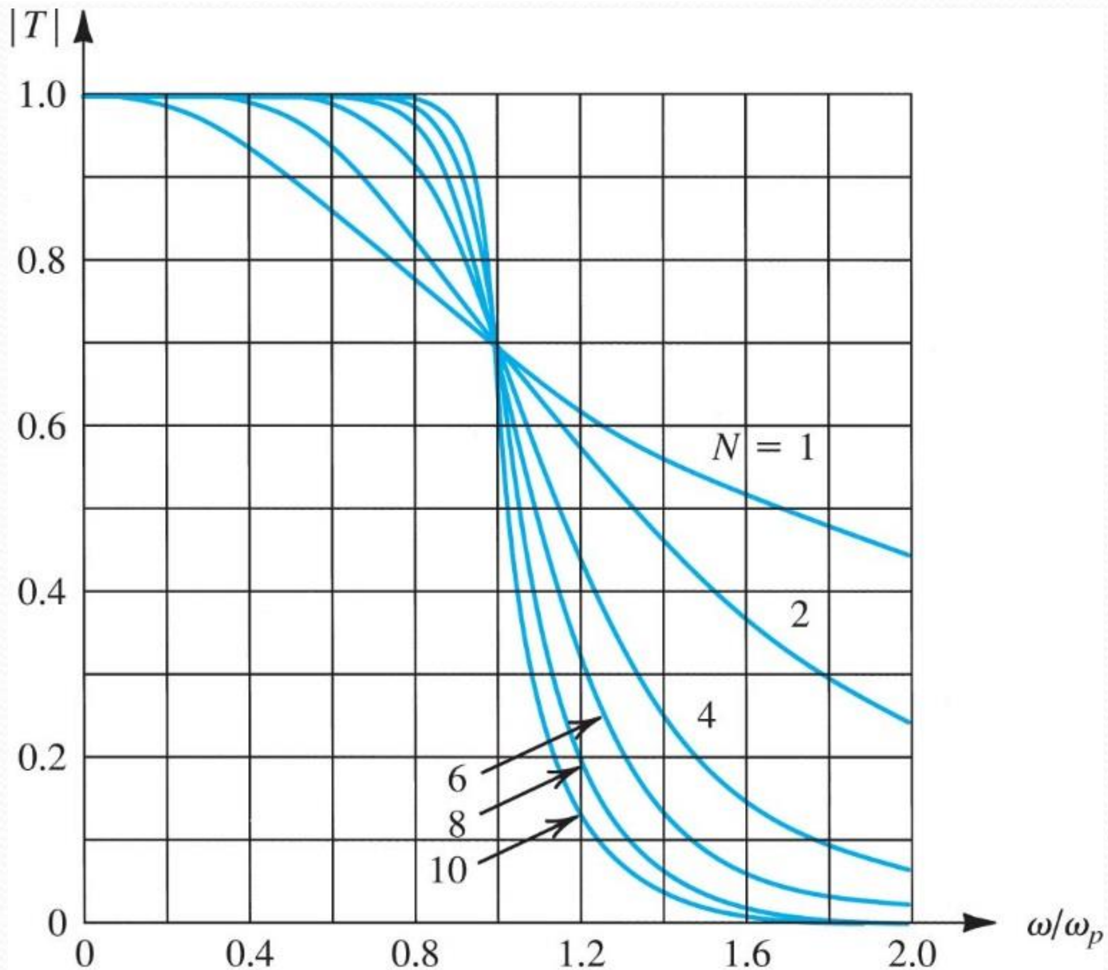


Figure 13.9 Magnitude response for Butterworth filters of various order with ε= 1. Note that as the order increases, the response approaches the ideal brick-wall type of transmission.

The Butterworth Filter(Cont'd)

- At the edge of the stopband, $\omega = \omega_s$, the attenuation of the Butterworth filter can be obtained by substituting $\omega = \omega_s$ in Eq. (13.11). The result is given by

  - $$A(\omega_s) = -20\log\left[1\Big/\sqrt{1+\epsilon^2(\omega_s/\omega_p)^{2N}}\right] = 10\log\left[1+\epsilon^2(\omega_s/\omega_p)^{2N}\right] \quad (13.15)$$

Eq. (13.11)

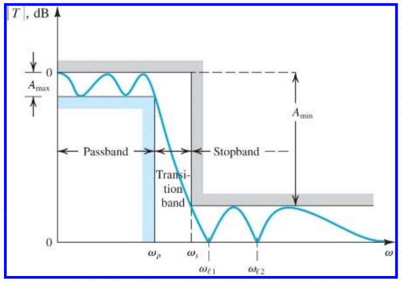
$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}}}$$

- This equation can be used to determine the filter order required, which is the lowest integer value of N that yields $A(\omega_s) \geq A_{\min}$.

- The natural modes of an Nth-order Butterworth filter can be determined from the graphical construction shown in Fig. 13.10(a).

- Figure 13.10(b), (c), and (d) shows the natural modes of Butterworth filters of order N = 2, 3, and 4, respectively. Once the N natural modes $p_1, p_2, \ldots, p_N$ have been found, the transfer function can be written as

  - $$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)} \quad (13.16)$$



The Butterworth Filter(Cont'd)

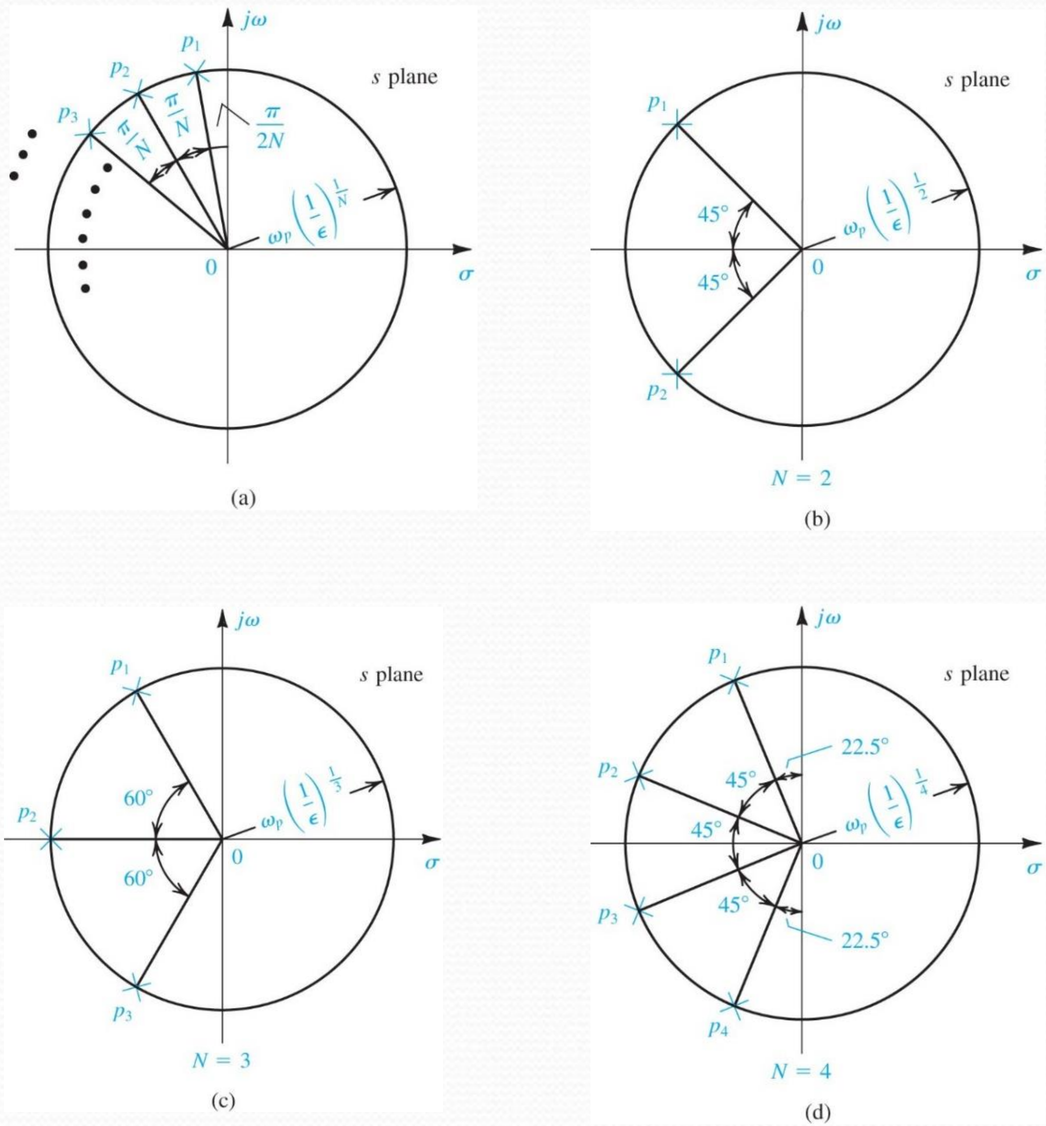


Figure 13.10 Graphical construction for determining the poles of a Butterworth filter of order *N*. All the poles lie in the left half of the *s* plane on a circle of radius $\omega_0 = \omega_p(1/\varepsilon)^{1/N}$, where ε is the passband deviation parameter ($\varepsilon=\sqrt{10^{A_{max}/10}-1}$): **(a)** the general case; **(b)** $N = 2$; **(c)** $N = 3$; **(d)** $N = 4$.

The Butterworth Filter (Cont'd)

- To summarize, to find a Butterworth transfer function that meets transmission specifications of the form in Fig. 13.3 we perform the following procedure:
  - **1.** Determine ε from Eq. (13.14).
  - **2.** Use Eq. (13.15) to determine the required filter order as the lowest integer value of N that results in $A(\omega_s) \geq A_{\min}$.
  - **3.** Use Fig. 13.10(a) to determine the N natural modes.
  - **4.** Use Eq. (13.16) to determine $T(s)$.

Example 13.1

Find the Butterworth transfer function that meets the following low-pass filter specifications: f_p = 10 kHz, A_{max} = 1 dB, f_s = 15 kHz, A_{min} = 25 dB, dc gain = 1.

Solution

Substituting A_{max} = 1 dB into Eq. (13.14) yields ε= 0.5088. Equation (13.15) is then used to determine the filter order by trying various values for N. We find that N =8 yields $A(\omega_s)$=22.3 dB and N =9 gives 25.8 dB. We thus select N = 9. Figure 13.11 shows the graphical construction for determining the poles. The poles all have the same frequency $\omega_0 = \omega_p(1/\varepsilon)^{1/N} = 2\pi \times 10\times10^3 (1/0.5088)^{1/9} = 6.773\times10^4$ rad/s. The first pole p_1 is given by

$$p_1 = \omega_0(-\cos 80^\circ + j \sin 80^\circ) = \omega_0(-0.1736 + j0.9848)$$

Combining p_1 with its complex conjugate p_9 yields the factor$(s^2 + s0.3472\omega_0 + \omega^2_0)$ in the denominator of the transfer function. The same can be done for the other complex poles, and the complete transfer function is obtained using Eq. (13.16),

$$T(s) = \frac{\omega_0^9}{(s+\omega_0)(s^2+s1.8794\omega_0+\omega_0^2)(s^2+s1.5321\omega_0+\omega_0^2)} \times \frac{1}{(s^2+s\omega_0+\omega_0^2)(s^2+s0.3472\omega_0+\omega_0^2)} \qquad (13.17)$$

Example 13.1(Cont'd)

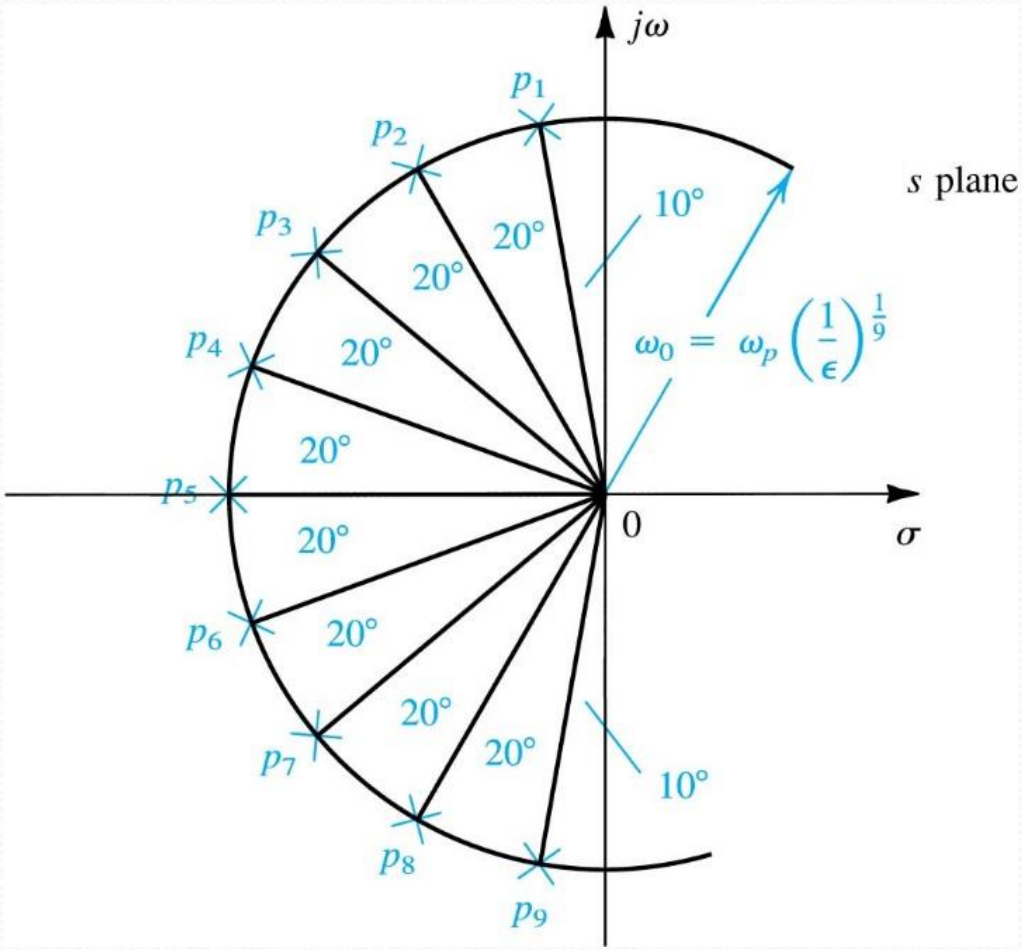


Figure 13.11 Poles of the ninth-order Butterworth filter of Example 13.1.

The Chebyshev Filter

- Figure 13.12 shows representative transmission functions for Chebyshev filters of even and odd orders.

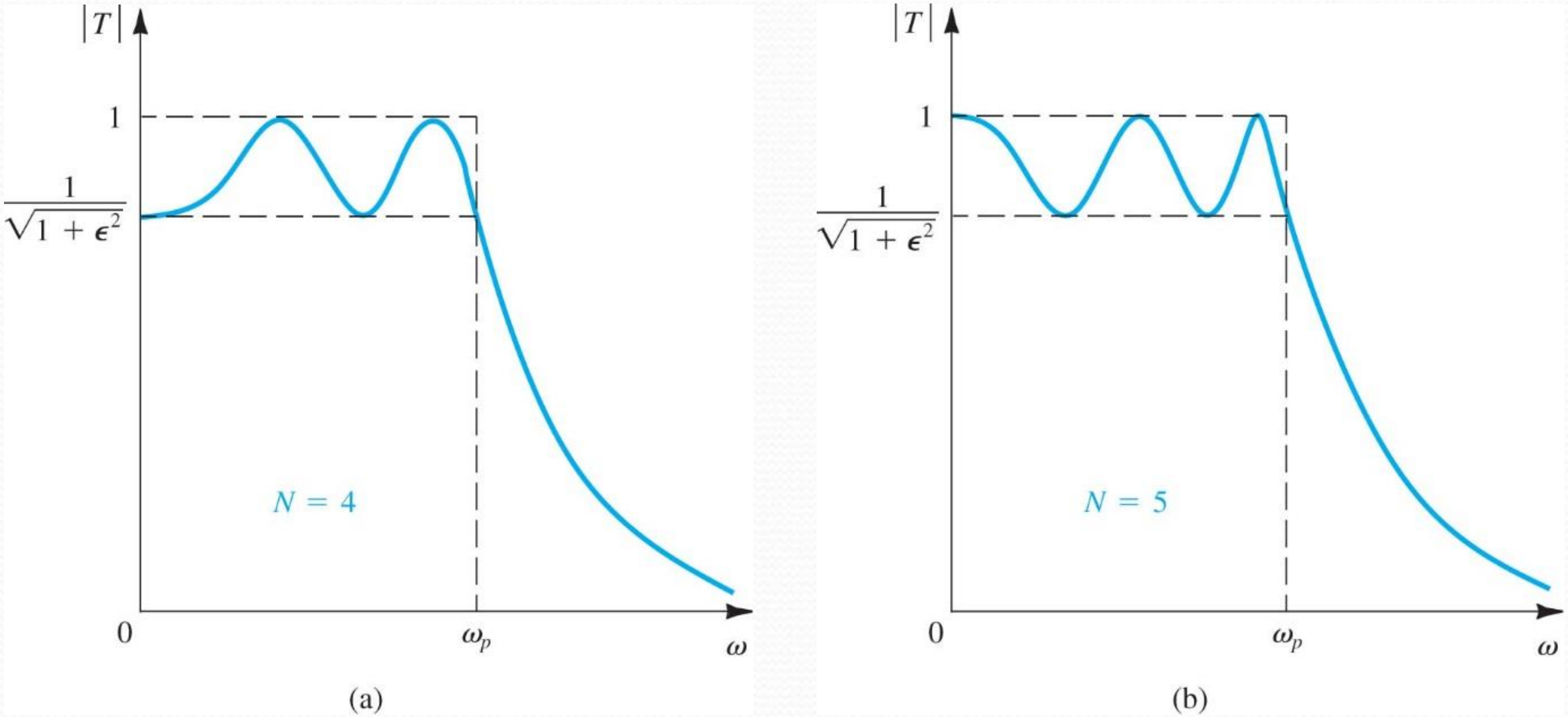


Figure 13.12 Sketches of the transmission characteristics of representative **(a)** even-order and **(b)** odd-order Chebyshev filters.

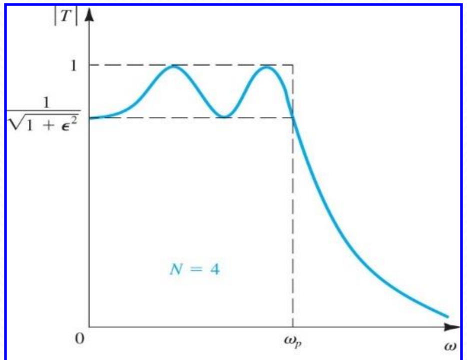
The Chebyshev Filter(Cont'd)

- The magnitude of the transfer function of an *N*th-order Chebyshev filter with a passband edge (ripple bandwidth) ω_p is given by
  - $$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\cos^2[N\cos^{-1}(\omega/\omega_p)]}} \quad \text{for} \quad \omega \le \omega_p \qquad (13.18)$$
  - $$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\cosh^2[N\cosh^{-1}(\omega/\omega_p)]}} \quad \text{for} \quad \omega \ge \omega_p \qquad (13.19)$$

Butterworth: Eq. (13.11)

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

- At the passband edge, $\omega = \omega_p$, the magnitude function is given by
  - $$|T(j\omega_p)| = \frac{1}{\sqrt{1+\epsilon^2}}$$
  - $A_{max} = 10\log(1+\varepsilon^2)$ (13.20)
  - $\varepsilon = \sqrt{10^{A_{max}/10} - 1}$ (13.21)
  - $A(\omega_s) = 10\log[1+\varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$ (13.22)



(a)

The Chebyshev Filter(Cont'd)

- The poles of the Chebyshev filter are given by
  - $$p_k = -\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) + j\omega_p \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) \qquad k = 1, 2, \ldots, N \tag{13.23}$$
- Finally, the transfer function of the Chebyshev filter can be written as
  - $$T(s) = \frac{K\omega_p^N}{\epsilon 2^{N-1}(s-p_1)(s-p_2)\cdots(s-p_N)} \tag{13.24}$$
- To summarize, given low-pass transmission specifications of the type shown in Fig. 13.3, the transfer function of a Chebyshev filter that meets these specifications can be found as follows:
  - **1.** Determine ε from Eq. (13.21).
  - **2.** Use Eq. (13.22) to determine the order required.
  - **3.** Determine the poles using Eq. (13.23).
  - **4.** Determine the transfer function using Eq. (13.24).

Example 13.2

Find the Chebyshev transfer function that meets the same low-pass filter specifications given in Example 13.1: namely, f_p = 10 kHz, A_{max}= 1 dB, f_s= 15 kHz, A_{min} = 25 dB, dc gain = 1.

Solution

Substituting A_{max}= 1 dB into Eq. (13.21) yields ε = 0.5088. By trying various values for N in Eq. (13.22) we find that N =4 yields $A(\omega_s)$=21.6 dB and N =5 provides 29.9 dB. We thus select N =5. Recall that we required a ninth-order Butterworth filter to meet the same specifications in Example 13.1.

The poles are obtained by substituting in Eq. (13.23) as

$$p_1,p_5 = \omega_p(-0.0895 \pm j0.9901)$$

$$p_2,p_4 = \omega_p(-0.2342 \pm j0.6119)$$

$$p_5 = \omega_p(-0.2895)$$

The transfer function is obtained by substituting these values in Eq. (13.24) as

$$T(s) = \frac{\omega_p^5}{8.1408(s+0.2895\,\omega_p)(s^2+s0.4684\,\omega_p+0.4293\,\omega_p^2)} \times \frac{1}{s^2+s\,0.1789\,\omega_p+0.9883\,\omega_p^2} \qquad (13.25)$$

where ω_p= $2\pi \times 10^4$ rad/s.

First-Order and Second-Order Filters

- First- and second-order filters can also be cascaded to realize a high-order filter.
- Because the filter poles occur in complex-conjugate pairs, a high-order transfer function $T(s)$ is factored into the product of second-order functions. If $T(s)$ is odd there will also be a first-order function in the factorization.

First-Order Filters

- The general first-order transfer function is given by
  - $$T(s) = \frac{a_1 s + a_0}{s + \omega_0} \quad (13.26)$$
- This **bilinear transfer function** characterizes a first-order filter with a natural mode at $s = -\omega_0$, a transmission zero at $s = -a_0/a_1$, and a high-frequency gain that approaches a_1.

First-Order Filters(Cont'd)

- Some special cases together with passive (RC) and active (op amp–RC) realizations are shown in Fig. 13.13 as below.

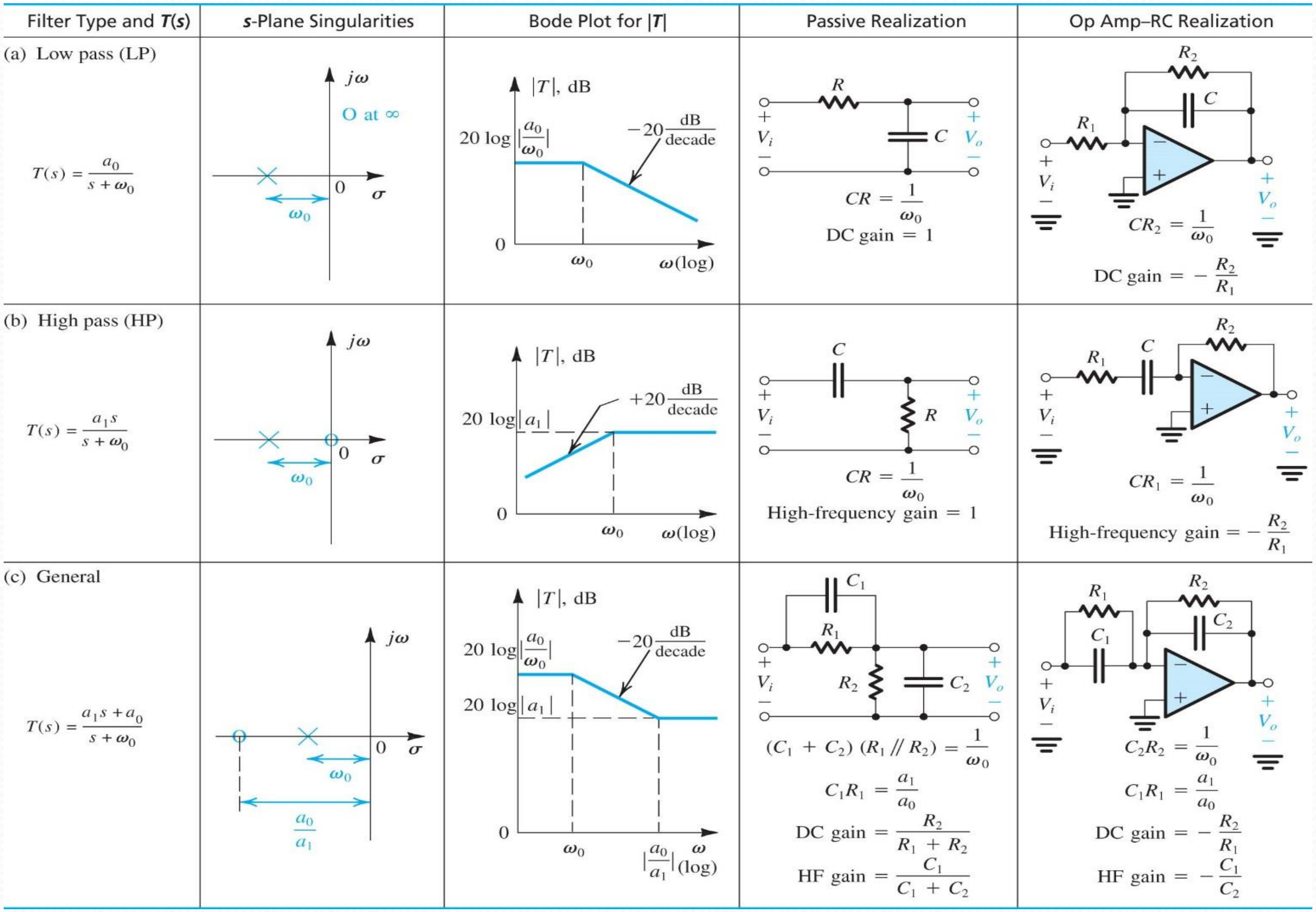


Fig. 13.13 First-order filters

All-pass First-Order Filters

- An important special case of the first-order filter function is the **all-pass filter** shown in Fig. 13.14.

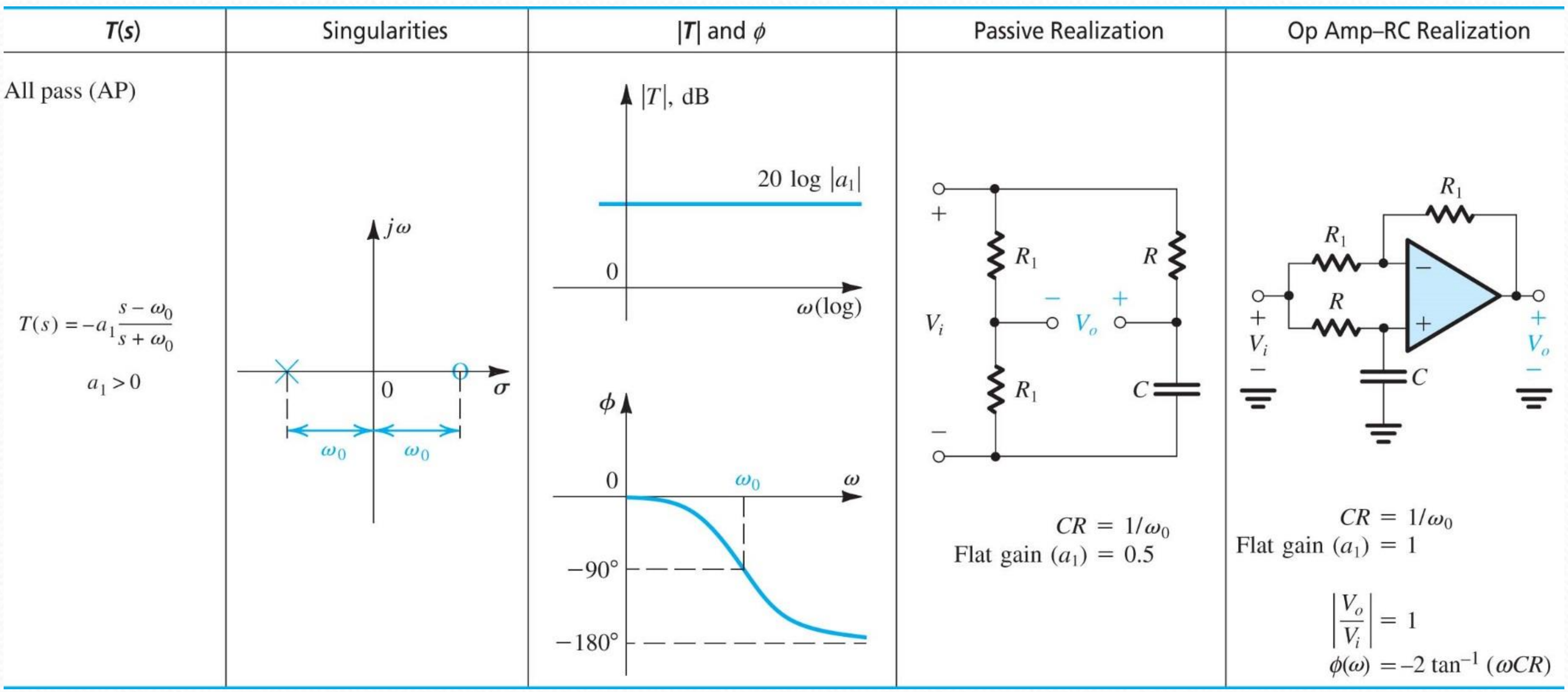


Fig. 13.14 First-order all-pass filters

Second-Order Filter Functions

- The general second-order (or **biquadratic**) filter transfer function is usually expressed in the standard form
  - $$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0/Q)s + \omega_0^2} \qquad (13.27)$$

  where ω_0 and Q determine the natural modes (poles) according to
  - $$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - (1/4Q^2)} \qquad (13.28)$$

Second-Order Filter Functions(Cont'd)

- Figure 13.15 shows the location of the pair of complex-conjugate poles in the *s* plane.
- The radial distance of the natural modes (from the origin) is equal to ω_0, which is known as the **pole frequency**.
- The parameter *Q* determines the distance of the poles from the *j*ω axis:
  - the higher the value of *Q*, the closer the poles are to the *j*ω axis, and the **more selective the filter response**
  - The parameter *Q* is called the **pole quality factor**, or simply, **pole *Q***.

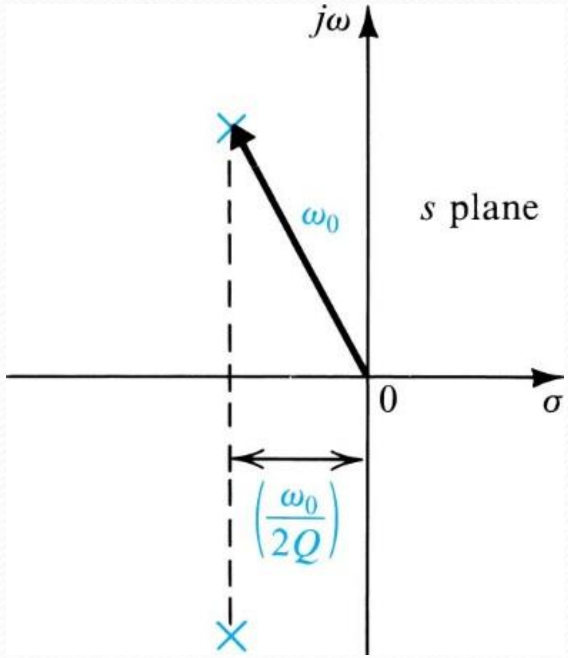


Figure 13.15 Definition of the parameters ω_0 and Q of a pair of complex-conjugate poles,

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - \left(1/4Q^2\right)}$$

Second-Order Filter Functions (Cont'd)

- The transmission zeros of the second-order filter are determined by the numerator coefficients, a_0, a_1, and a_2.
- It follows that the numerator coefficients determine the type of second-order filter function (i.e., LP, HP, etc.).

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - (1/4Q^2)}$$

Second-Order Filter Functions(Cont'd)

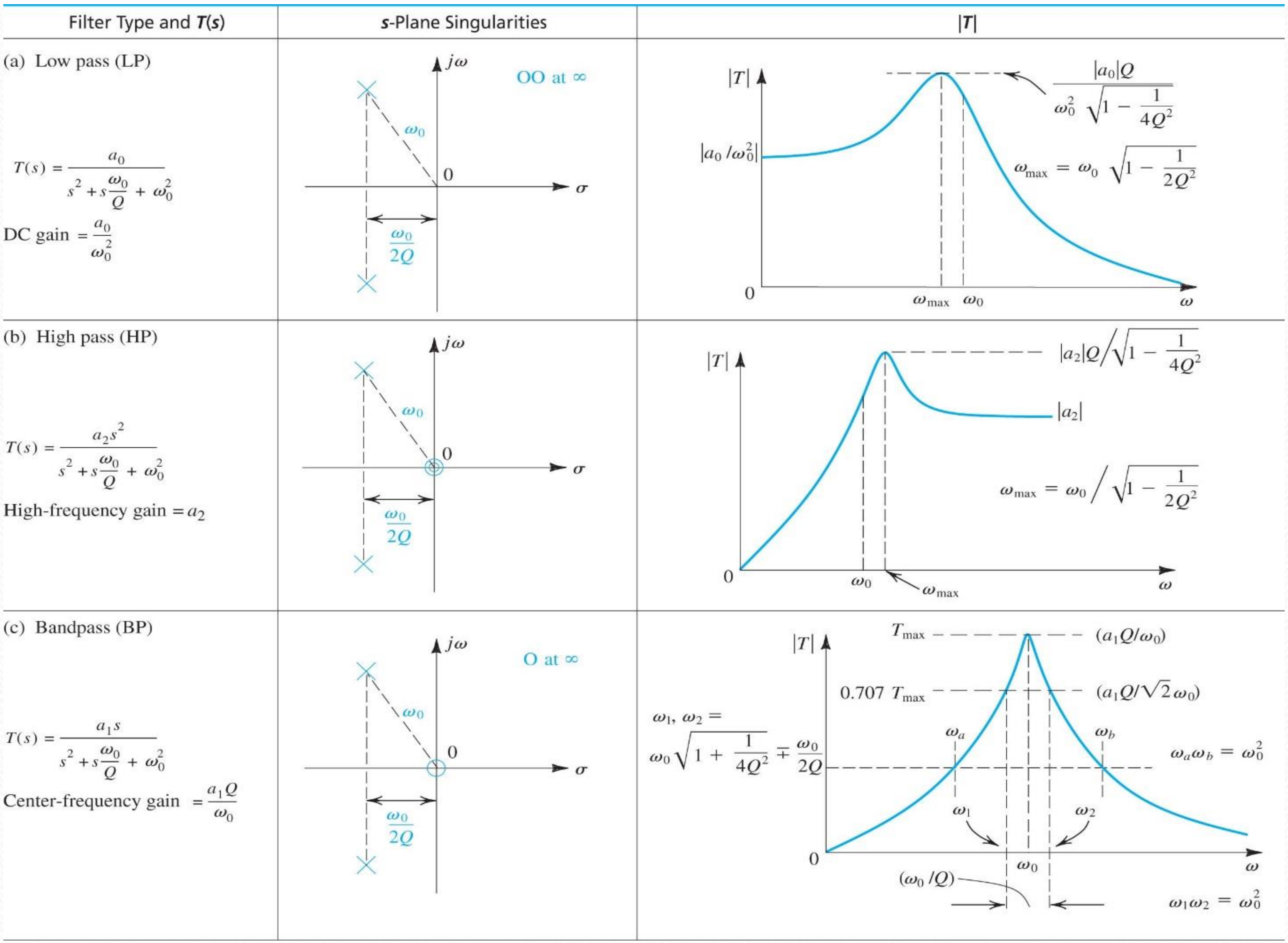


Figure 13.16 Second-order filtering functions.

Second-Order Filter Functions(Cont'd)

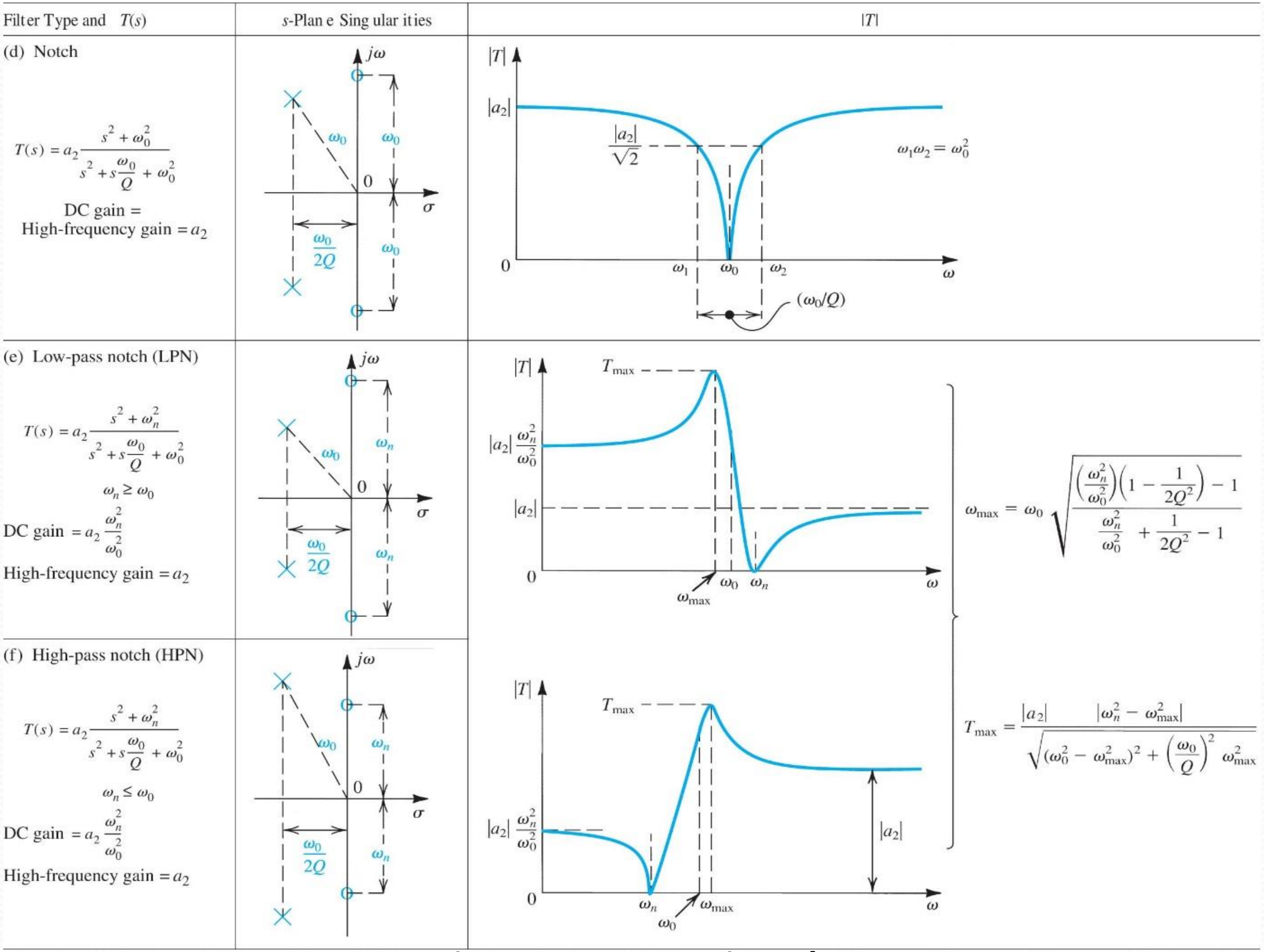
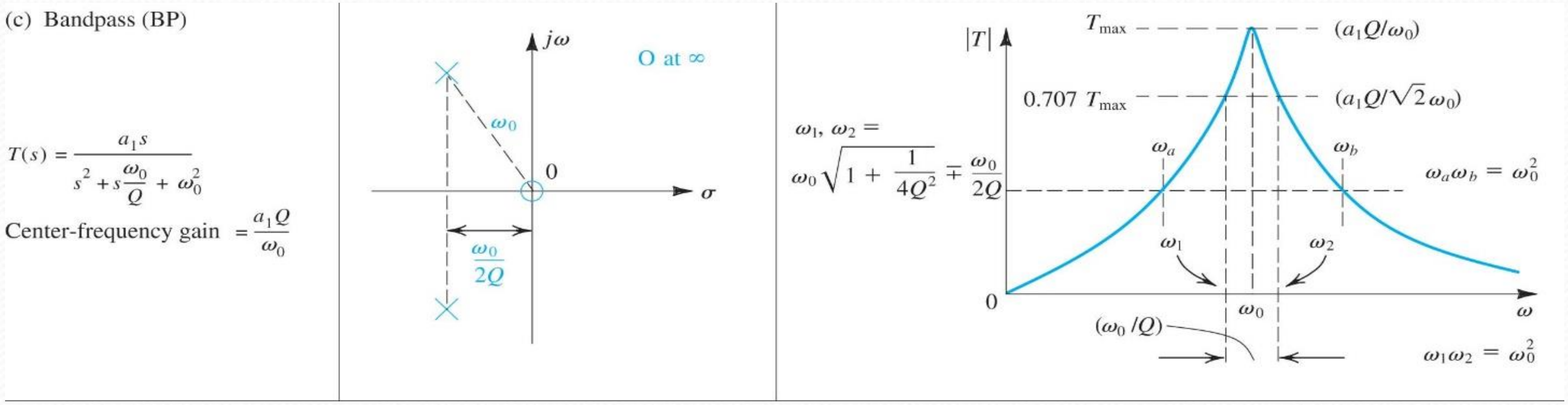


Figure 13.16 continued

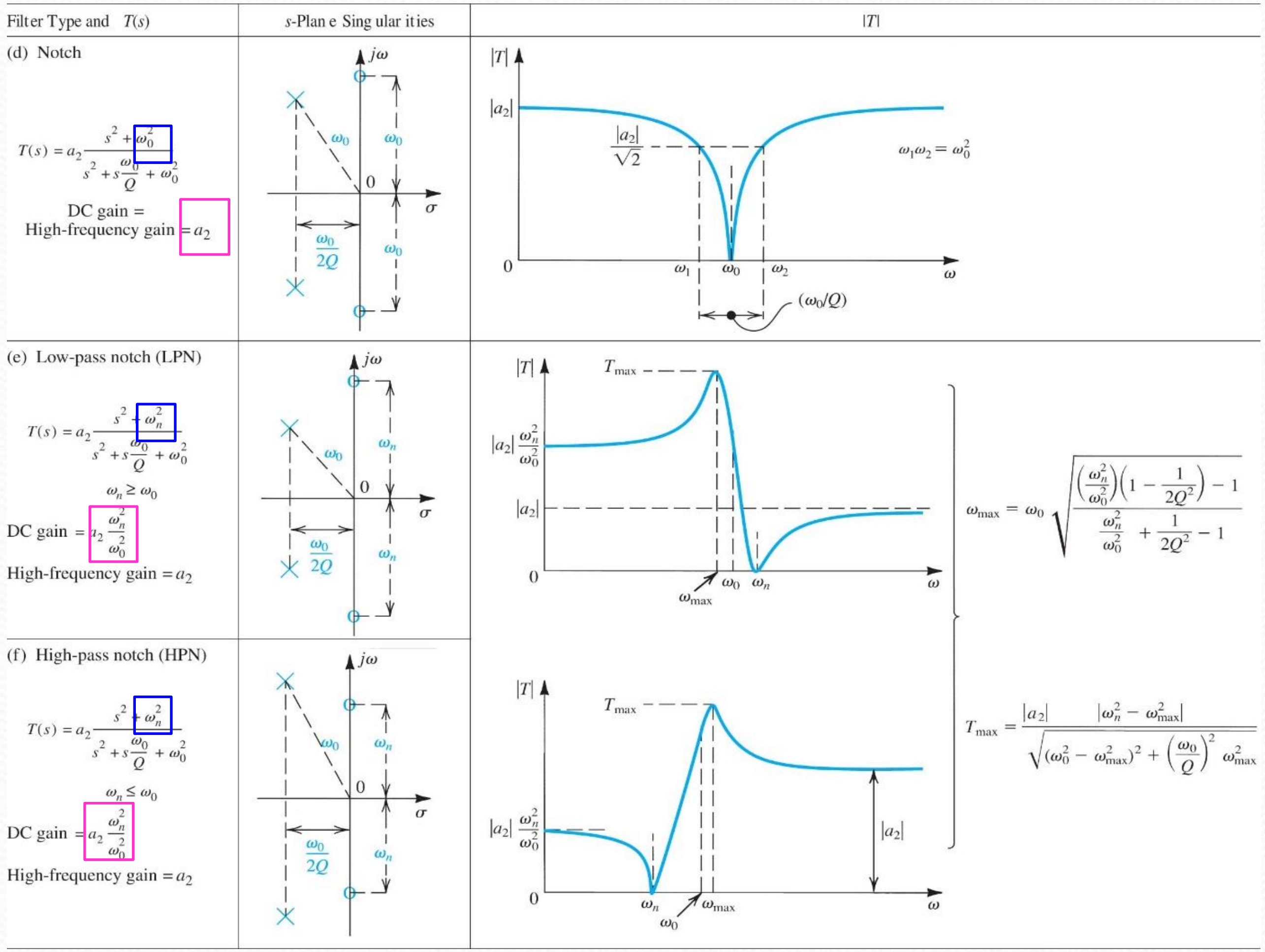
Second-Order Filter Functions(Cont'd)



- From the bandpass in Fig. 13.16, it can be shown that
  - $$\omega_1, \omega_2 = \omega_0\sqrt{1 + (1/4Q^2)} \pm \frac{\omega_0}{2Q} \qquad (13.29)$$
- Thus, $BW \equiv \omega_2 - \omega_1 = \omega_0/Q$ (13.30)
- Observe that as *Q* increases, the bandwidth decreases and the bandpass filter becomes more selective.

Notch Filters

- If the transmission zeros are located on the $j\omega$ axis, at the complex-conjugate locations $\pm j\omega_n$, then the magnitude response exhibits zero transmission at $\omega = \omega_n$.
- Thus a **notch** in the magnitude response occurs at $\omega = \omega_n$, and ω_n is known as the **notch frequency**. Three cases of the second-order notch filter are possible in previous figures.



The Second-Order LCR Resonator

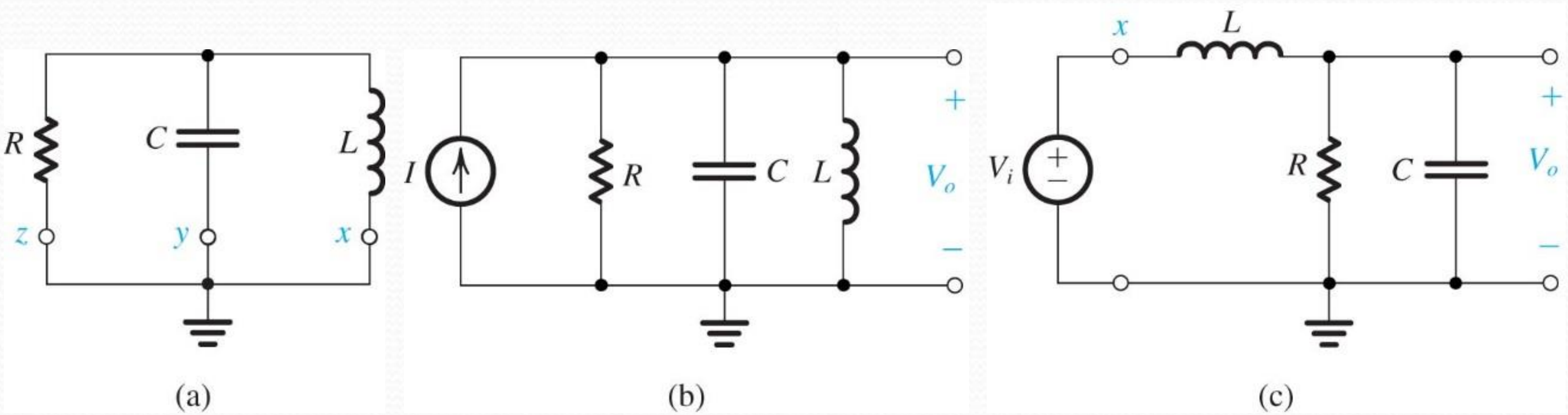
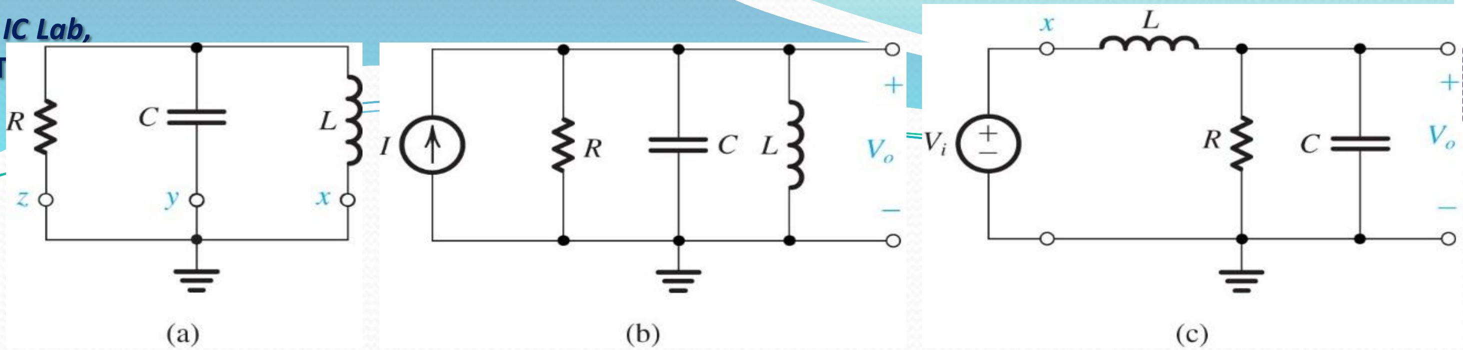
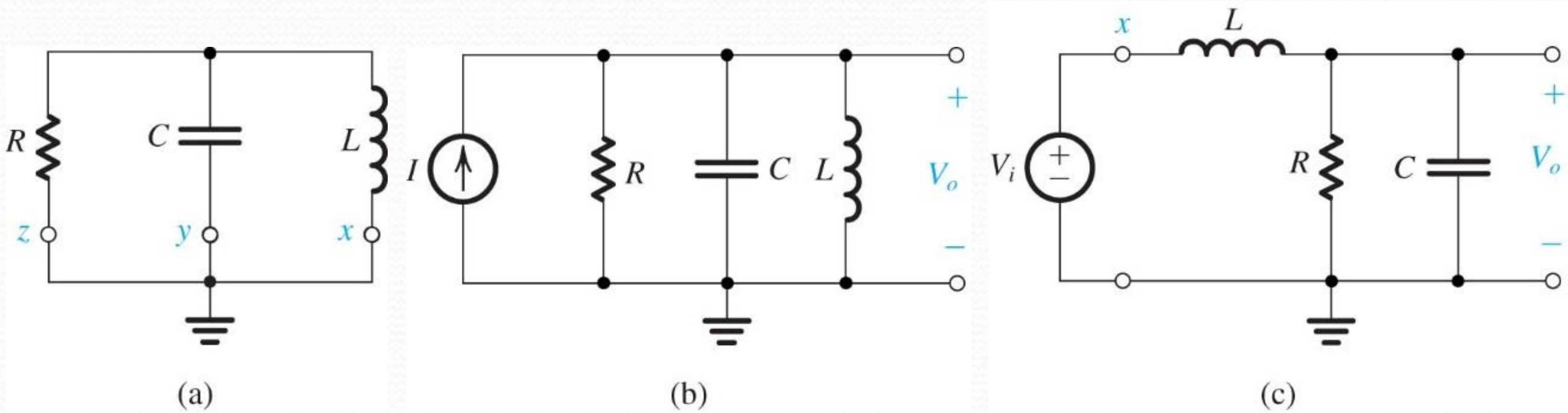


Figure 13.17 (a) The second-order parallel LCR resonator. **(b, c)** Two ways of exciting the resonator of **(a)** without changing its *natural structure:* resonator poles are those poles of Vo/I and Vo/V_i.



- In Fig. 13.17(b), to work in terms of the admittance *Y*; thus,
  - $$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{(1/sL) + sC + (1/R)} = \frac{s/C}{s^2 + s(1/CR) + (1/LC)} \qquad (13.31)$$
- Equating the denominator to the standard form $[s^2 + s(\omega_0/Q) + \omega_0{}^2)$ leads to $\omega_0{}^2 = 1/LC$ (13.32)
- and $\omega_0/Q = 1/CR$ (13.33)
- Thus $\omega_0 = 1/\sqrt{LC}$ (13.34)
- $Q = \omega_0 CR$ (13.35)

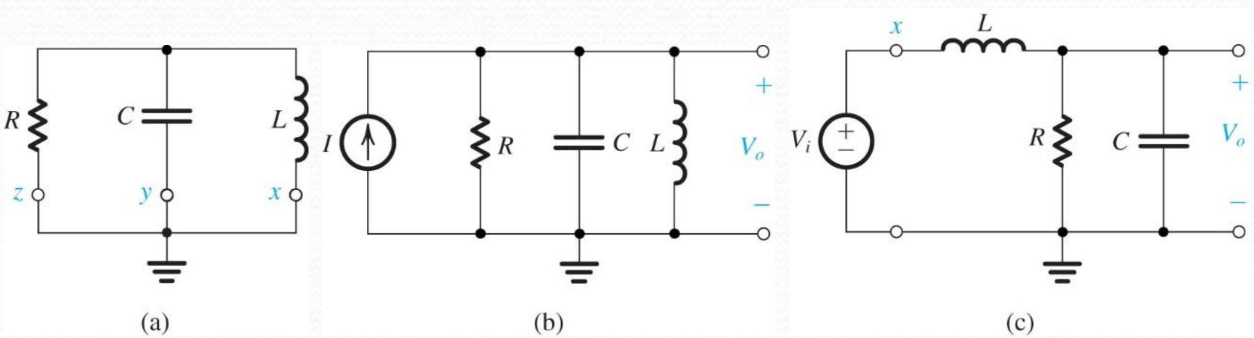


- An alternative way of exciting the parallel LCR resonator for the purpose of determining its natural modes is shown in Fig. 13.17(c). Here, node *x* of inductor *L* has been disconnected from ground and connected to an ideal voltage source *Vi*.

- In a design problem, we will be given ω_0 and *Q* and will be asked to determine *L, C,* and *R*. Equations (13.34) and (13.35) are two equations in the three unknowns. The one available degree of freedom can be utilized to set the impedance level of the circuit to a value that results in practical component values.

Realization of Transmission Zeros

- Any of the nodes labeled *x*, *y*, or *z* can be disconnected from ground and connected to Vi without altering the circuit's natural modes. When this is done, the circuit takes the form of a voltage divider, as shown in Fig. 13.18(a). Thus the transfer function realized is
  - $$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} \qquad (13.36)$$



$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

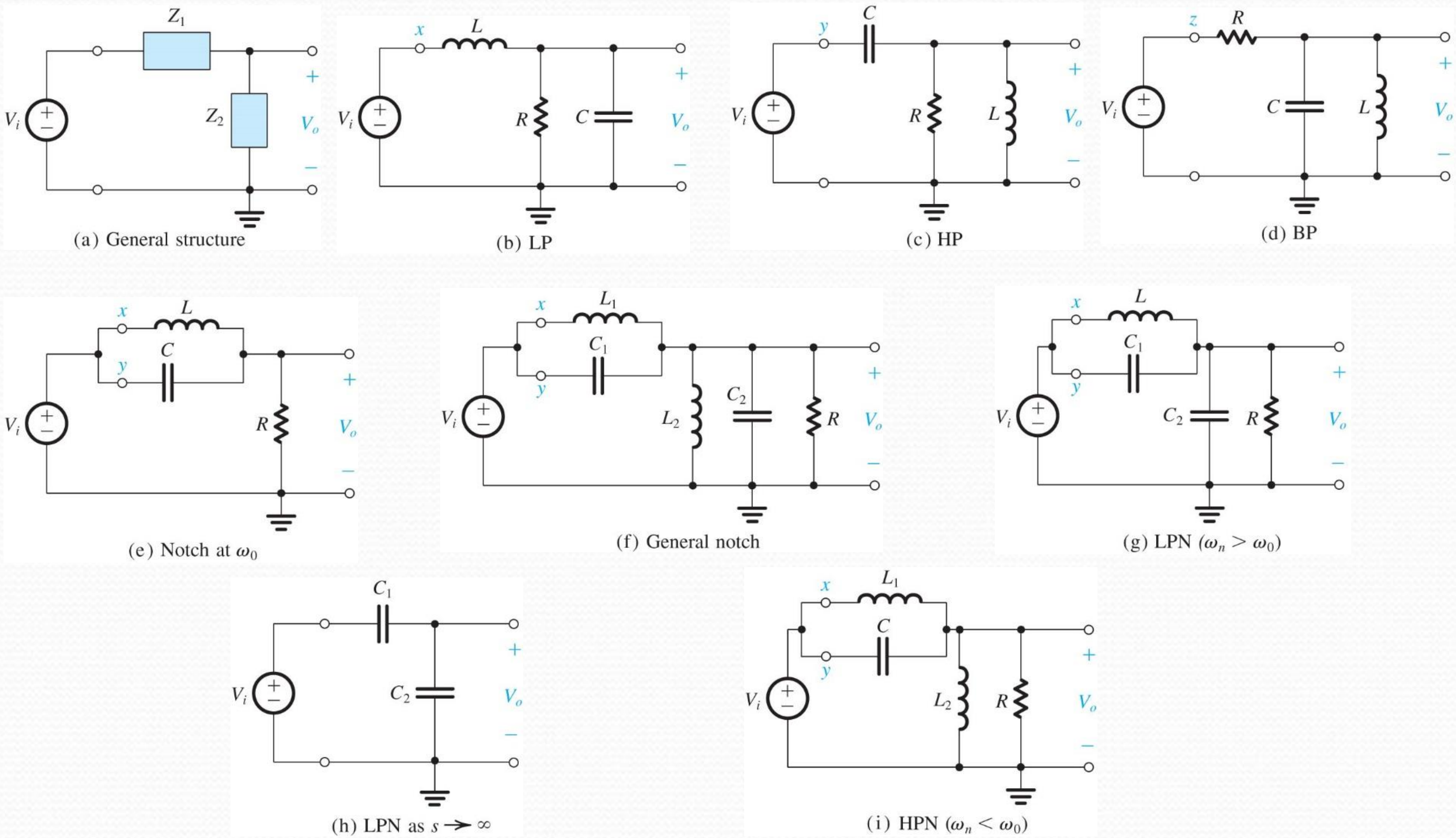
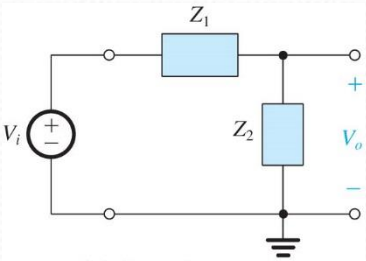


Figure 13.18 Realization of various second-order filter functions using the LCR resonator of Fig. 13.17(b): **(a)** general structure, **(b)** LP, **(c)** HP**, (d)** BP, **(e)** notch at ω_0, **(f)** general notch, **(g)** LPN ($\omega_n \geqq \omega_0$), **(h)** LPN as $s \rightarrow \infty$, **(i)** HPN ($\omega_n < \omega_0$).

Realization of Transmission Zeros (Cont'd)

- From equation (13.36) and Fig. 13.18(a), we observe that *the transmission zeros are the values of s at which $Z_2(s)$ is zero, provided $Z_1(s)$ is not simultaneously zero, and the values of s at which $Z_1(s)$ is infinite, provided $Z_2(s)$ is not simultaneously infinite*.

equation (13.36) $$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$



(a) General structure

Fig. 13.18(a)

Realization of the Low-Pass Function

- To realize a low-pass function, node *x* is disconnected from ground and connected to Vi, as shown in Fig. 13.18(b). This circuit has two transmission zeros at *s* = ∞, as a second-order LP is supposed to. The transfer function can be written either by inspection or by using the voltage divider rule. Following the latter approach, we obtain

$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/sL}{(1/sL) + sC + (1/R)} = \frac{1/LC}{s^2 + s(1/CR) + (1/LC)} \qquad (13.37)$$

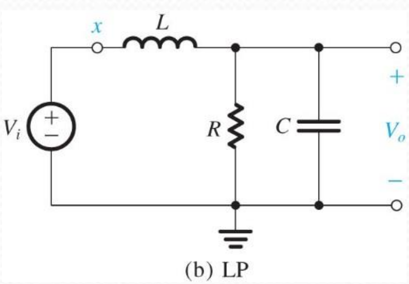
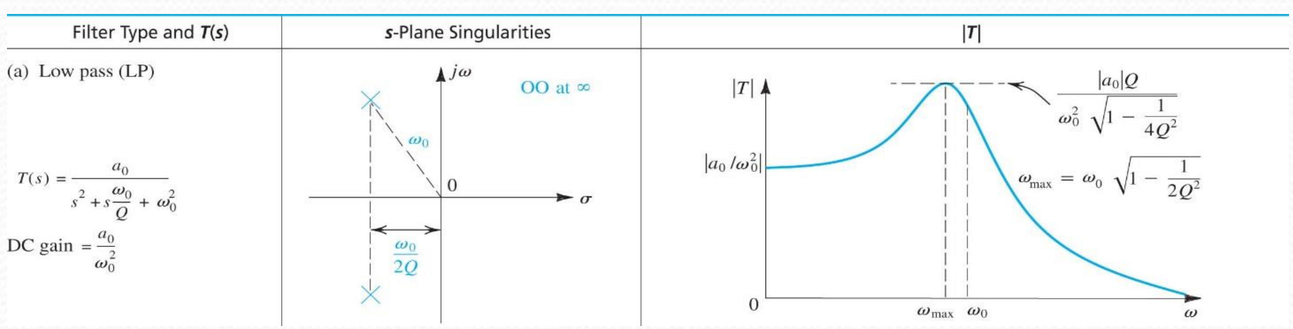


Fig. 13.18(b)



Realization of the High-Pass Function

- To realize the second-order high-pass function, node *y* is disconnected from ground and connected to Vi, as shown in Fig. 13.18(c). By inspection, the transfer function may be written as

$$T(s) \equiv \frac{V_o}{V_i} = \frac{a_2 s^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (13.38)$$

where ω_0 and Q are the natural mode parameters given by Eqs. (13.34) and (13.35) and a_2 is the high-frequency transmission.

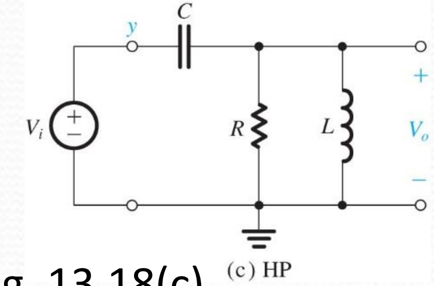
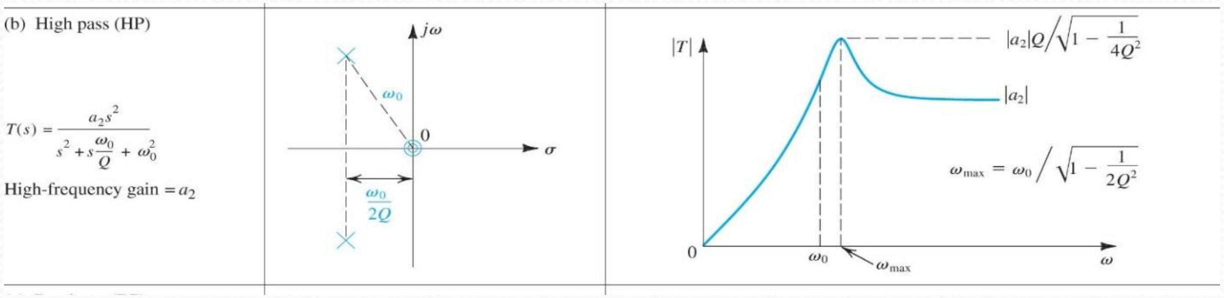


Fig. 13.18(c)



- $\omega_0 = 1/\sqrt{LC}$ (13.34)

- $Q = \omega_0 CR$ (13.35)

Realization of the Bandpass Function

- The bandpass function is realized by disconnecting node *z* from ground and connecting it to *Vi*, as shown in Fig. 13.18(d). Its transfer function can be obtained as follows:

$$T(s) = \frac{Y_R}{Y_R + Y_L + Y_C} = \frac{1/R}{(1/R) + (1/sL) + sC} = \frac{s(1/CR)}{s^2 + s(1/CR) + (1/LC)} \qquad (13.39)$$

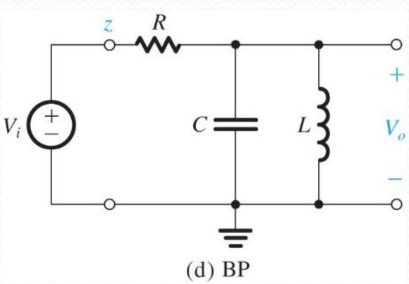
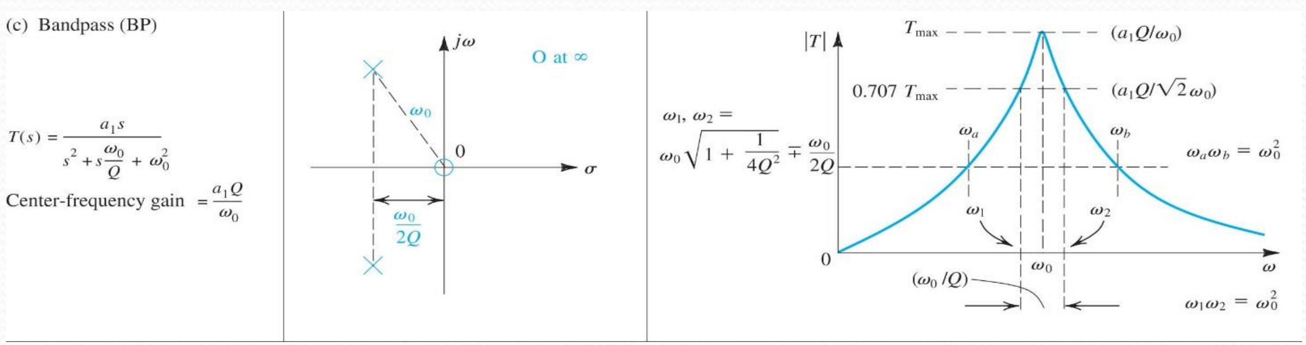
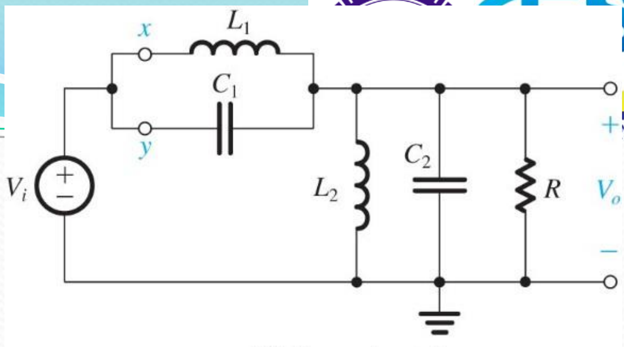


Fig. 13.18(d)

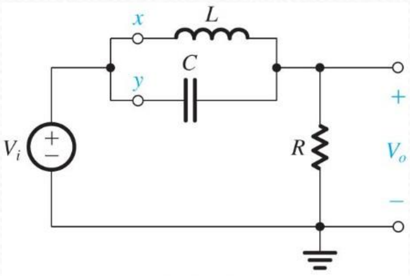


Realization of the Notch Functions

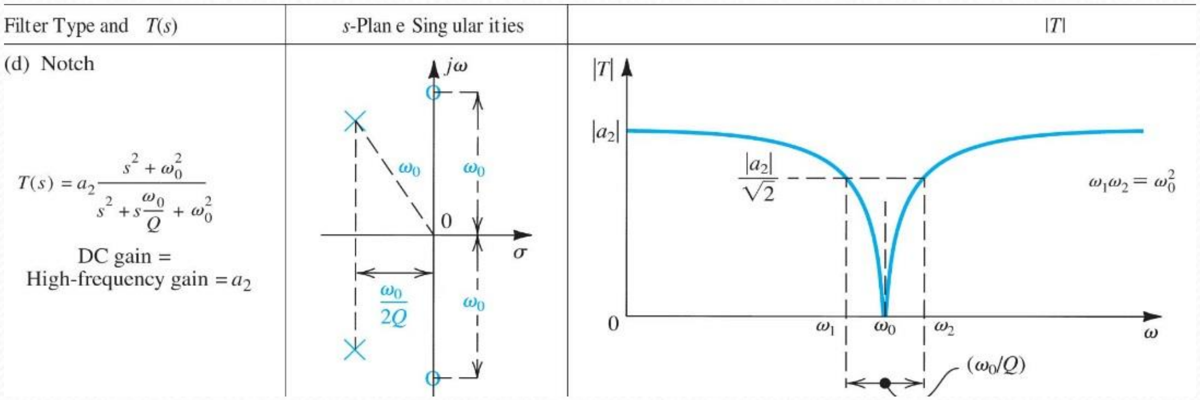


(f) General notch

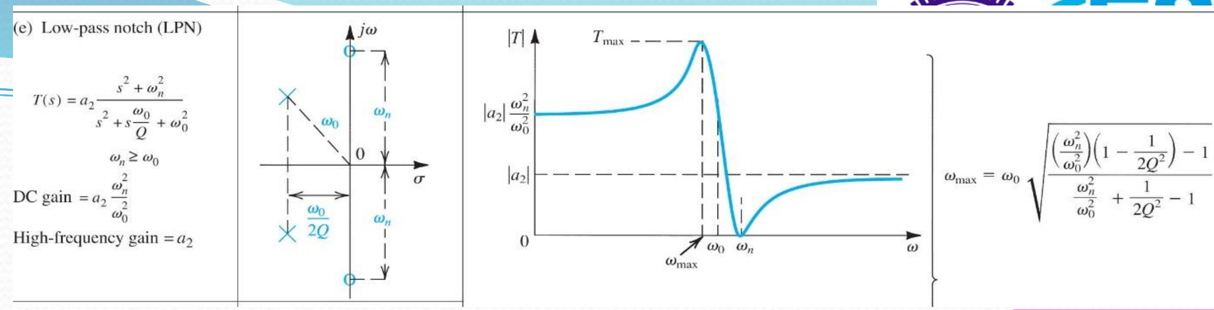
- Will realize the notch transfer function
  - $$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (13.40)$$
- We still use a parallel LC circuit in the series branch, as shown in Fig. 13.18(f) where L_1 and C_1 are selected so that
  - $$L_1 C_1 = 1/\omega_n^2 \qquad (13.41)$$
- The values of L_2 and C_2 must be selected to ensure that the natural modes have not been altered; thus,
  - $C_1 + C_2 = C$ (13.42)
  - $L_1 \,||\, L_2 = L$ (13.43)



(e) Notch at ω_0



LPN realization



- For the LPN, $\omega_n > \omega_0$

  thus and $L_1C_1 < (L_1 \,||\, L_2)(C_1 + C_2)$, since $(L_1 \,||\, L_2)(C_1 + C_2) = LC = 1/\omega_0^2$, $\boxed{L_1C_1 = 1/\omega_n^2}$

- This condition can be satisfied with L_2 eliminated (i.e., $L_2 = \infty$ and $L_1 = L$), resulting in the LPN circuit in Fig. 13.18(g). The transfer function can be written by inspection as

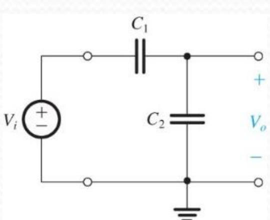
- $$T(s) \equiv \frac{V_o}{V_i} = a_2 \frac{s^2+\omega_n^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (13.44)$$

- where $\omega^2{}_n = 1/LC_1, \omega^2{}_o = 1/L(C_1+C_2), \omega_o/Q = 1/CR$, and a_2 is the high-frequency gain.

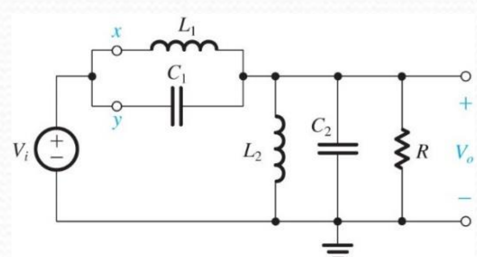
- As $s \to \infty$, the circuit reduces to that in Fig. 13.18(h), for which

- Thus, $$\frac{V_o}{V_i} = \frac{C_1}{C_1+C_2}$$

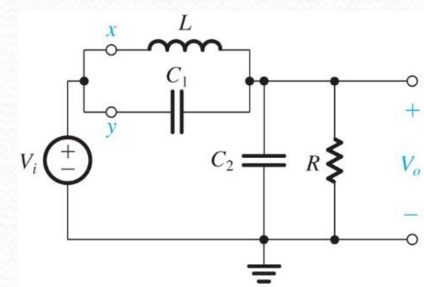
$$\boxed{a_2 = \frac{C_1}{C_1+C_2}} \qquad (13.45)$$



(h) LPN as $s \to \infty$

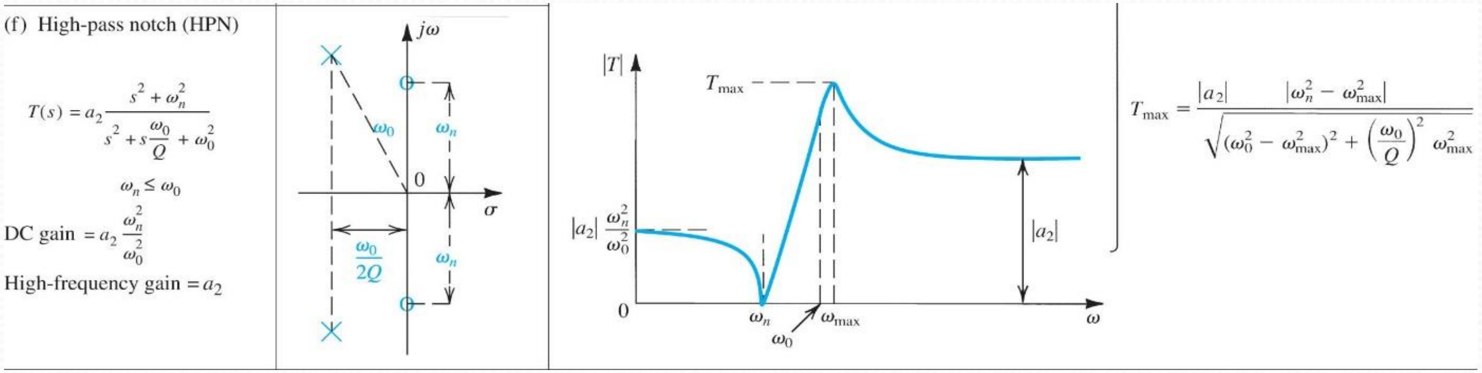


(f) General notch

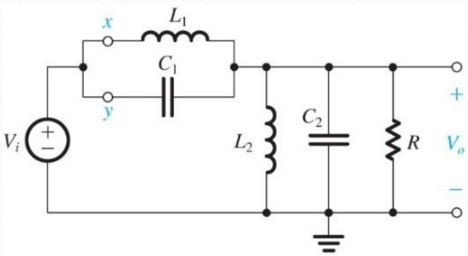


(g) LPN ($\omega_n > \omega_0$)

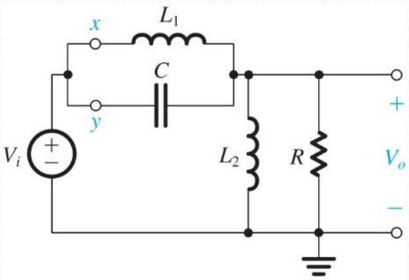
HPN realization



- To obtain an HPN realization we start with the circuit of Fig. 13.18(f) and use the fact That $\omega_n < \omega_0$ to obtain $L_1C_1 > (L_1 \,||\, L_2)(C_1 + C_2)$, which can be satisfied while selecting $C_2 = 0$ (i.e., $C_1 = C$).



(f) General notch



(i) HPN ($\omega_n < \omega_0$)

The transfer function can be expressed as

- $$T(s) \equiv \frac{V_o}{V_i} = \frac{s^2 + (1/L_1C)}{s^2 + s(1/CR) + [1/(L_1 \| L_2)C]} \quad (13.46)$$

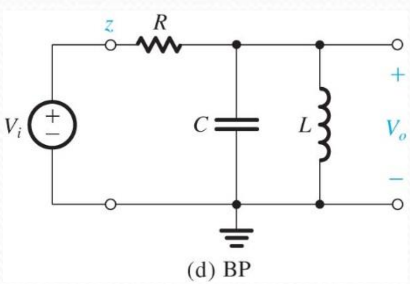
Realization of the All-Pass Function

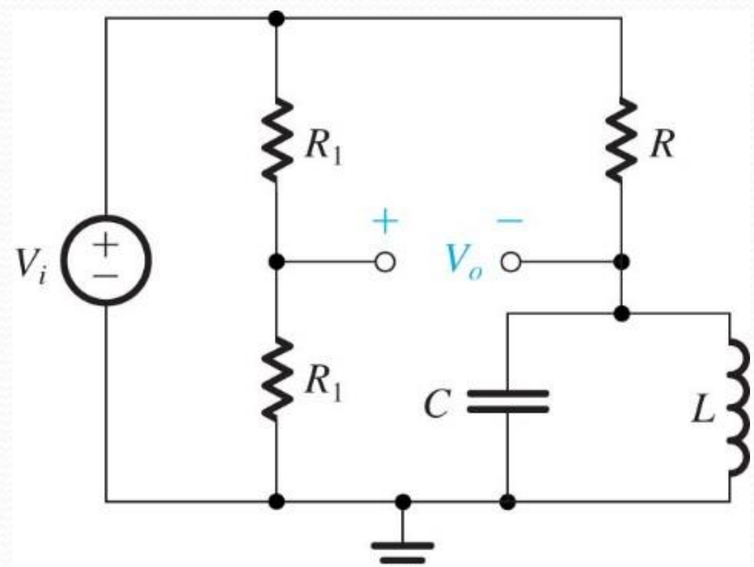
- The all-pass transfer function
  - $$T(s) = \frac{s^2 - s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (13.47)$$
- can be written as
  - $$T(s) = 1 - \frac{s2(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (13.48)$$

Realization of the All-Pass Function(Cont'd)

- We shall therefore attempt an all-pass realization with a flat gain of 0.5, that is, $$T(s) = 0.5 - \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$
- This function can be realized using a voltage divider with a transmission ratio of 0.5 together with the bandpass circuit of Fig. 13.18(d).
- To effect the subtraction, the output of the all-pass circuit is taken between the output terminal of the voltage divider and that of the bandpass filter, as shown in Fig. 13.19.

Figure 13.19 Realization of the second-order all-pass transfer function using a voltage divider and an LCR resonator





Second-Order Active Filters Based on Inductor Replacement

- In this section, we study a family of op amp–RC circuits that realize the various second-order filter functions.

The Antoniou Inductance-Simulation Circuit

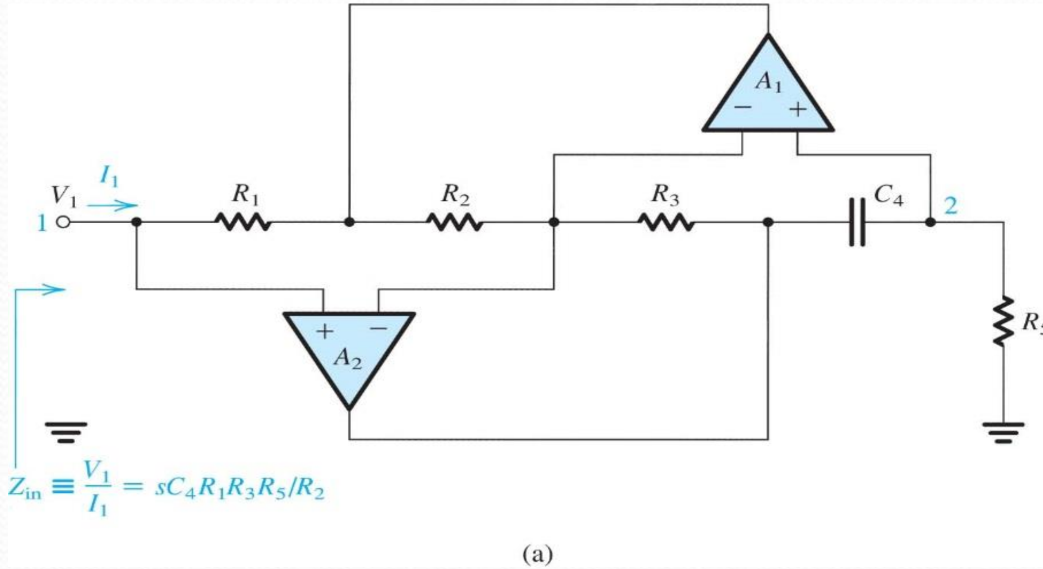
- Figure 13.20(a) shows the Antoniou inductance-simulation circuit. If the circuit is fed at its input (node 1) with a voltage source V_1 and the input current is denoted I_1, then for ideal op amps the input impedance can be shown to be
  - $Z_{in} \equiv V_1/I_1 = sC_4R_1R_3R_5/R_2$ (13.49)

  which is that of an inductance L given by
- $$L = C_4R_1R_3R_5/R_2 \qquad (13.50)$$
- The design of this circuit is usually based on selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C$, which leads to $L = CR^2$. Convenient values are then selected for C and R to yield the desired inductance value L.

The Antoniou Inductance-Simulation Circuit(Cont'd)

$$Z_{in} \equiv V_1/I_1 = sC_4R_1R_3R_5/R_2$$

$$L_{in} = C_4R_1R_3R_5/R_2$$
$$= CR^2$$



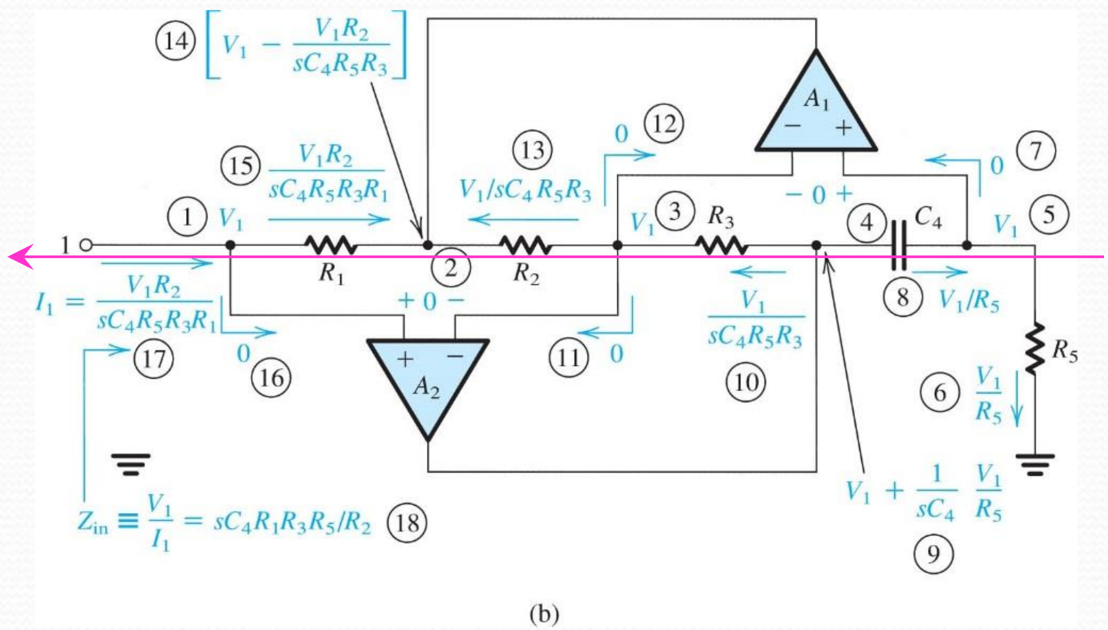
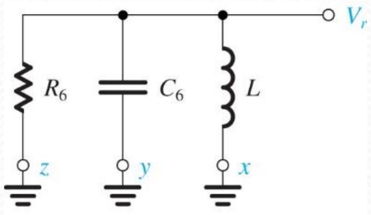


Figure 13.20 (a) The Antoniou inductance-simulation circuit. **(b)** Analysis of the circuit assuming ideal op amps. The order of the analysis steps is indicated by the circled numbers.

The Op Amp-RC Resonator

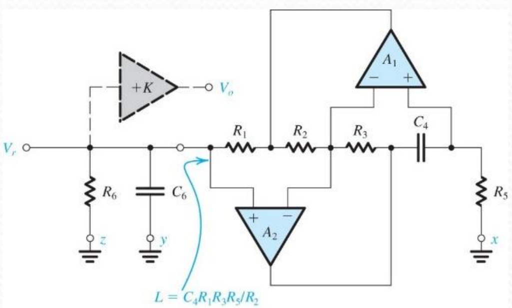
- Figure 13.21(a) shows the LCR resonator. Replacing the inductor *L* with a simulated inductance realized by the Antoniou circuit of Fig. 13.20(a) results in the op amp–RC resonator of Fig. 13.21(b). (Ignore for the moment the additional amplifier drawn with broken lines.) The circuit of Fig. 13.21(b) is a second-order resonator having a pole frequency



  - $\omega_0 = 1/\sqrt{LC_6} = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 \ / \ R_2}$ (13.51)

- The pole *Q* factor can be obtained using the expression in Eq. (13.35) with $C = C_6$ and $R = R_6$; thus, $Q = \omega_0 C_6 R_6$.

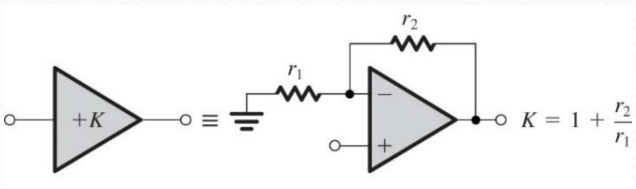
  - $$Q = \omega_0 C_6 R_6 = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$$ (13.52)



(b)

- Usually one selects $C_4 = C_6 = C$ and $R_1 = R_2 = R_3 = R_5 = R$, which results in

  - $\omega_0 = 1/CR$ (13.53)
  - $Q = R_6/R$ (13.54)



(c)

The Op Amp-RC Resonator(Cont'd)

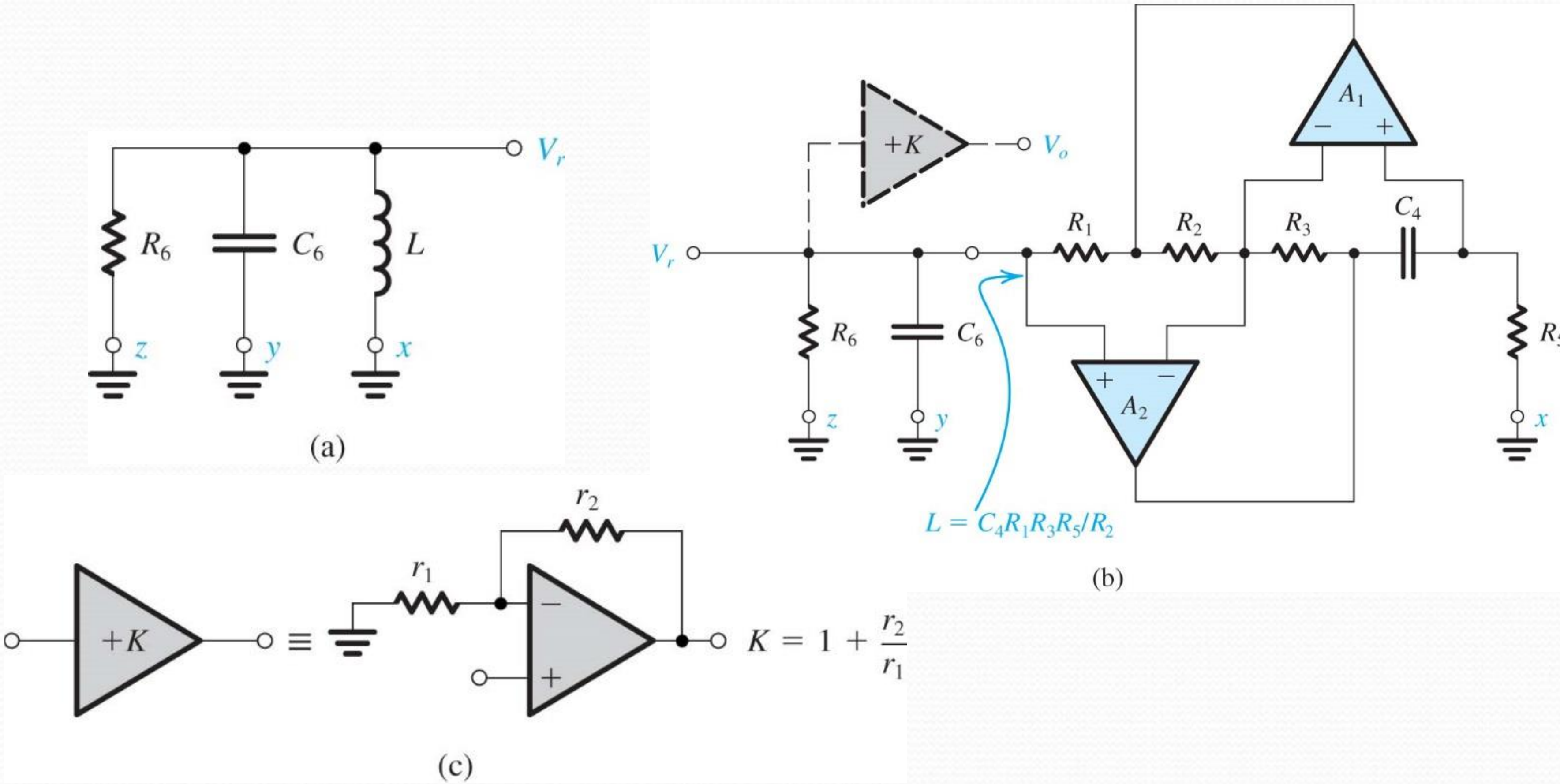
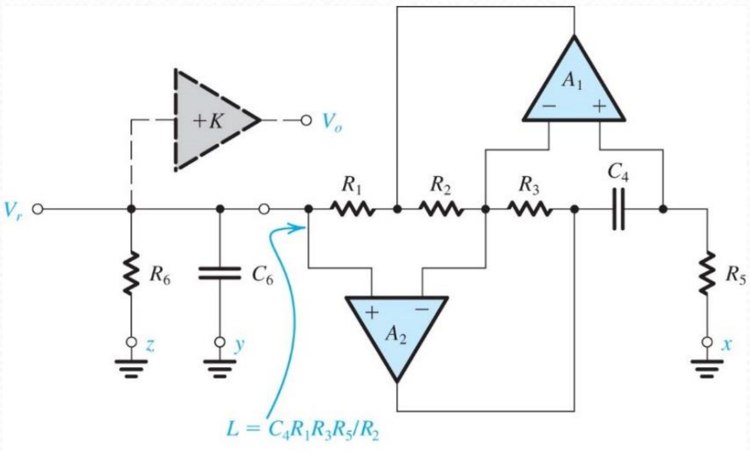


Figure 13.21 (a) An LCR resonator. **(b)** An op amp–RC resonator obtained by replacing the inductor L in the LCR resonator of **(a)** with a simulated inductance realized by the Antoniou circuit of Fig. 13.20(a). **(c)** Implementation of the buffer amplifier K.

Realization of the Various Filter Types



(b)

- The op amp–RC resonator of Fig. 13.21(b) can be used to generate circuit realizations for the various second-order filter functions by following the approach described in detail in Section 13.5 in connection with the LCR resonator. Figure 13.21(c) shows how a buffer amplifier can be simply implemented using an op amp connected in the noninverting configuration.
- Figure 13.22 shows the various second-order filter circuits obtained from the resonator of Fig. 13.21(b). The transfer functions and design equations for these circuits are given in Table 13.1.

Realization of the Various Filter Types(Cont'd)

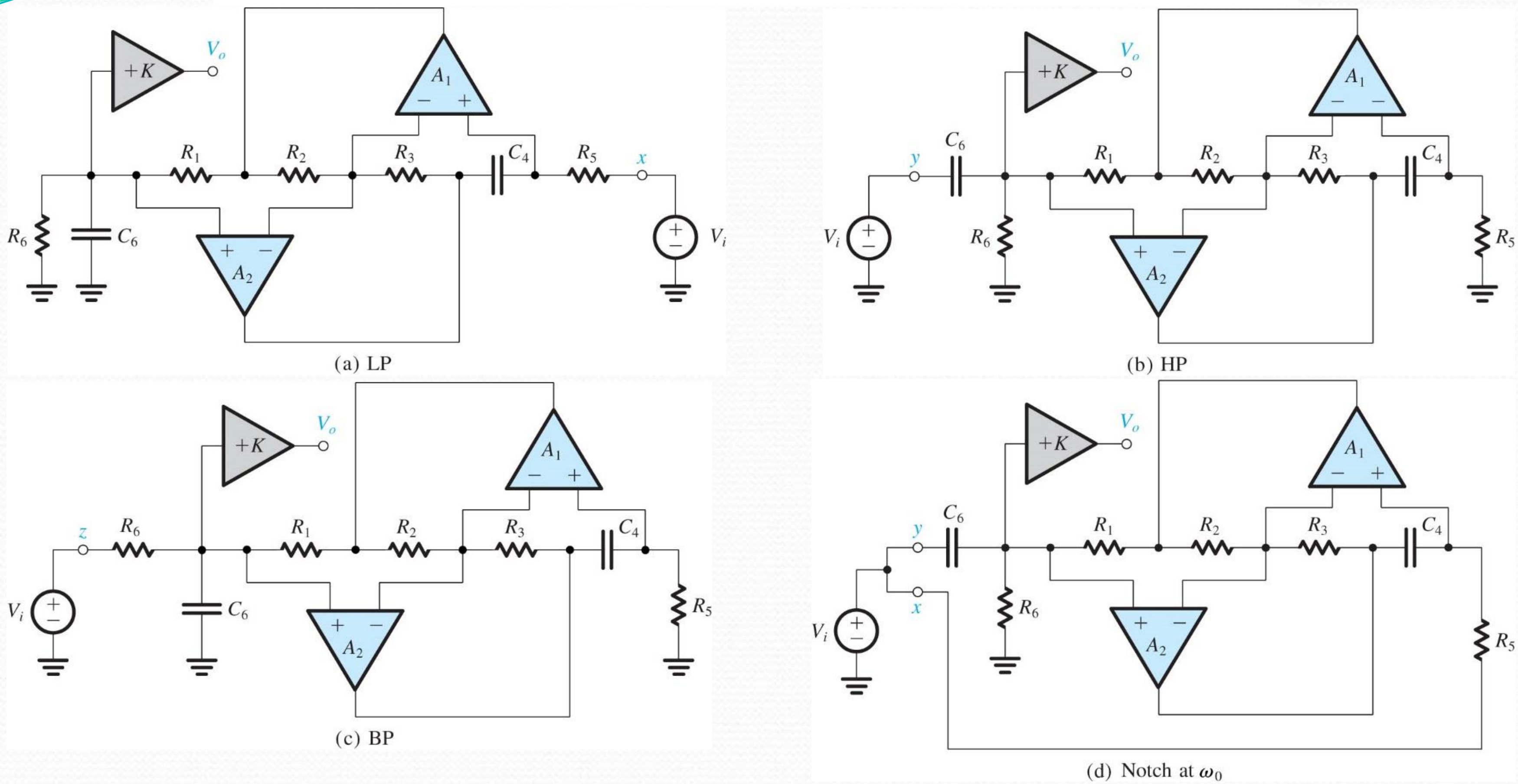
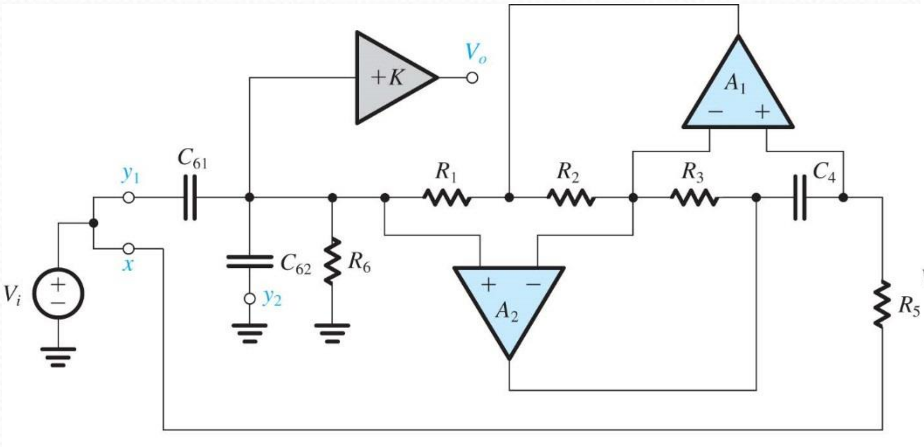


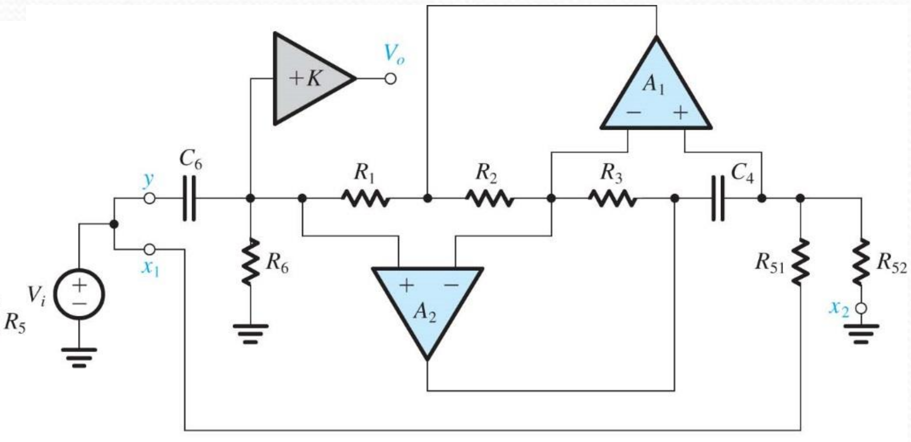
Figure 13.22 Realizations for the various second-order filter functions using the op amp–RC resonator of Fig. 13.21(b): **(a)** LP, **(b)** HP, **(c)** BP. The circuits are based on the LCR circuit in Fig. 13.18. Design considerations are given in Table 13.1. **(d)** Notch at ω_0; **(e)** LPN, $\omega_n \geq \omega_0$; **(f)** HPN, $\omega_n \leq \omega_0$. **(g)** All pass.

Realization of the Various Filter Types (Cont'd)

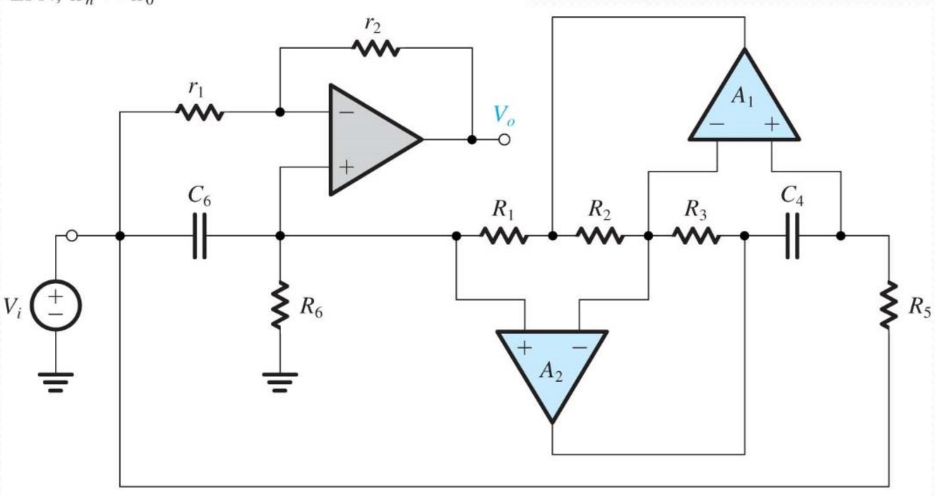
(e) LPN, $\omega_n \geq \omega_0$; **(f)** HPN, $\omega_n \leq \omega_0$. **(g)** All pass



(e) LPN, $\omega_n \geq \omega_0$



(f) HPN, $\omega_n \leq \omega_0$



(g) All-pass

Figure 13.22 *continued*

Design Data for the Circuit

Table 13.1 Design Data for the Circuits of Figs. 13.21(b) and 13.22

Circuit	Transfer Function and Other Parameters	Design Equations
Resonator Fig. 13.21(b)	$\omega_0 = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}$ $Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$	$C_4 = C_6 = C$ (practical value) $R_1 = R_2 = R_3 = R_5 = 1/\omega_0 C$ $R_6 = Q/\omega_0 C$
Low-pass (LP) Fig. 13.22(a)	$T(s) = \dfrac{K R_2 / C_4 C_6 R_1 R_3 R_5}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ DC gain
High-pass (HP) Fig. 13.22(b)	$T(s) = \dfrac{K s^2}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ High-frequency gain
Bandpass (BP) Fig. 13.22(c)	$T(s) = \dfrac{K s / C_6 R_6}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ Center-frequency gain
Regular notch (N) Fig. 13.22(d)	$T(s) = \dfrac{K\left[s^2 + (R_2 / C_4 C_6 R_1 R_3 R_5)\right]}{s^2 + s\frac{1}{C_6 R_6} + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ Low- and high-frequency gain

Design Data for the Circuit

Low-pass notch (LPN) Fig. 13.22(e)	$T(s) = K\dfrac{C_{61}}{C_{61}+C_{62}} \times \dfrac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{(C_{61}+C_{62})R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}}$ $\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2}$ $\omega_0 = 1/\sqrt{C_4(C_{61}+C_{62})R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\dfrac{C_{61}+C_{62}}{C_4}\dfrac{R_2}{R_1R_3R_5}}$	$K =$ DC gain $C_{61}+C_{62} = C_6 = C$ $C_{61} = C(\omega_0/\omega_n)^2$ $C_{62} = C - C_{61}$
High-pass notch (HPN) Fig. 13.22(f)	$T(s) = K\dfrac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$ $\omega_n = 1/\sqrt{C_4C_6R_1R_3R_{51}/R_2}$ $\omega_0 = \sqrt{\dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$ $Q = R_6\sqrt{\dfrac{C_6}{C_4}\dfrac{R_2}{R_1R_3}\left(\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}}\right)}$	$K =$ High-frequency gain $\dfrac{1}{R_{51}}+\dfrac{1}{R_{52}} = \dfrac{1}{R_5} = \omega_0 C$ $R_{51} = R_5(\omega_0/\omega_n)^2$ $R_{52} = R_5\Big/\left[1-(\omega_n/\omega_0)^2\right]$
All-pass (AP) Fig. 13.22(g)	$T(s) = \dfrac{s^2 - s\dfrac{1}{C_6R_6}\dfrac{r_2}{r_1} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}$ $\omega_z = \omega_0 \quad Q_z = Q(r_1/r_2)$ Flat gain $= 1$	$r_1 = r_2 = r$ (arbitrary) Adjust r_2 to make $Q_z = Q$

The All-Pass Circuit

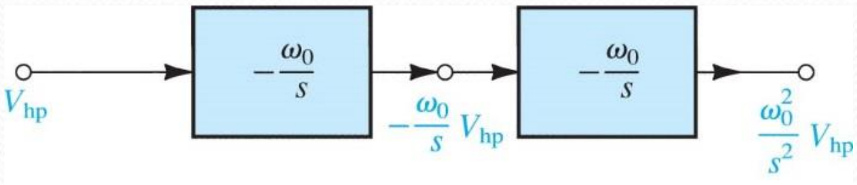
- From Eq. (13.48) we see that an all-pass function with a flat gain of unity can be written as
  - AP = 1 − (BP with a center-frequency gain of 2) (13.55)
- Two circuits whose transfer functions are related in this fashion are said to be **complementary**.
- A simple procedure exists for obtaining the complement of a given linear circuit: Disconnect all the circuit nodes that are connected to ground and connect them to *Vi*, and disconnect all the nodes that are connected to *Vi* and connect them to ground. That is, *interchanging input and ground in a linear circuit generates a circuit whose transfer function is the complement of that of the original circuit.*
- in addition to being simple to design, the circuits in Fig. 13.22 exhibit excellent performance. They can be used on their own to realize second-order filter functions, or they can be cascaded to implement high-order filters.

Second-Order Active Filters Based on the Two-Integrator-Loop Topology

- In this section, we study another family of op amp–RC circuits that realize second-order filter functions.

Two-Integrator-Loop Biquad

- To derive the two-integrator-loop biquadratic circuit, or **biquad** as it is commonly known, consider the second-order high-pass transfer function
  - $$\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (13.56)$$
  - $$V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) + \left(\frac{\omega_0^2}{s^2}V_{hp}\right) = KV_i \qquad (13.57)$$
- Figure 13.23(a) shows a block diagram for such a two-integrator arrangement.



(a)

Two-Integrator-Loop Biquad

- Toward that end, we rearrange Eq. (13.57), expressing V_{hp} in terms of its single- and double-integrated versions and of V_i as

Eq. (13.57),

$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) + \left(\frac{\omega_0^2}{s^2}V_{hp}\right) = KV_i$$

- $$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp} \qquad (13.58)$$

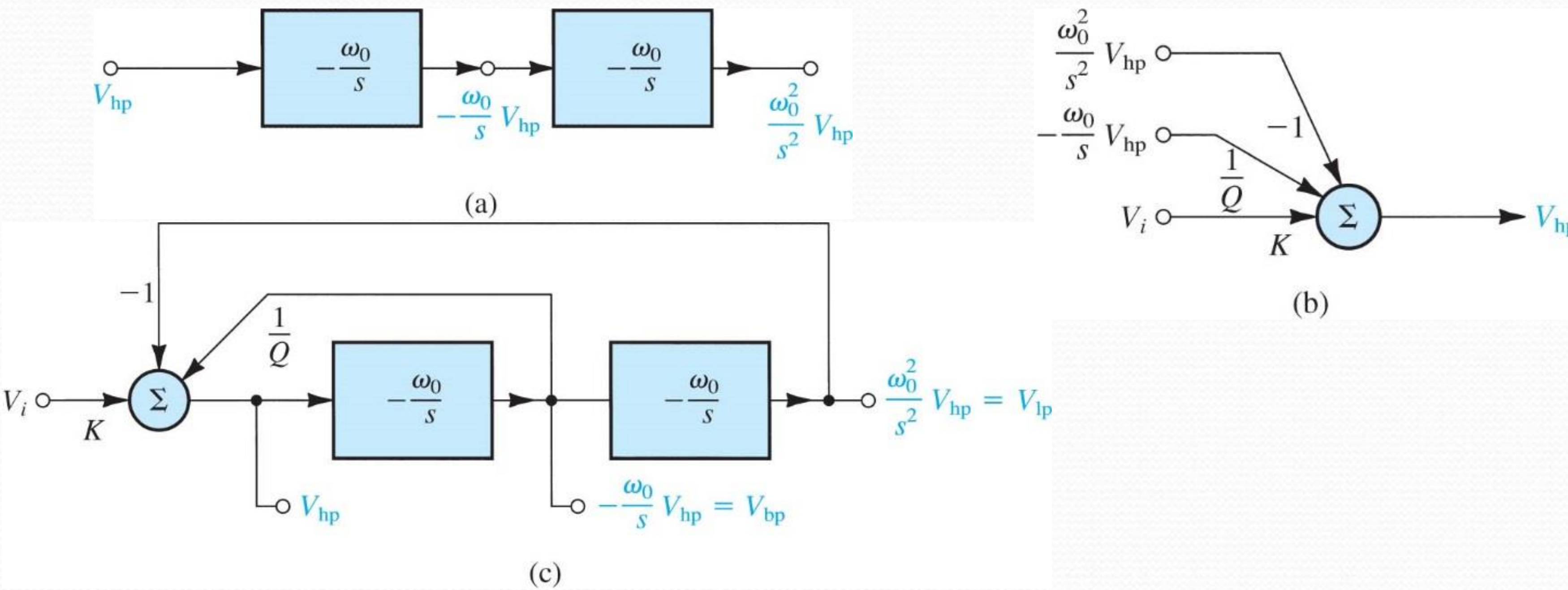
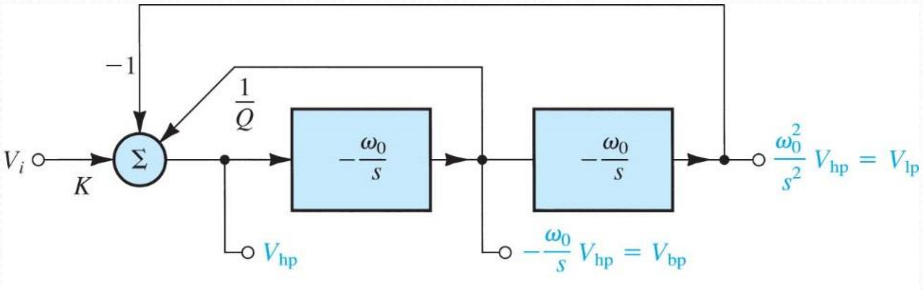
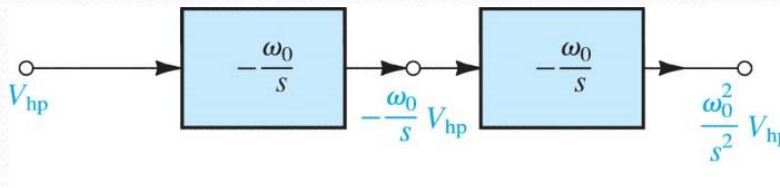


Figure 13.23 Derivation of a block diagram realization of the two-integrator-loop biquad.

Two-Integrator-Loop Biquad (Cont'd)



(c)



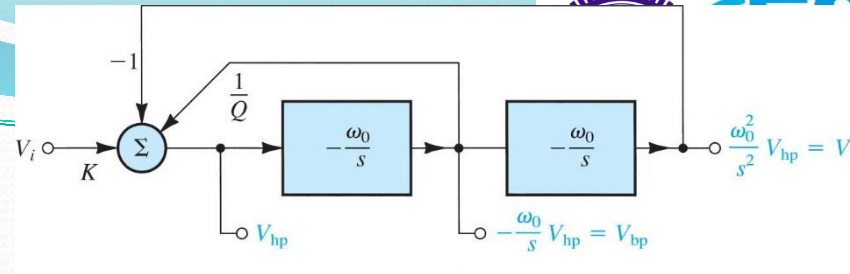
(a)

- In Fig. 13.23(c), the signal at the output of the first integrator is − $(\omega_0/s)\, V_{hp}$, which is a bandpass function
  - $$\frac{(-\omega_0/s)V_{hp}}{V_i} = -\frac{K\omega_0 s}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{bp}(s) \quad (13.59)$$

$$\boxed{V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s}V_{hp}\right) + \left(\frac{\omega_0^2}{s^2}V_{hp}\right) = KV_i}$$

- We can show that the transfer function realized at the output of the second integrator is the low-pass function,
  - $$\frac{(\omega_0^2/s^2)V_{hp}}{V_i} = \frac{K\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{lp}(s) \quad (13.60)$$
- The two-integrator-loop biquad shown in block diagram form in Fig. 13.23(c) realizes the three basic second-order filtering functions, LP, BP, and HP, *simultaneously*. This versatility has made the circuit very popular and has given it the name ***universal active filter***.

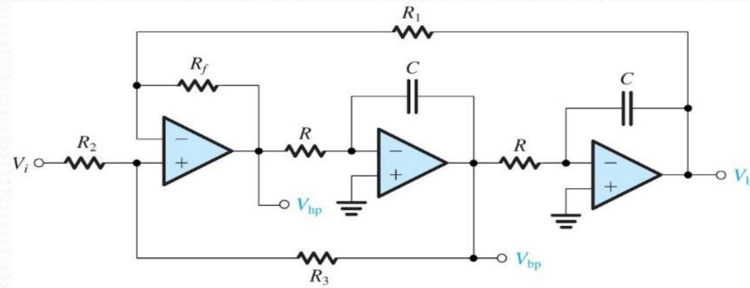
Circuit Implementation



(c)

- The circuit, known as the Kerwin–Huelsman–Newcomb or **KHN biquad**, after its inventors, is shown in Fig. 13.24(a). To determine the values of the resistors associated with the summer, we first use *superposition* to express the output of the summer V_{hp} in terms of its inputs, V_i, V_{bp} and V_{lp} as

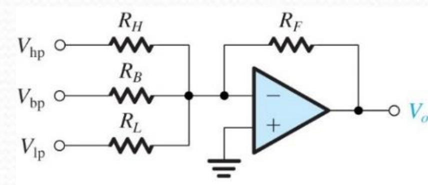
$$V_{hp} = V_i \frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) + V_{bp}\frac{R_2}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) - V_{lp}\frac{R_f}{R_1}$$



(a)

- Substituting $V_{bp} = -(\omega_0/s)V_{hp}$ and $V_{lp} = (\omega_0^2/s^2)V_{hp}$ gives

$$V_{hp} = \frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right)V_i + \frac{R_2}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right)\left(-\frac{\omega_0}{s}V_{hp}\right) - \frac{R_f}{R_1}\left(\frac{\omega_0^2}{s^2}V_{hp}\right) \quad (13.61)$$



(b)

- Equating the last right-hand-side terms of Eqs. (13.61) and (13.58) gives
  - $R_f/R_1 = 1$ (13.62)
- Equating the second-to-last terms on the right-hand side of Eqs. (13.61) and (13.58) and setting $R_1 = R_f$ yields the ratio required to realize a given Q as
  - $R_3/R_2 = 2Q-1$ (13.63)

$$\boxed{V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}}$$

Circuit Implementation (Cont'd)

- Equating the coefficients of V_i in Eqs. (13.61) and (13.58) and substituting $R_f = R_1$ and for R_3/R_2 from Eq. (13.63) results in
  - $K = 2 - (1/Q)$ (13.64)
- Thus the gain parameter K is fixed to this value.

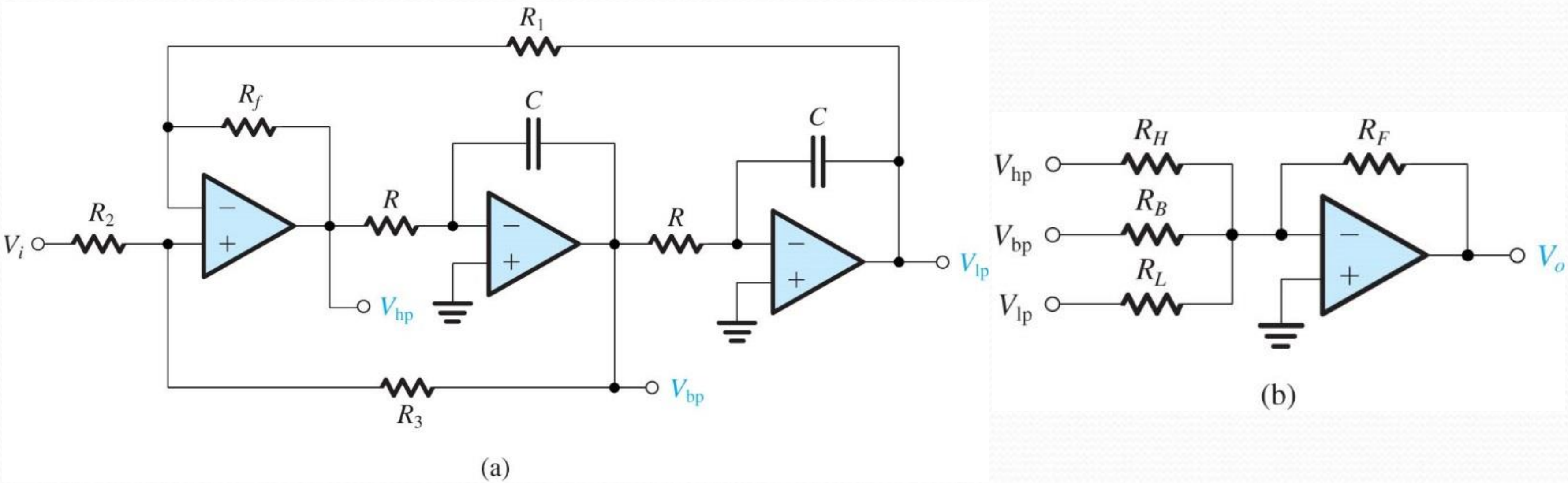
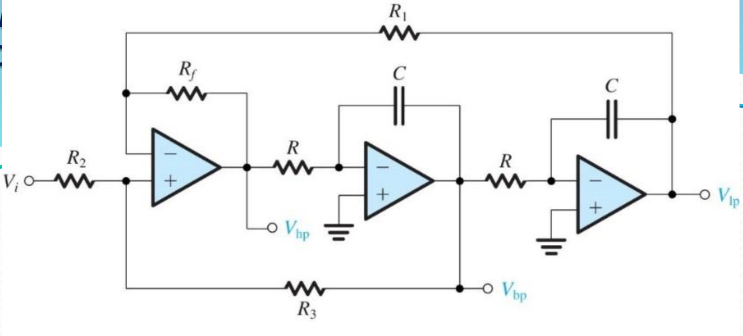
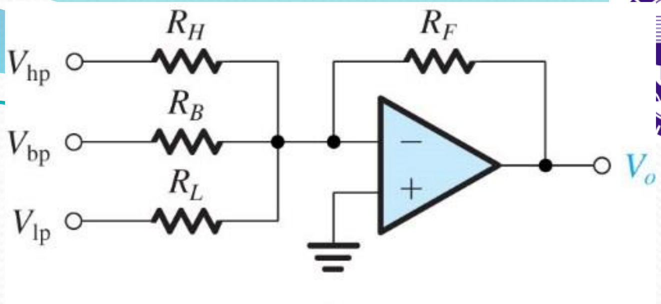


Figure 13.24 (a) The KHN biquad circuit, obtained as a direct implementation of the block diagram of Fig. 13.23(c). The three basic filtering functions, HP, BP, and LP, are simultaneously realized. **(b)** To obtain notch and all-pass functions, the three outputs are summed with appropriate weights using this op-amp summer.



(a)



(b)

- An op-amp summer is shown in Fig. 13.24(b); for this summer we can write

  - $$V_o = -\left(\frac{R_F}{R_H}V_{\mathrm{hp}} + \frac{R_F}{R_B}V_{\mathrm{bp}} + \frac{R_F}{R_L}V_{\mathrm{lp}}\right)$$
  $$= -V_i\left(\frac{R_F}{R_H}T_{\mathrm{hp}} + \frac{R_F}{R_B}T_{\mathrm{bp}} + \frac{R_F}{R_L}T_{\mathrm{lp}}\right)$$ (13.65)

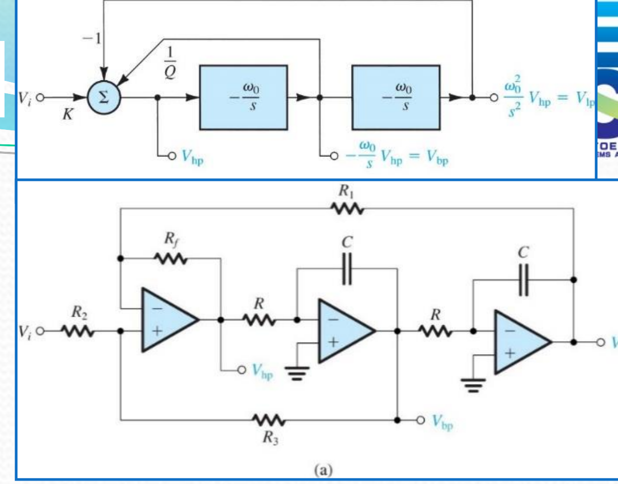
- Substituting for T_{hp}, T_{bp}, and T_{lp} from Eqs. (13.56), (13.59), and (13.60), respectively, gives the overall transfer function

  - $$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$ (13.66)

- A notch is obtained from eqa. (13.66) by selecting $R_B = \infty$ and

  - $$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2$$ (13.67)

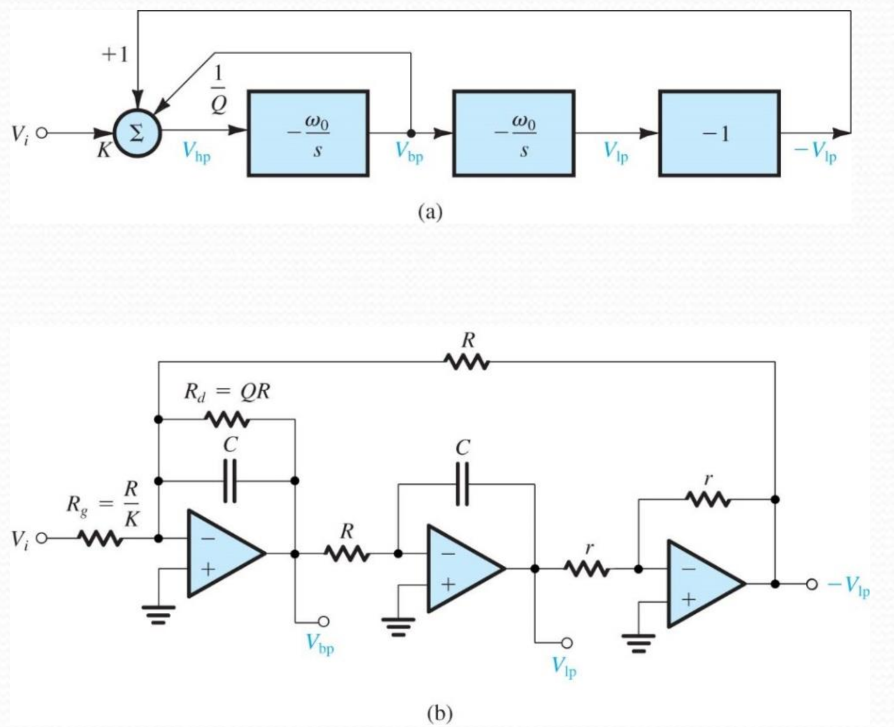
$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$



An Alternative Tow-Thomas Biquad Circuit

- An alternative two-integrator-loop biquad -**Tow–Thomas biquad**, after its originators.

- the virtual ground available at the input of each of the three op amps in the Tow–Thomas circuit permits the input signal to be fed to all three op amps, as shown in Fig. 13.26. If V_o is taken at the output of the damped integrator, straightforward analysis yields the filter transfer function



$$\frac{V_o}{V_i} = -\frac{s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{1}{QCR} + \frac{1}{C^2R^2}} \qquad (13.68)$$

which can be used to obtain the design data given in Table 13.2.

The **Tow–Thomas** biquad can realize all special second-order functions.

$$\frac{V_o}{V_i} = -\frac{s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{1}{QCR} + \frac{1}{C^2R^2}}$$

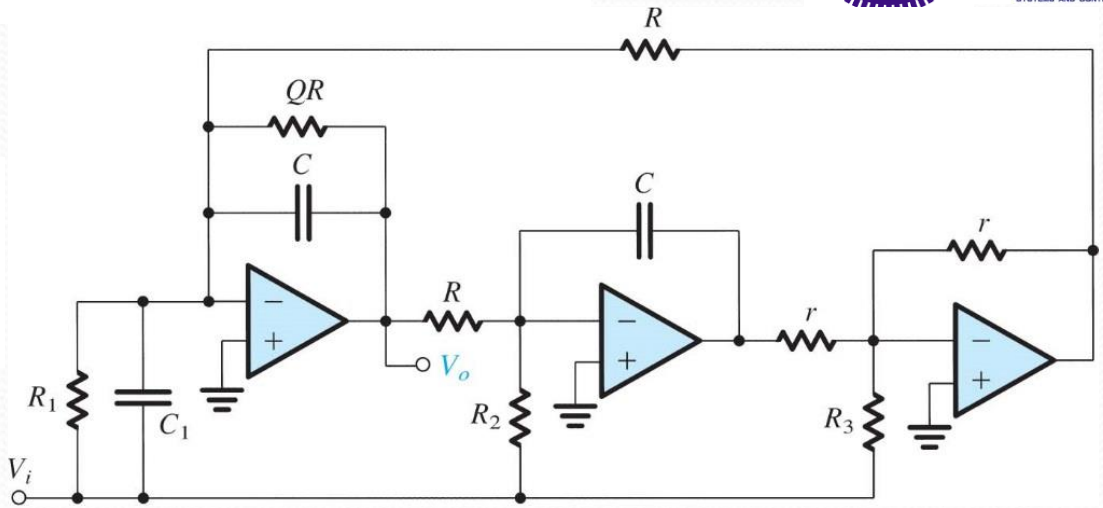


Table 13.2 Design Data for the Circuit in Fig. 13.26	
All cases	$C =$ arbitrary, $R = 1/\omega_0 C$, $r =$ arbitrary
LP	$C_1 = 0$, $R_1 = \infty$, $R_2 = R$/dc gain, $R_3 = \infty$
Positive BP	$C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr$/center-frequency gain
Negative BP	$C_1 = 0$, $R_1 = QR$/center-frequency gain, $R_2 = \infty$, $R_3 = \infty$
HP	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$
Notch (all types)	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = R(\omega_0/\omega_n)^2$/high-frequency gain, $R_3 = \infty$
AP	$C_1 = C \times$ flat gain, $R_1 = \infty$, $R_2 = R$/gain, $R_3 = Qr$/gain

Single-Amplifier Biquadratic Active Filters

- The op amp–RC biquadratic circuits are not economic in their use of op amps, requiring three or four amplifiers per second-order section. In this section we shall study a class of second-order filter circuits that requires **only one op amp per biquad**.
- The **single-amplifier biquads** (SABs) are therefore limited to the less stringent filter specifications—for example, pole *Q* factors less than about 10.
- The synthesis of SABs follows a two-step process:
  - 1. Synthesis of a feedback loop that realizes a pair of complex-conjugate poles characterized by a frequency ω_0 and a Q factor.
  - 2. Injecting the input signal in a way that realizes the desired transmission zeros.

Synthesis of the Feedback Loop

- A two-port RC network *n* placed in the negative-feedback path of an op amp. We shall denote by *t*(*s*) the open-circuit voltage transfer function of the RC network *n*, where the definition of *t*(*s*) is illustrated in Fig. 13.27(b). The transfer function *t*(*s*) can in general be written as the ratio of two polynomials *N*(*s*) and *D*(*s*):

$$t(s) = \frac{N(s)}{D(s)}$$

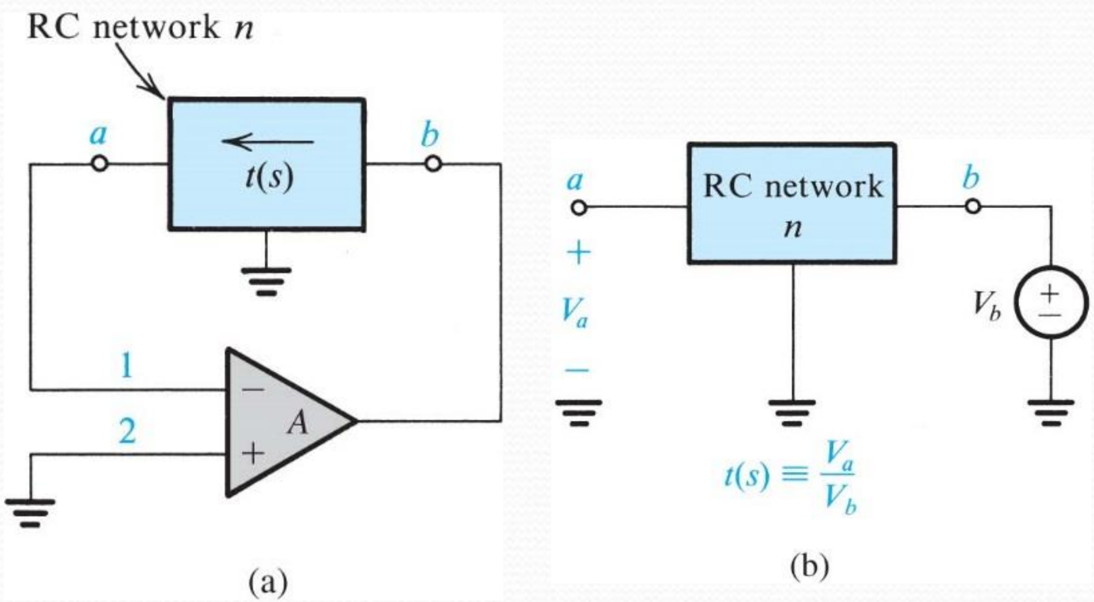


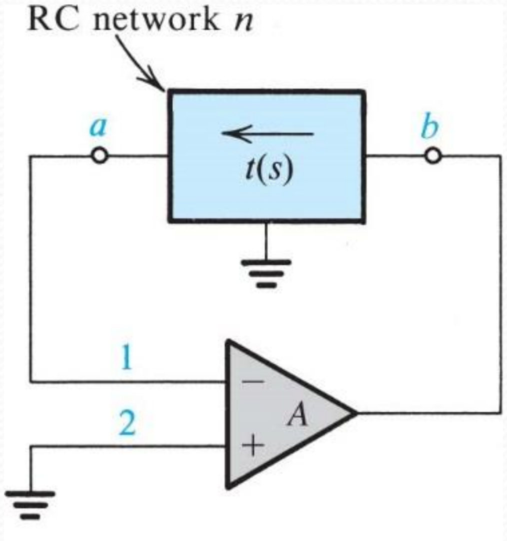
Figure 13.27 (a) Feedback loop obtained by placing a two-port RC network *n* in the feedback path of an op amp. **(b)** Definition of the open-circuit transfer function *t*(*s*) of the RC network.

Synthesis of the Feedback Loop (Cont'd)

- The loop gain $L(s)$ is simply the product of the op-amp gain A and the transfer function $t(s)$,
  - $$L(s) = At(s) = \frac{AN(s)}{D(s)} \qquad (13.69)$$
- Substituting for $L(s)$ into the characteristic equation
  - $1 + L(s) = 1 + A\,t(s) = 0 \qquad (13.70)$

results in the poles s_P of the closed-loop circuit obtained as solutions to the equation

  - $$t(s_P) = -\frac{1}{A} \qquad (13.71)$$

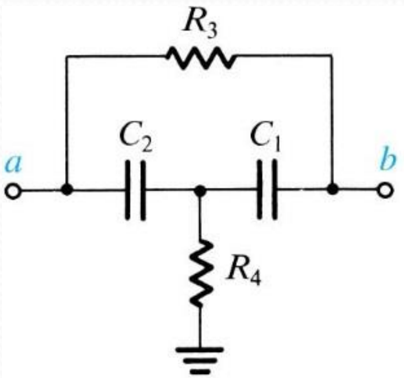


(a)

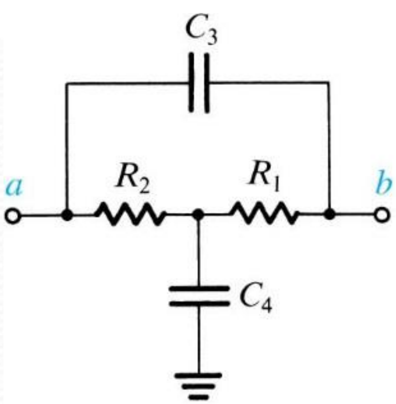
- In the ideal case, $A = \infty$ and the poles are obtained from
  - $N(s_p) = 0 \qquad (13.72)$
- That is, *the filter poles are identical to the zeros of the RC network with $A = \infty$* .

Synthesis of the Feedback Loop (Cont'd)

- Aims to have complex-conjugate transmission zeros.
- The simplest such networks are the **bridged-T networks** shown in Fig. 13.28 together with their transfer functions *t*(*s*) from *b* to *a*, with *a* open-circuited.



$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}}{s^2 + s\left(\frac{1}{C_1 R_3} + \frac{1}{C_2 R_3} + \frac{1}{C_1 R_4}\right) + \frac{1}{C_1 C_2 R_3 R_4}}$$



$$t(s) = \frac{s^2 + s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_4} + \frac{1}{C_3 C_4 R_1 R_2}}{s^2 + s\left(\frac{1}{C_4 R_1} + \frac{1}{C_4 R_2} + \frac{1}{C_3 R_2}\right) + \frac{1}{C_3 C_4 R_1 R_2}}$$

(b)

Synthesis of the Feedback Loop (Cont'd)

- Using the negative-feedback path of an op amp, as shown in Fig. 13.29.
- The pole polynomial of the active-filter circuit will be equal to the numerator polynomial of the bridged-T network; thus,

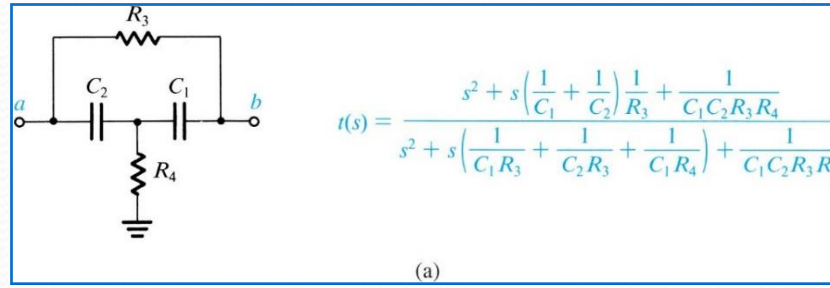
$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}$$

- which enables us to obtain ω_0 and Q as

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$$

$$Q = \left[\frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{-1}$$

- Choose C1=C2=C, R3=R, R4=R/m, then
  - $m = 4Q^2$ (13.75)
  - $CR = 2Q/\omega_0$ (13.76)



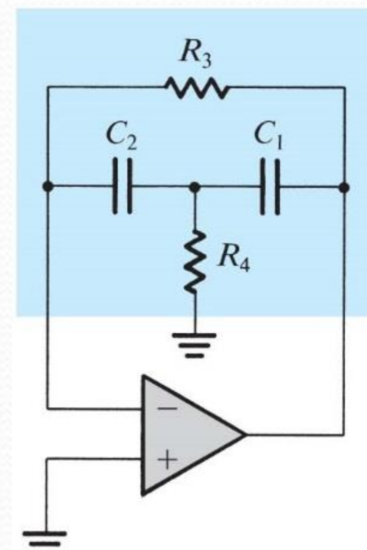
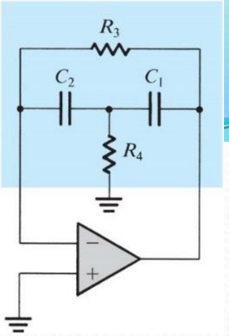
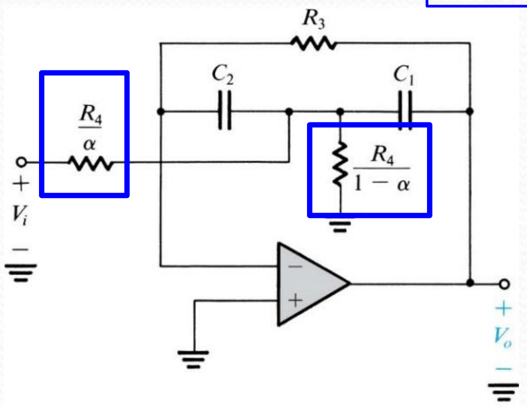


Figure 13.29 An active-filter feedback loop generated using the bridged-T network of Fig. 13.28(a).

Injecting the Input Signal



(R4/α)||(R4/1-α)
=R4

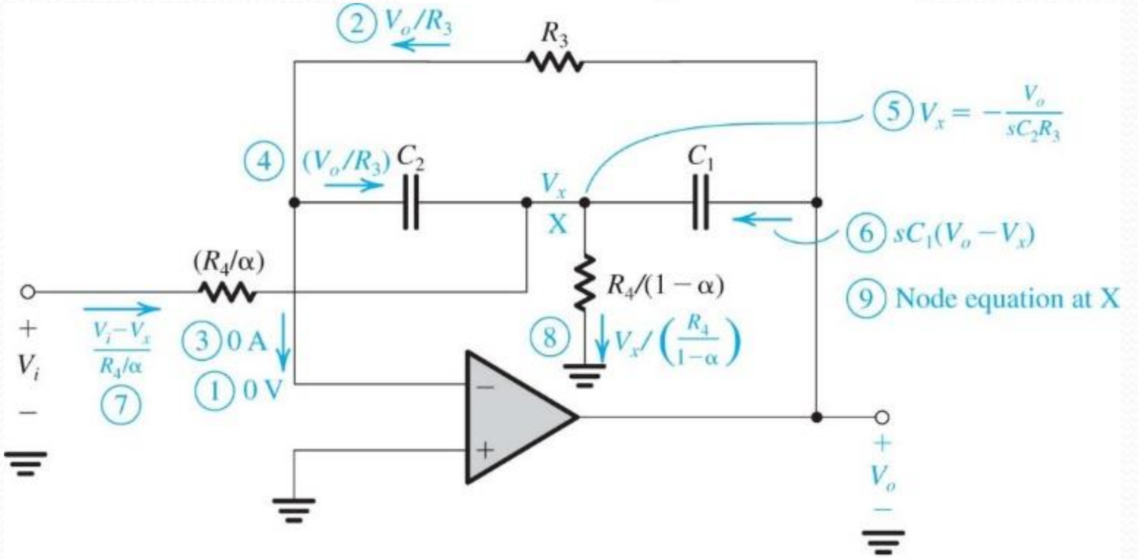


- Injecting the input voltage signal into the feedback.
- As an example at right.
- Analysis of the circuit to determine its voltage transfer function T(s) ≡ $V_o(s)/V_i(s)$ is illustrated in Fig. 13.30(b).
- The transfer function (KCL at X)

$$\frac{V_o}{V_i} = \frac{-s(\alpha/C_1R_4)}{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}$$

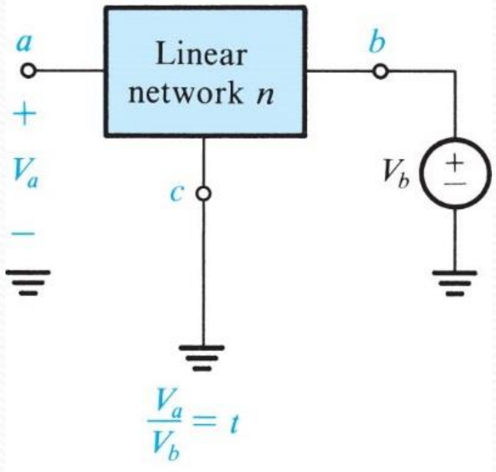
- Actually a **bandpass**

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2 = s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}$$

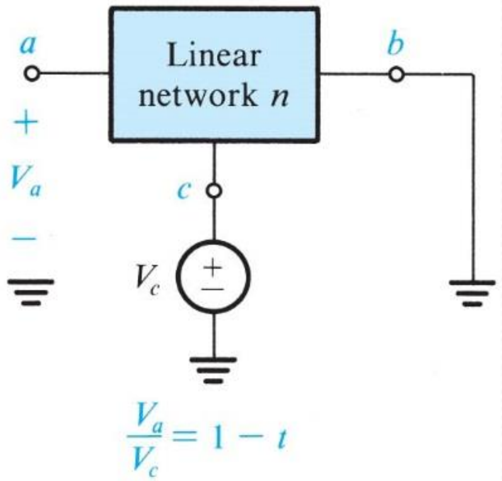


Generation of Equivalent Feedback Loops

- The **complementary transformation** at right for the two-port (three-terminal) network *n*.
- a two-step process:
  - **1.** Nodes of the feedback network and any of the op-amp inputs that are connected to ground should be disconnected from ground and connected to the op-amp output. Conversely, those nodes that were connected to the op-amp output should be now connected to ground. That is, we simply interchange the op-amp output terminal with ground.
  - **2.** The two input terminals of the op amp should be interchanged.



(a)



(b)

Generation of Equivalent Feedback Loops (Cont'd)

- Application of the complementary transformation to this loop in (a) results in the feedback loop of Fig. 13.32(b).
- The characteristic equation of the circuit in Fig. 13.32(b):

$$1-\frac{A}{A+1}(1-t)=0$$

- This equation can be manipulated to the form

$$1+At=0$$

which is the characteristic equation of the loop in Fig. 13.32(a).

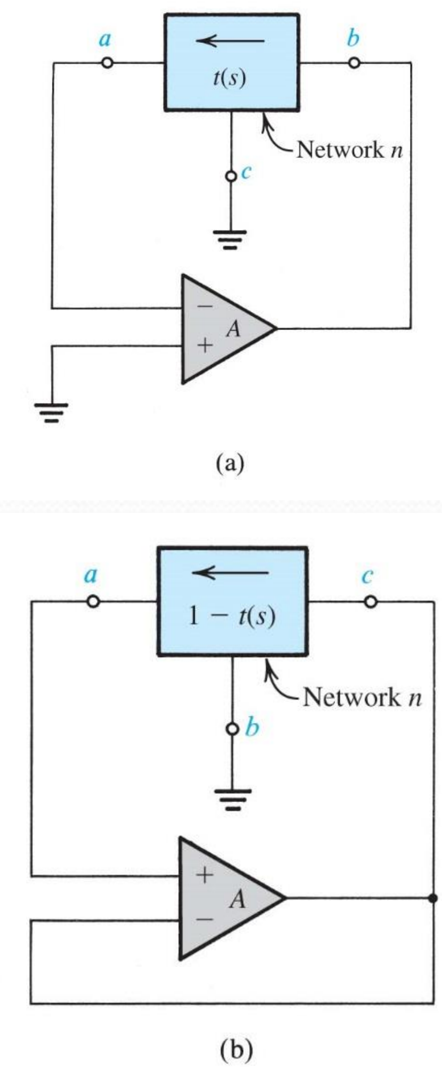
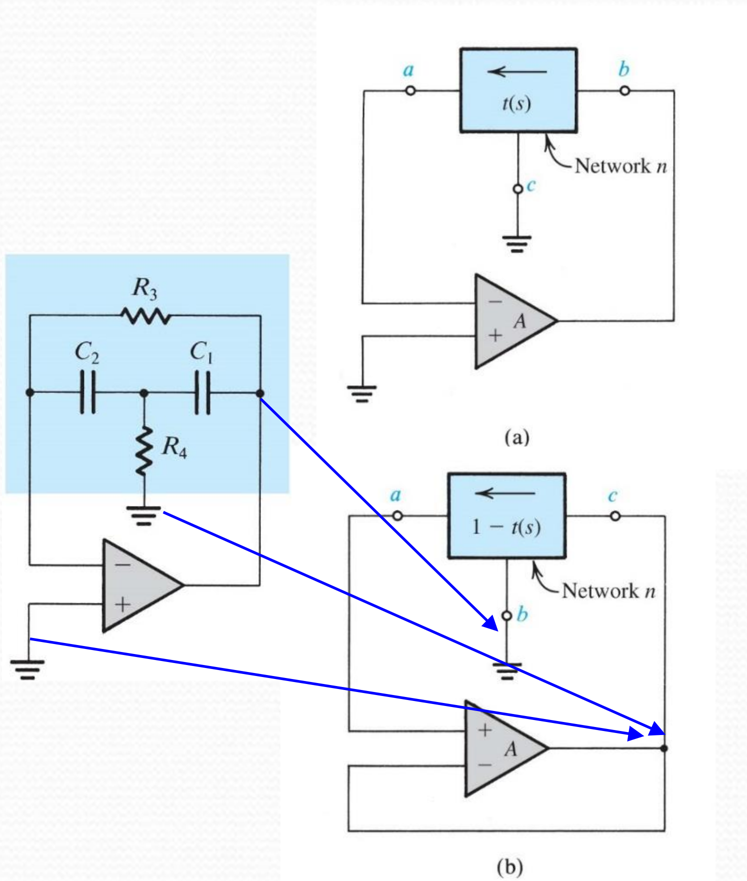


Figure 13.32 Application of the complementary transformation to the feedback loop in **(a)** results in the equivalent loop (same poles) shown in **(b)**.

Generation of Equivalent Feedback Loops(Cont'd)

- Application of the complementary transformation
- The design of the circuit in Fig. 13.33(b) is based on Eqs. (13.73) through (13.76): namely, $R_3 = R$, $R_4 = R/4Q^2$, $C_1 = C_2 = C$, $CR = 2Q/\omega_0$ and the value of C is arbitrarily chosen to be practically convenient --- a **high-pass** function.



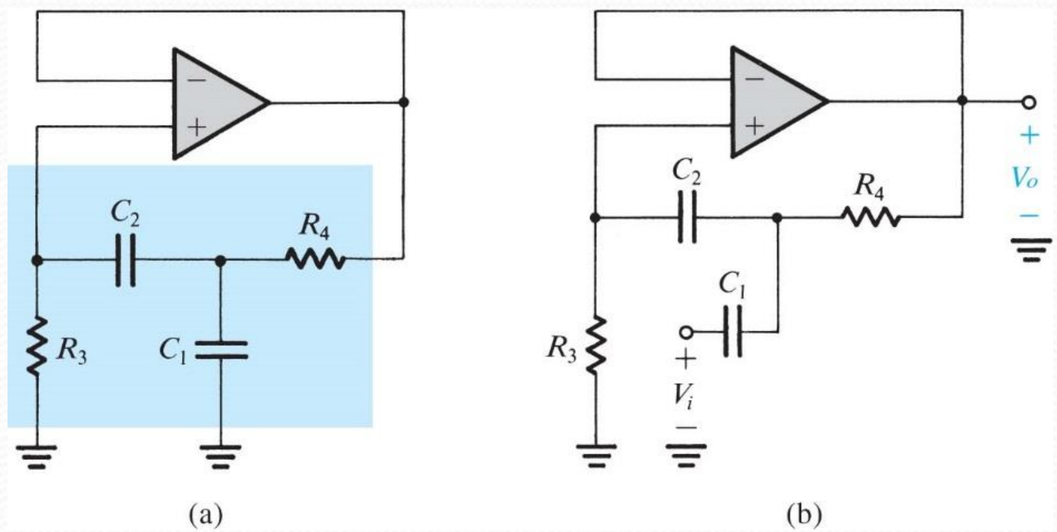


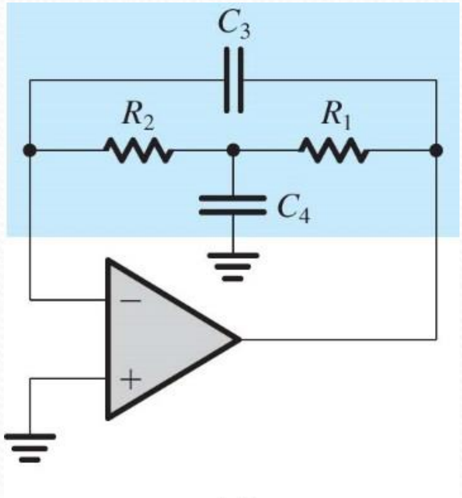
Figure 13.33 **(a)** Feedback loop obtained by applying the complementary transformation to the loop in Fig. 13.29. **(b)** Injecting the input signal through C_1 realizes the high-pass function. This is one of the **Sallenand-Key** family of circuits.

Generation of Equivalent Feedback Loops (Cont'd)

- As another example, Fig. 13.34(a) shows the feedback loop generated by placing the two-port RC network of Fig. 13.28(b) in the negative-feedback path of an op amp. Using the expression for $t(s)$ given in Fig. 13.28(b), we can write for the active-filter poles

  - $$\omega_0 = 1/\sqrt{C_3 C_4 R_1 R_2} \quad (13.77)$$
  - $$Q = \left[\frac{\sqrt{C_3 C_4 R_1 R_2}}{C_4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{-1} \quad (13.78)$$

    (the only different result from the previous in page 75)

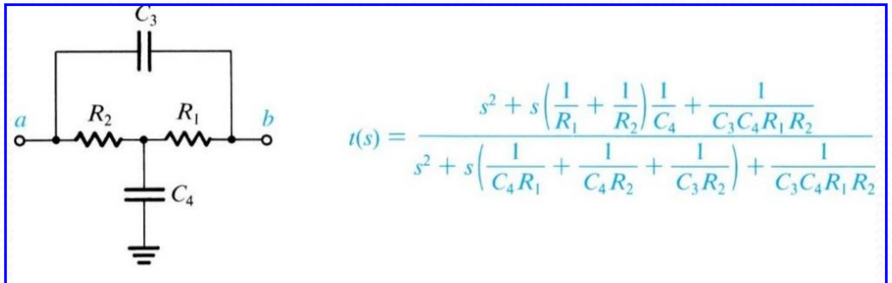


(a)

- Normally the design of this circuit is based on selecting $R_1 = R_2 = R$, $C_4 = C$, and $C_3 = C/m$. When substituted in Eqs. (13.77) and (13.78), these yield

  - $m = 4Q^2$ (13.79)
  - $CR = 2Q/\omega_0$ (13.80)

  with the remaining degree of freedom (the value of C or R) left to the designer to choose.



(b)

Generation of Equivalent Feedback Loops (Cont'd)

- If we apply the complementary transformation to the feedback loop in Fig. 13.34(a), we obtain the equivalent loop in Fig. 13.34(b). The new loop in Fig. 13.34(b) can be used to realize a low-pass function by injecting the input signal as shown in Fig. 13.34(c).

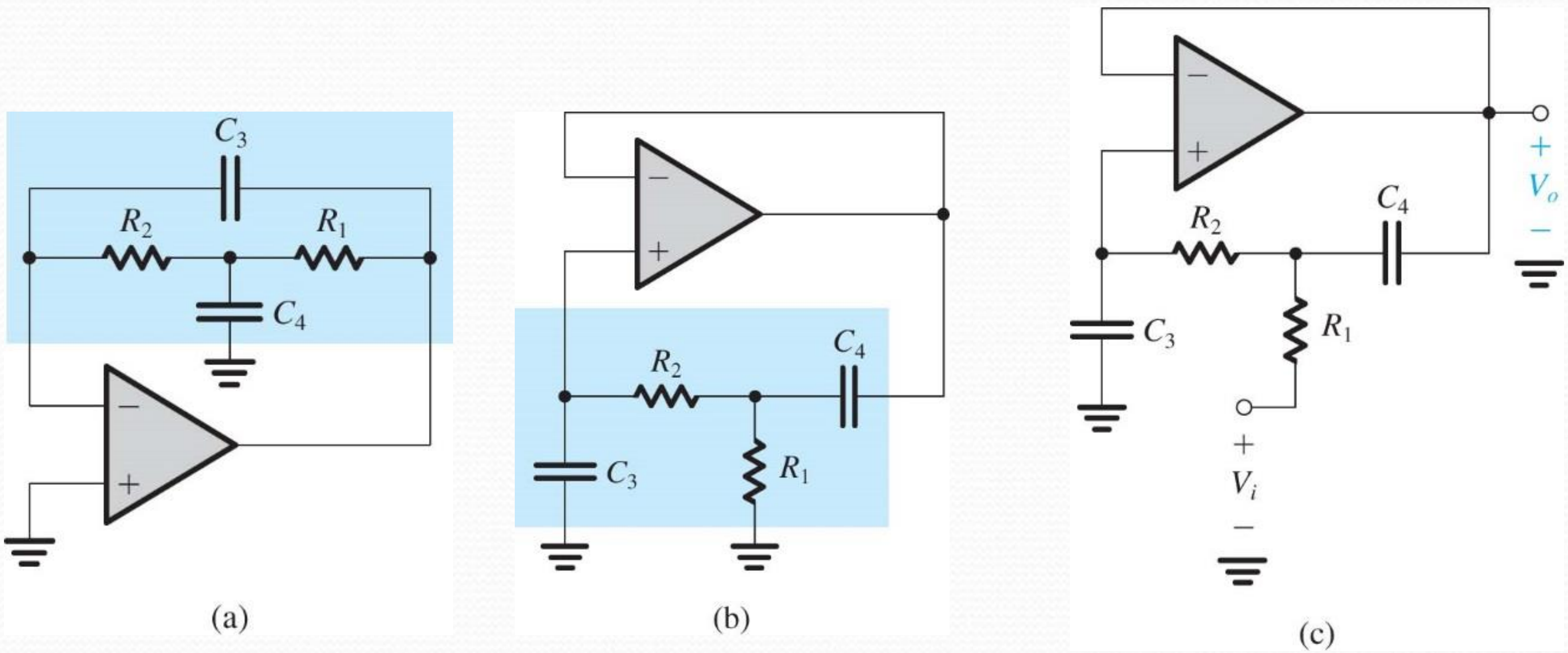
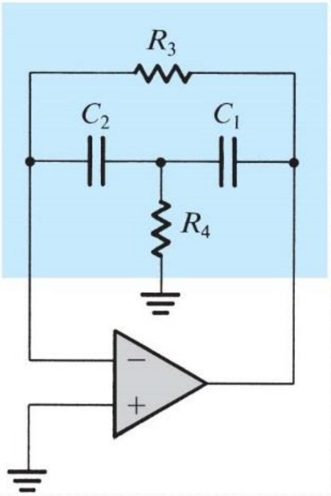


Figure 13.34 **(a)** Feedback loop obtained by placing the bridged-T network of Fig. 13.28(b) in the negative-feedback path of an op amp. **(b)** Equivalent feedback loop generated by applying the complementary transformation to the loop in **(a)**. **(c)** A low-pass filter obtained by injecting V_i through R_1 into the loop in **(b)**.

Sensitivity

- As a means for predicting the deviations of the actual assembled filter from the ideal response, the filter designer employs the concept of **sensitivity**. These sensitivities can be quantified using the **classical sensitivity function** S_x^y, defined as
  - $$S_x^y \equiv \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} \qquad (13.81)$$
- Thus,
  - $$S_x^y = \frac{\partial y}{\partial x} \frac{x}{y} \qquad (13.82)$$
- For small changes
  - $$S_x^y \simeq \frac{\Delta y / y}{\Delta x / x} \qquad (13.83)$$
- Thus we can use the value of S_x^y to determine the per-unit change in *y* due to a given per-unit change in *x*.
  - For instance, if the sensitivity of *Q* relative to a particular resistance R_1 is 5, then a 1% increase in R_1 results in a 5% increase in the value of *Q*.

Example 13.3



$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$$

$$Q = \left[\frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right]^{-1}$$

- Choose C1=C2=C, R3=R, R4=R/m, then
  - m = $4Q^2$
  - CR = $2Q/\omega_0$

For the feedback loop of Fig. 17.29, find the sensitivities of ω_0 and Q relative to all the passive components and the op-amp gain. Evaluate these sensitivities for the design considered in the preceding section for which $C_1 = C_2$.

Solution

To find the sensitivities with respect to the passive components, called **passive sensitivities**, we assume that the op-amp gain is infinite. In this case, ω_0 and Q are given by Eqs. (17.73) and (17.74). Thus for ω_0 we have

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$$

which can be used together with the sensitivity definition of Eq. (17.82) to obtain

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_4}^{\omega_0} = -\frac{1}{2}$$

For Q we have

$$Q = \left[\sqrt{C_1 C_2 R_3 R_4} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3} \right]^{-1}$$

to which we apply the sensitivity definition to obtain

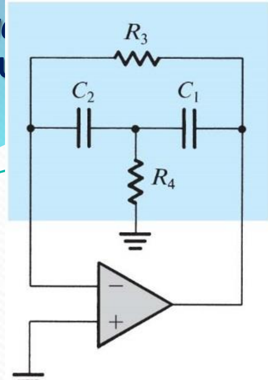
$$S_{C_1}^{Q} = \frac{1}{2} \left(\sqrt{\frac{C_2}{C_1}} - \sqrt{\frac{C_1}{C_2}} \right) \left(\sqrt{\frac{C_2}{C_1}} + \sqrt{\frac{C_1}{C_2}} \right)^{-1}$$

For the design with $C_1 = C_2$ we see that $S_{C_1}^{Q} = 0$. Similarly, we can show that

$$S_{C_2}^{Q} = 0, \quad S_{R_3}^{Q} = \frac{1}{2}, \quad S_{R_4}^{Q} = -\frac{1}{2}$$

It is important to remember that the sensitivity expression should be derived *before* values corresponding to a particular design are substituted.

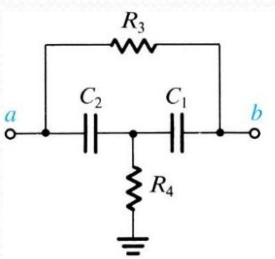
Example 13.3 (Cont'd)



$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$$

$$Q = \left[\frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right]^{-1}$$

- Choose C1=C2=C, R3=R, R4=R/m, then
  - m = $4Q^2$
  - CR = $2Q/\omega_0$



$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}}{s^2 + s\left(\frac{1}{C_1 R_3}+\frac{1}{C_2 R_3}+\frac{1}{C_1 R_4}\right) + \frac{1}{C_1 C_2 R_3 R_4}}$$

Next we consider the sensitivities relative to the amplifier gain. If we assume the op amp to have a finite gain A, the characteristic equation for the loop becomes

$$1 + At(s) = 0 \tag{17.84}$$

where $t(s)$ is given in Fig. 17.28(a). To simplify matters we can substitute for the passive components by their design values. This causes no errors in evaluating sensitivities, since we are now finding the sensitivity with respect to the amplifier gain. Using the design values obtained earlier—namely, $C_1 = C_2 = C$, $R_3 = R$, $R_4 = R/4Q^2$, and $CR = 2Q/\omega_0$—we get

$$t(s) = \frac{s^2 + s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q)(2Q^2+1) + \omega_0^2} \tag{17.85}$$

where ω_0 and Q denote the nominal or design values of the pole frequency and Q factor. The actual values are obtained by substituting for $t(s)$ in Eq. (17.84):

$$s^2 + s\frac{\omega_0}{Q}\left(2Q^2+1\right) + \omega_0^2 + A\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right) = 0$$

Assuming the gain A to be real and dividing both sides by $A+1$, we get

$$s^2 + s\frac{\omega_0}{Q}\left(1 + \frac{2Q^2}{A+1}\right) + \omega_0^2 = 0 \tag{17.86}$$

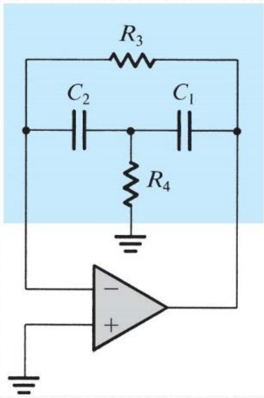
From this equation we see that the actual pole frequency, ω_{0a}, and the pole Q, Q_a, are

$$\omega_{0a} = \omega_0 \tag{17.87}$$

$$\boxed{Q_a = \frac{Q}{1 + 2Q^2/(A+1)}} \tag{17.88}$$

Q is for A= ∞;
Qa is for A= finite

Example 13.3 (Cont'd)



$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$$

$$Q = \left[\frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{-1}$$

$$s^2 + s\frac{\omega_0}{Q}\left(1 + \frac{2Q^2}{A+1}\right) + \omega_0^2 = 0 \qquad (17.86)$$

From this equation we see that the actual pole frequency, ω_{0a}, and the pole Q, Q_a, are

$$\omega_{0a} = \omega_0 \qquad (17.87)$$

$$Q_a = \frac{Q}{1 + 2Q^2/(A+1)} \qquad (17.88)$$

Thus

$$S_A^{\omega_{0a}} = 0$$

$$S_A^{Q_a} = \frac{A}{A+1}\,\frac{2Q^2/(A+1)}{1 + 2Q^2/(A+1)}$$

For $A \gg 2Q^2$ and $A \gg 1$ we obtain

$$S_A^{Q_a} \simeq \frac{2Q^2}{A}$$

It is usual to drop the subscript a in this expression and write

$$S_A^{Q} \simeq \frac{2Q^2}{A} \qquad (17.89)$$

Note that if Q is high ($Q \geq 5$), its sensitivity relative to the amplifier gain can be quite high.[10]

[10]Because the open-loop gain A of op amps usually has wide tolerance, it is important to keep $S_A^{\omega_0}$ and S_A^{Q} very small.

Transconductance-C Filters

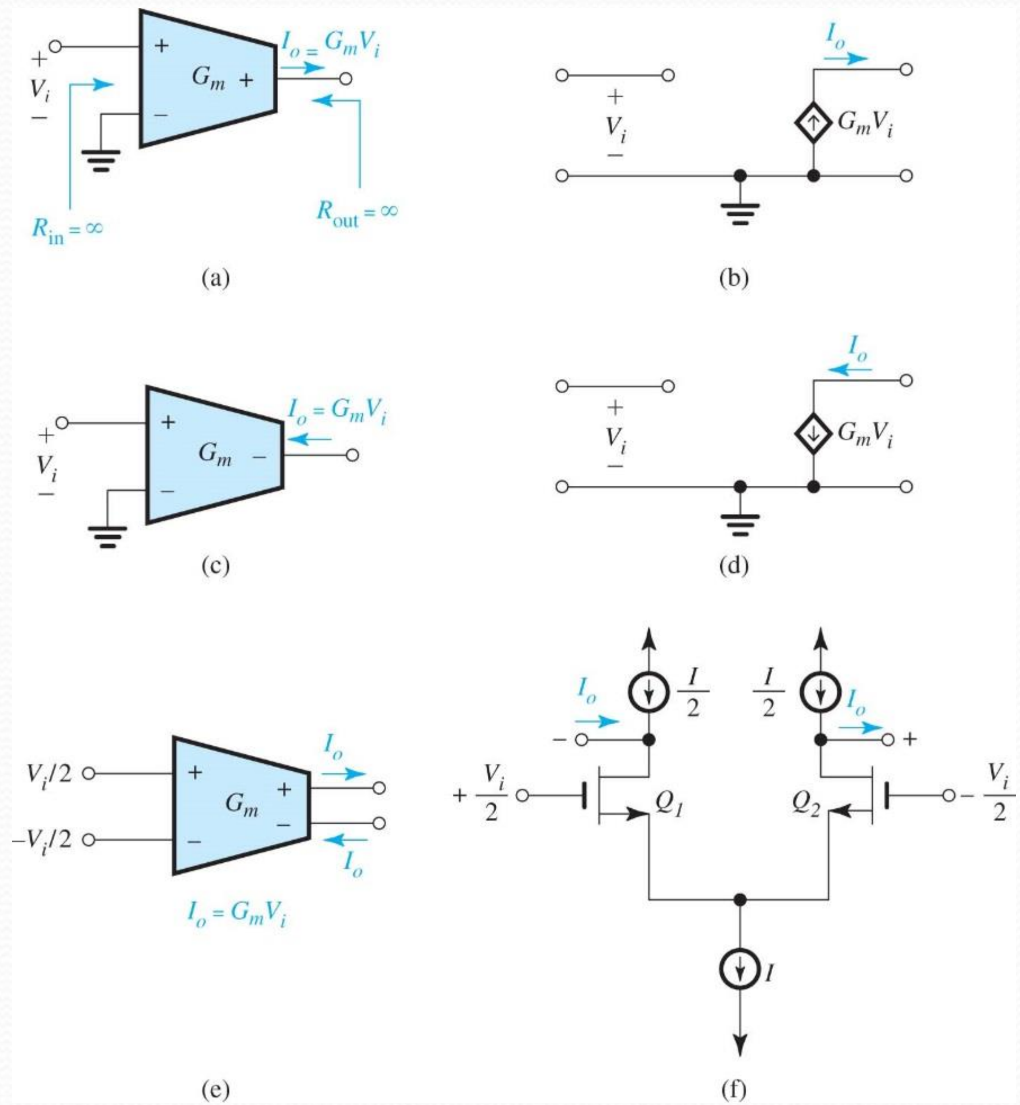
- The limitation of op amp–RC filters to low-frequency applications is a result of the relatively low bandwidth of general-purpose op amps. The lack of suitability of these filter circuits for implementation in IC form stems from:
  - **1.** The need for large-valued capacitors, which would require impractically large chip areas;
  - **2.** The need for very precise values of RC time constants. This is impossible to achieve on an IC without resorting to expensive trimming and tuning techniques; and
  - **3.** The need for op amps that can drive resistive and large capacitive loads. As we have seen, CMOS op amps are usually capable of driving only small capacitances.

Methods for IC Filter Implementation

- We now introduce the three approaches currently in use for implementing filters in monolithic form.
- **Transconductance-C Filters** These utilize transconductance amplifiers or simply transconductors together with capacitors and are hence called G_m–C filters.
- **MOSFET-C Filters** These utilize the two-integrator-loop circuits of Section 13.7 but with the resistors replaced with MOSFETs operating in the triode region.
- **Switched-Capacitor Filters** These are based on the ingenious technique of obtaining a large resistance by switching a capacitor at a relatively high frequency. The switched-capacitor approach is ideally suited for implementing low-frequency filters in IC form using CMOS technology.

Methods for IC Filter Implementation(Cont'd)

- Figure 13.35 (a) shows the circuit symbol for a transconductor, and Fig. (b) shows its equivalent circuit.
  - Assuming the transconductor to be ideal, with infinite input and output impedances.
- The transconductor of Fig. 13.35(a) has a positive output.
- Transconductors with a negative output are in (c), with its ideal model in (d). we show in (e) a differential-input–differential-output transconductor.
- Fig. (f), is simply a differential amplifier loaded with two current sources.
- Many elaborate transconductor circuits have been proposed and utilized in the design of G_m–C filters.



GM-C filter

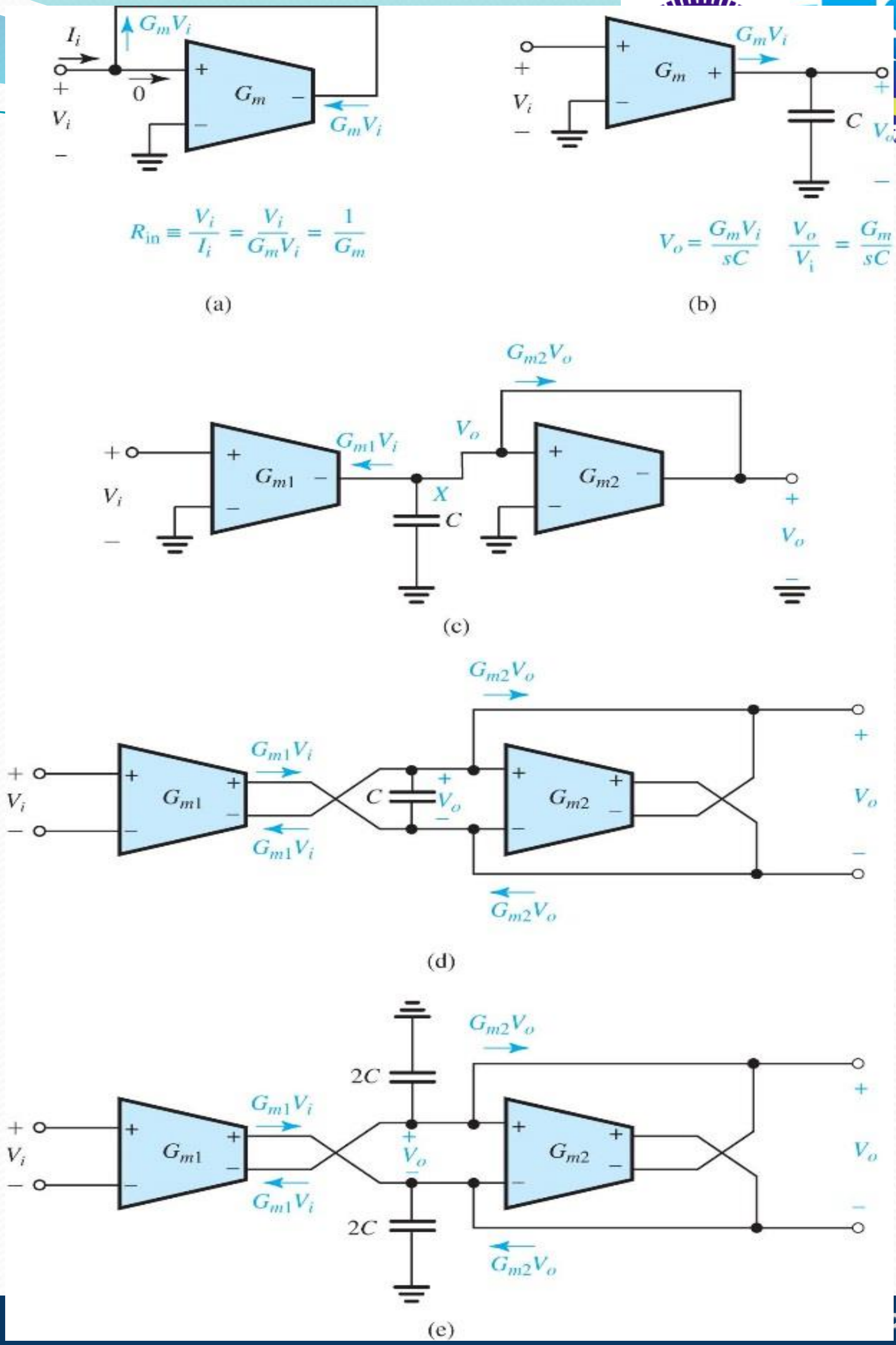


Figure 13.36

Realization of

(**a**) a resistance using a negative transconductor;

(**b**) an ideal noninverting integrator;

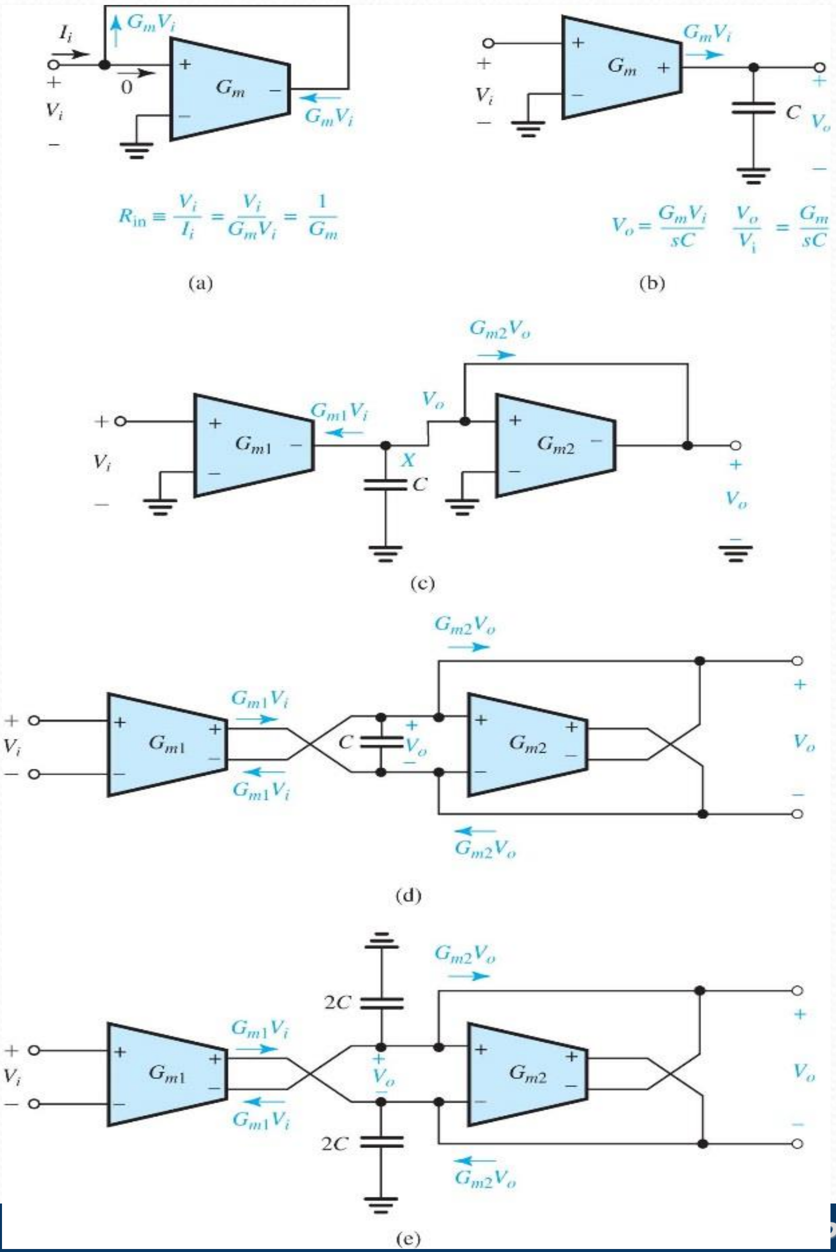
(**c**) a first-order low-pass filter (a damped integrator); and

(**d**) a fully differential first-order low-pass filter.

(**e**) Alternative realization of the fully differential first-order low-pass filter.

Basic Building Blocks

- An integrator is obtained by feeding the output current of a transconductor, $G_m V_i$, to a grounded capacitor.
- $$\frac{V_o}{V_i} = \frac{G_m}{sC} \qquad (13.90)$$
- we connect a resistance of the type in Fig. 17.36(a) in parallel with the capacitor C in the integrator of Fig. 17.36(b). Then in Fig. 17.36(c). The transfer function is
  - $$\frac{V_o}{V_i} = -\frac{G_{m1}}{sC + G_{m2}} \qquad (13.91)$$
- Thus, the pole frequency is (G_{m2}/C) and the dc gain is $(-G_{m1}/G_{m2})$.
- In 13.36(c) can be easily converted to the fully differential form shown in Fig. 13.36(d). An alternative implementation of the fully differential first-order low-pass filter is shown in Fig. 13.36(e).



Second-Order G_m–C Filter

- To derive the transfer functions (V_1/V_i) and (V_2/V_i) we first note that V_2 and V_1 are related by

$$V_2 = \frac{G_{m2}}{sC_2} V_1 \qquad (13.92)$$

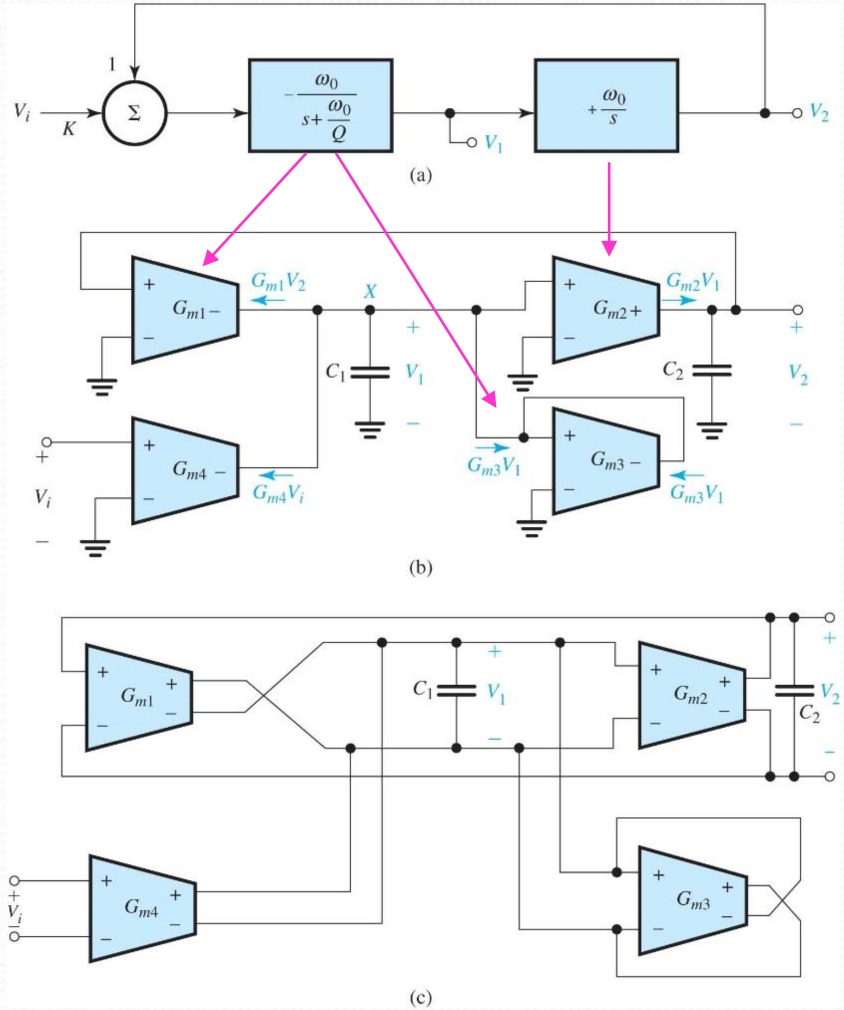
$$\frac{V_1}{V_i} = -\frac{s(G_{m4}/C_1)}{s^2 + s\frac{G_{m3}}{C_1} + \frac{G_{m1}G_{m2}}{C_1C_2}} \qquad (13.93)$$

$$\frac{V_2}{V_i} = -\frac{G_{m2}G_{m4}/C_1C_2}{s^2 + s\frac{G_{m3}}{C_1} + \frac{G_{m1}G_{m2}}{C_1C_2}} \qquad (13.94)$$

- Thus, the circuit in Fig. 13.37(b) is capable of realizing simultaneously a bandpass function (V_1/V_i) and a low-pass function (V_2/V_i). For both

$$\omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}} \qquad (13.95) \qquad \text{and}$$

$$Q = \frac{\sqrt{G_{m1}G_{m2}}}{G_{m3}} \sqrt{\frac{C_1}{C_2}} \qquad (13.96)$$



Second-Order G_m–C Filter(Cont'd)

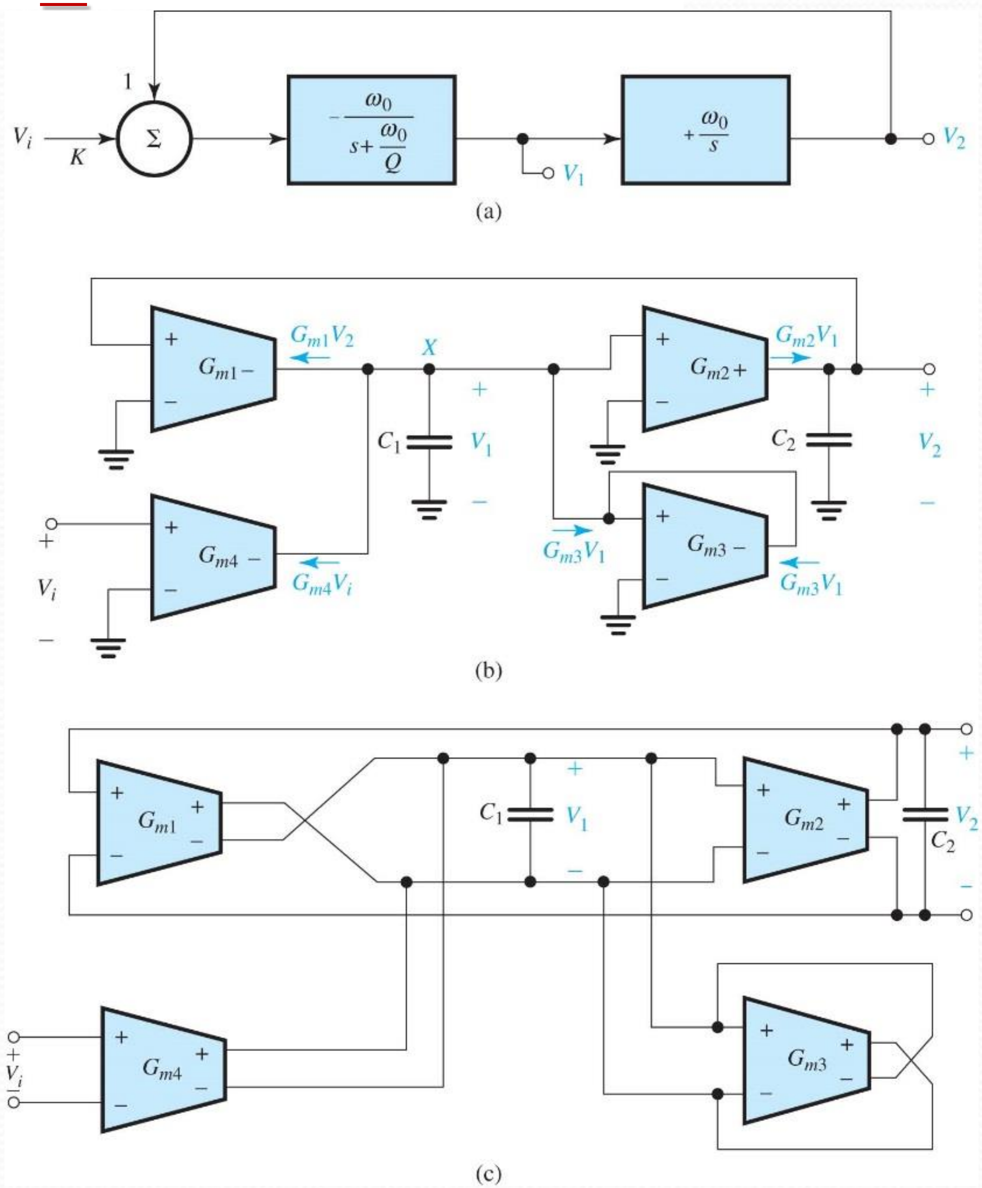
Figure 13.37

(a) Block diagram of the two-integrator-loop biquad. This is a somewhat modified version of Fig. 13.25.

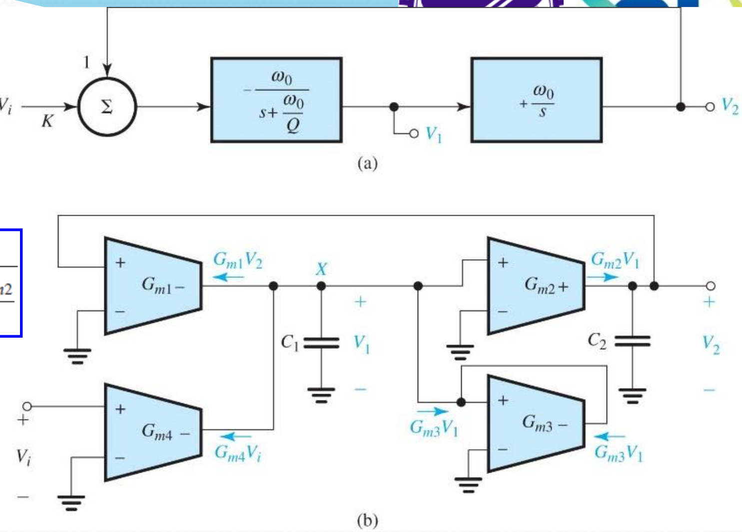
(**b**) G_m –C implementation of the block diagram in (**a**).

(**c**) Fully differential G_m –C implementation of the block diagram in (**a**).

In all parts, V_1/V_i is a bandpass function and V_2/V_i is a low-pass function.



2nd-Order G_m–C Filter(Cont'd)



- For the bandpass function,
  - Center-frequency gain= $-\dfrac{G_{m4}}{G_{m3}}$

$$\boxed{\frac{V_1}{V_i} = -\frac{s(G_{m4}/C_1)}{s^2 + s\frac{G_{m3}}{C_1} + \frac{G_{m1}G_{m2}}{C_1C_2}}}$$

- and for the low-pass function,
  - DC gain= $-\dfrac{G_{m4}}{G_{m1}}$ (13.98)

$$\boxed{\frac{V_2}{V_i} = -\frac{G_{m2}G_{m4}/C_1C_2}{s^2 + s\frac{G_{m3}}{C_1} + \frac{G_{m1}G_{m2}}{C_1C_2}}}$$

- There are a variety of possible designs. The most common is to make the time constants of the integrators equal [which is the case in the block diagram of Fig. 13.37(a)]. Doing this and selecting $G_{m1}= G_{m2}= G_m$ and $C_1= C_2= C$ results in the following design equation
  - $\dfrac{G_m}{C} = \omega_0$ (13.99)
  - $G_{m3} = \dfrac{G_m}{Q}$ (13.100)
  - For the BP: $G_{m4} = \dfrac{G_m}{Q}|\mathrm{Gain}|$ (13.101)
  - For the LP: $G_{m4} = G_m|\mathrm{Gain}|$ (13.102)

$$\boxed{\omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}}}$$

$$\boxed{Q = \frac{\sqrt{G_{m1}G_{m2}}}{G_{m3}}\sqrt{\frac{C_1}{C_2}}}$$

Example 13.4

Design the G_m–C circuit of Fig. 17.37(b) to realize a bandpass filter with a center frequency of 10 MHz, a 3-dB bandwidth of 1 MHz, and a center-frequency gain of 10. Use equal capacitors of 5 pF.

Solution

Using the equal-integrator-time-constants design, Eq. (17.99) yields

$$G_m = \omega_0 C = 2\pi \times 10 \times 10^6 \times 5 \times 10^{-12} = 0.314 \text{ mA/V}$$

Thus,

$$G_{m1} = G_{m2} = 0.314 \text{ mA/V}$$

To obtain G_{m3}, we first note that $Q = f_0/\text{BW} = 10/1 = 10$, and then use Eq. (17.100) to obtain

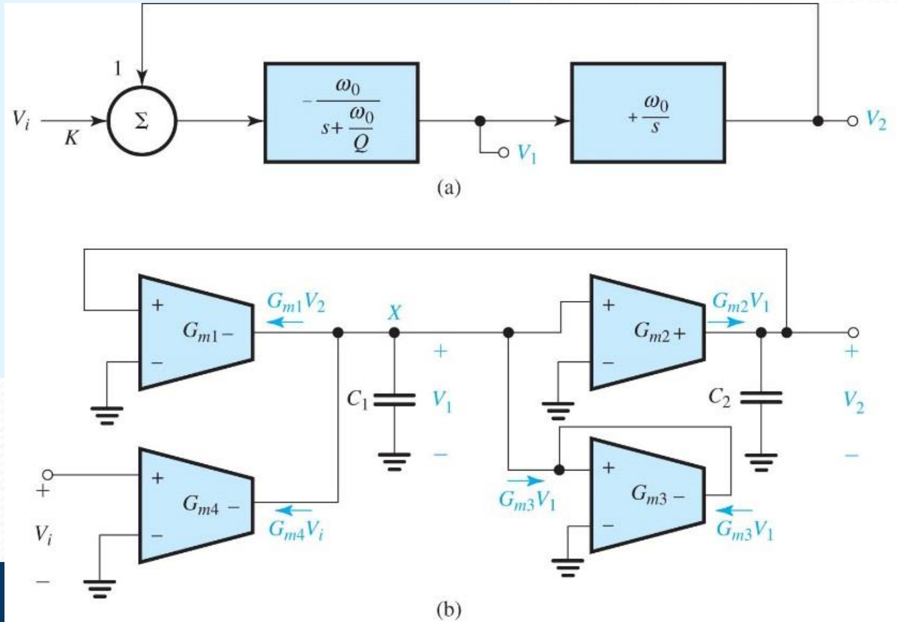
$$G_{m3} = \frac{G_m}{Q} = \frac{0.314}{10} = 0.0314 \text{ mA/V}$$

or

$$G_{m3} = 31.4\ \mu\text{A/V}$$

Finally, G_{m4} can be found by using Eq. (17.101) as

$$G_{m4} = \frac{G_m}{10} \times 10 = 0.314 \text{ mA/V}$$



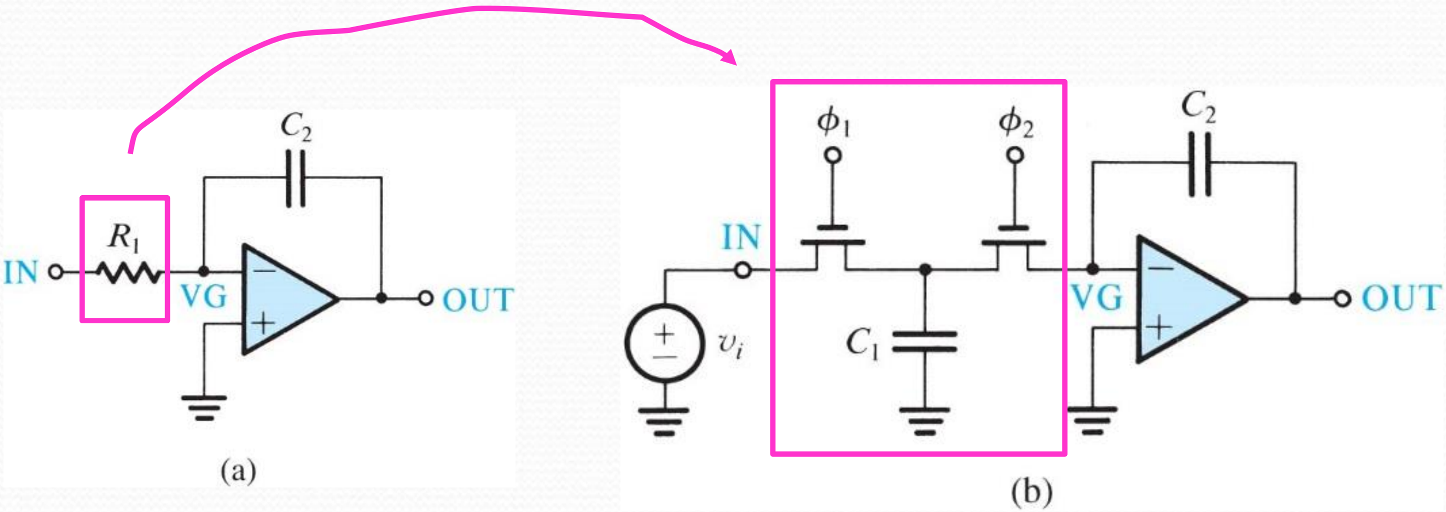
Switched-Capacitor Filters

- Switched-capacitor filters, which require only small capacitors, analog switches, and op amps that need to drive only small capacitive loads, are ideally suited for implementation in CMOS.

The Basic Principle

- The switched-capacitor filter technique is based on the realization that a capacitor switched between two circuit nodes at a sufficiently high rate is equivalent to a resistor connecting these two nodes. Consider the active-RC integrator of Fig. 13.38 (a). In Fig. 13.38(b) we have replaced the input resistor R_1 by a grounded capacitor C_1 together with two MOS transistors acting as switches.

- The two MOS switches in Fig. 13.38(b) are driven by a *nonoverlapping* two-phase clock.

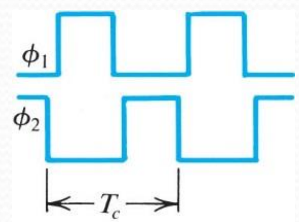


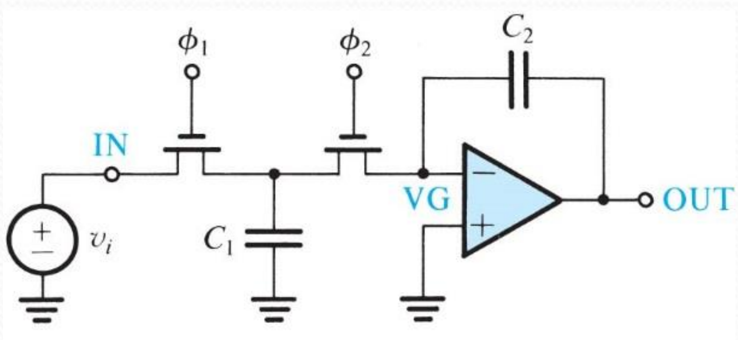
The Basic Principle (Cont'd)

- We see that during each clock period T_c an amount of charge $q_{C1} = C_1 v_i$ is extracted from the input source and supplied to the integrator capacitor C_2. Thus the average current flowing between the input node (IN) and the virtual-ground node (VG) is

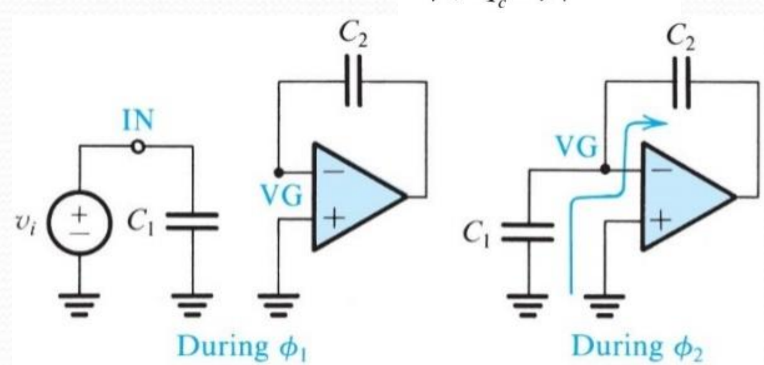
$$\boxed{i_{av} = \frac{C_1 v_i}{T_c}}$$

- If T_c is sufficiently short, one can think of this process as almost continuous and define an equivalent resistance R_{eq} that is in effect present between nodes IN and VG:
- $R_{eq} \equiv v_i / i_{av}$
- Thus, $R_{eq} = T_c / C_1$ (13.103)





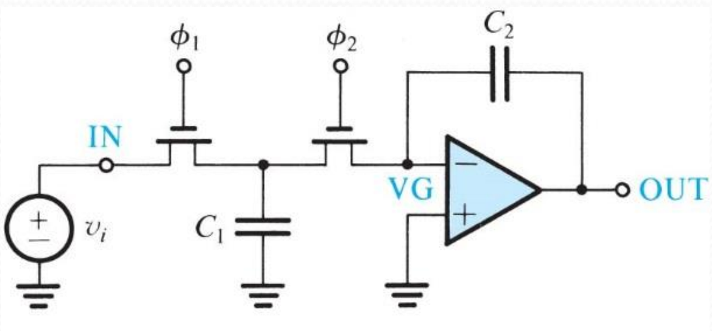
(b)



(d)

The Basic Principle(Cont'd)

- Using R_{eq} we obtain an equivalent time constant for the integrator:
  - Time constant = C_2R_{eq} = $T_c\frac{C_2}{C_1}$ (13.104)
- Thus the time constant that determines the frequency response of the filter is established by the clock period T_c and the capacitor ratio C_2/C_1. Both these parameters can be well controlled in an IC process.



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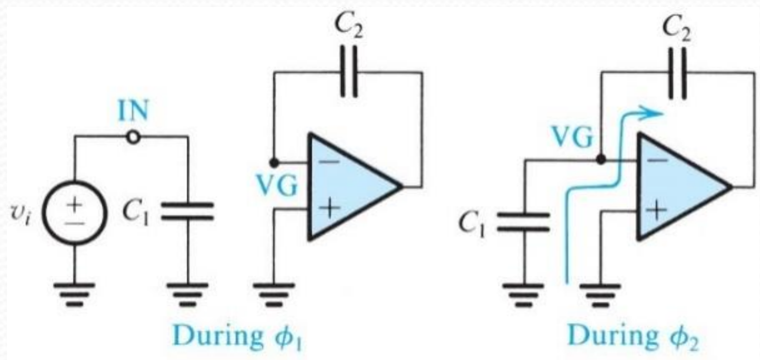
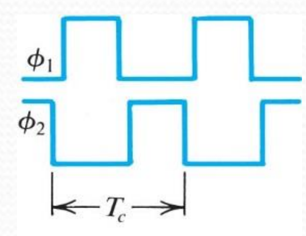


(d)



(c)

Practical Circuits

- To realize a switched-capacitor biquad filter, we therefore need a pair of complementary switched-capacitor integrators.
- Figure 13.39(a) shows a noninverting, or positive, integrator circuit. The complementary integrators of Fig. 13.39 have become the standard building blocks in the design of switched-capacitor filters.

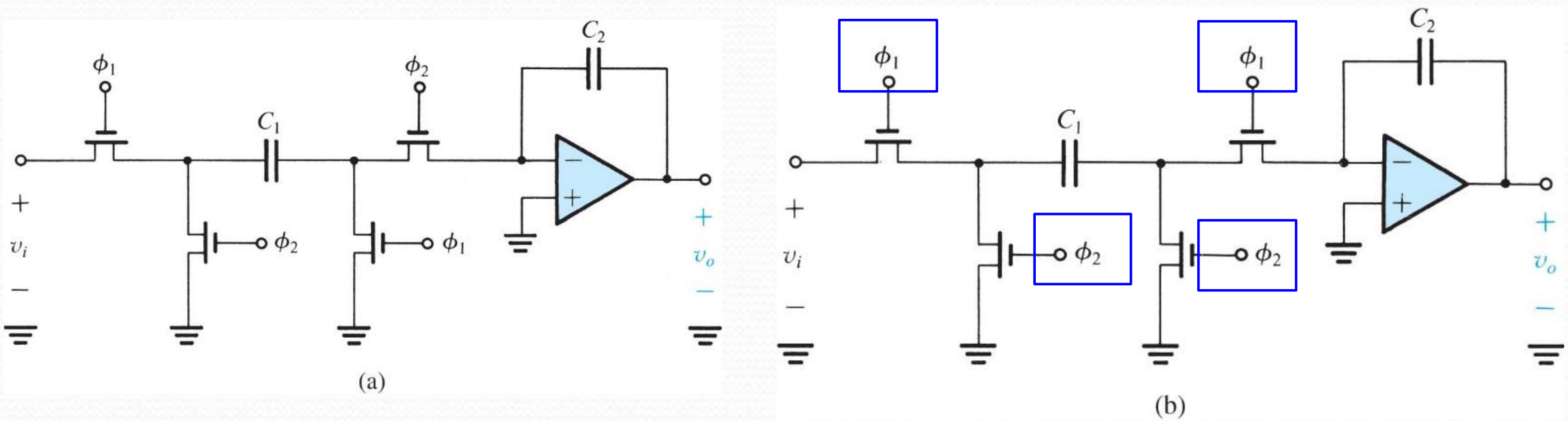


Figure 13.39 A pair of complementary stray-insensitive, switched-capacitor integrators. **(a)** Noninverting switched-capacitor integrator. **(b)** Inverting switched-capacitor integrator.

Realizing Switched-C Filters

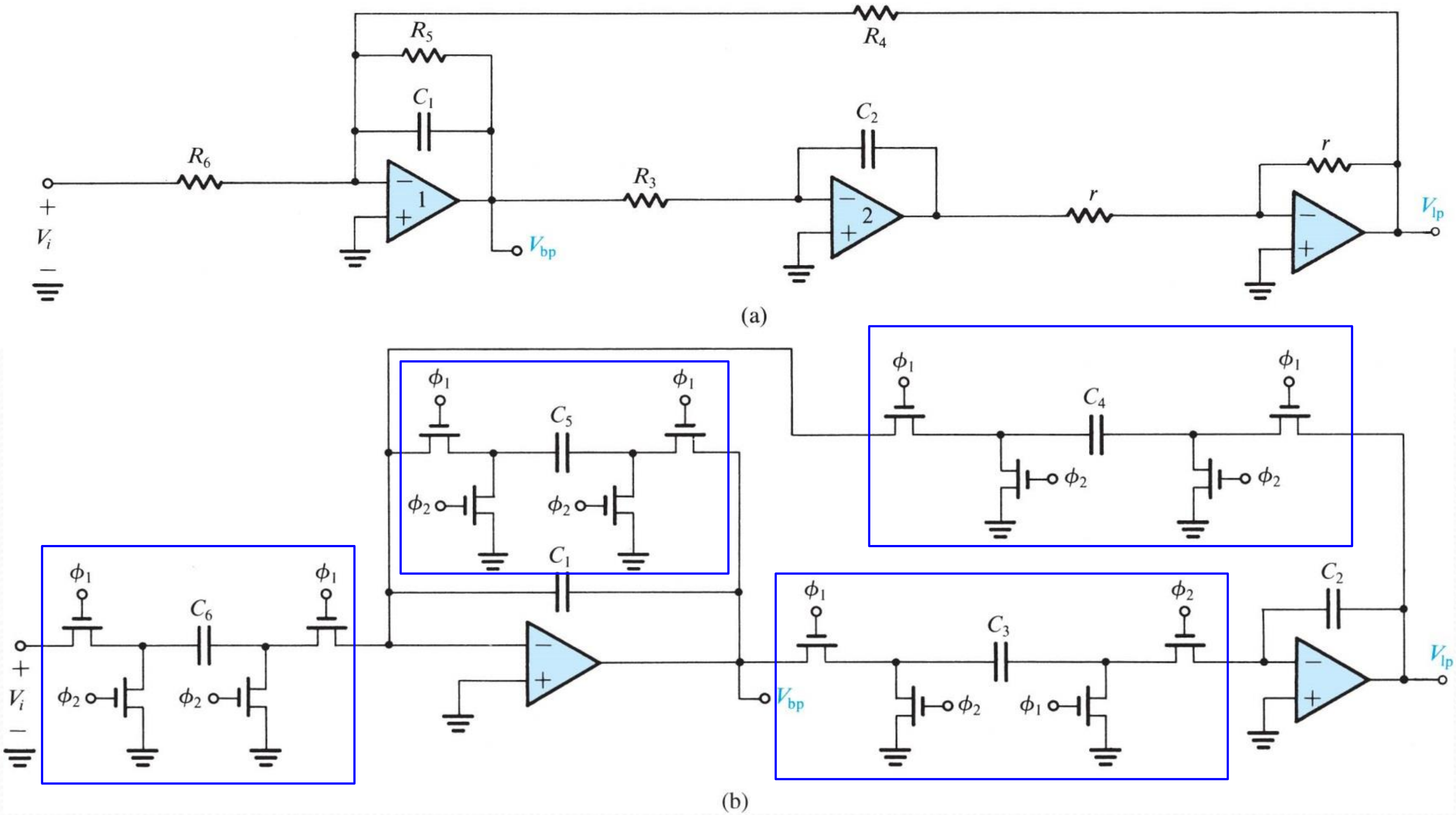
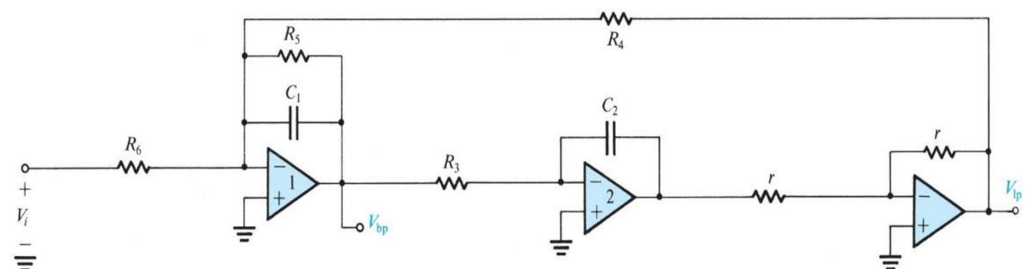


Figure 13.40 (a) A two-integrator-loop, active-RC biquad and **(b)** its switched-capacitor counterpart.

Realizing Switched-C Filters

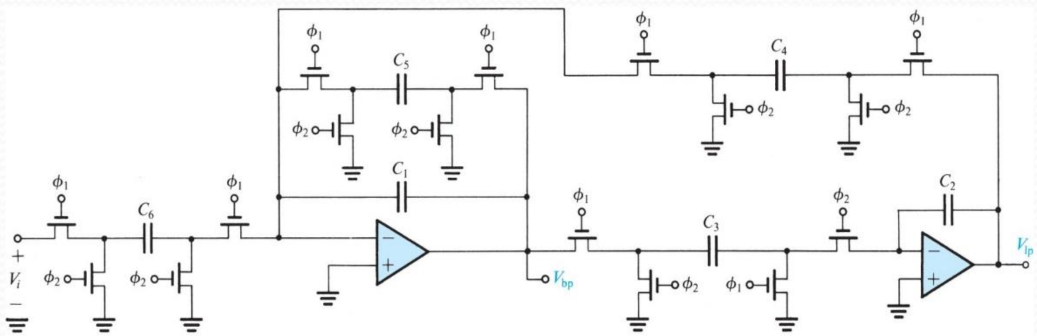
- Analysis of the circuit in Fig. 13.40(a) yields

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} \qquad (13.105)$$



(a)

- Replacing R_2 and R_4 with their switched-capacitor equivalent values, that is, $R_3 = T_c / C_3$ and $R_4 = T_c / C_4$
  - $$\omega_0 = \frac{1}{T_c}\sqrt{\frac{C_3}{C_2}\frac{C_4}{C_1}} \qquad (13.106)$$
- It is usual to select the time constants of the two integrators to be equal; that is,
  - $$\frac{T_c}{C_3}C_2 = \frac{T_c}{C_4}C_1 \qquad (13.107)$$



(b)

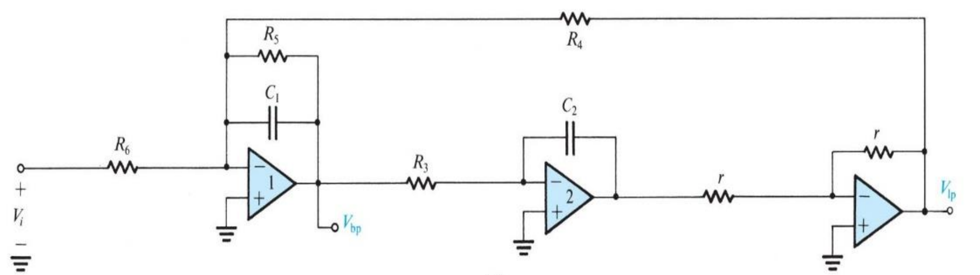
- If, further, we select the two integrating capacitors C_1 and C_2 to be equal, $C_1 = C_2 = C$ (13.108),
- then $C_3 = C_4 = KC$ (13.109)

Realizing Switched-C Filters

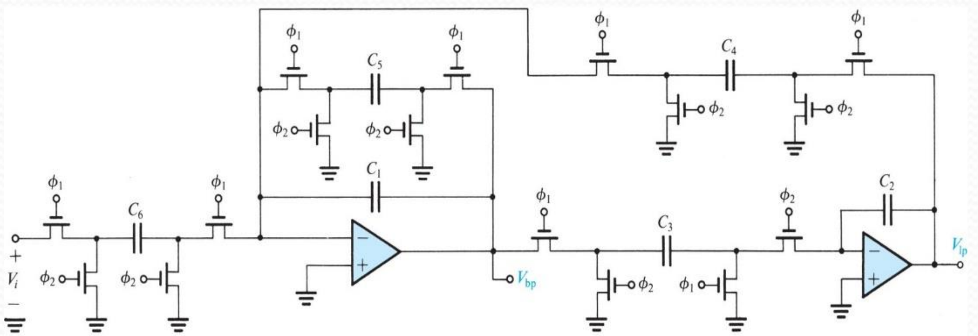
- where from Eq.(13.106), $K = \omega_0 T_c$ (13.110)
- For the case of equal time constants, the Q factor of the circuit in Fig. 13.40(a) is given by R_5/R_4. Thus the Q factor of the corresponding switched-capacitor circuit in Fig. 13.40(b) is given by
  - $$C_5 = \frac{C_4}{Q} = \frac{KC}{Q} = \omega_0 T_c \frac{C}{Q}$$ (13.111)
- Thus C_5 should be selected from

$$Q = \frac{T_c/C_5}{T_c/C_4} = \frac{C_4}{C_5}$$ (13.112)

- Finally, the center-frequency gain of the bandpass function is given by
  - $$\text{Center-frequency gain} = \frac{C_6}{C_5} = Q\frac{C_6}{\omega_0 T_c C}$$
  - (13.113)



(a)



(b)

Tuned Amplifiers

- Figure 13.41 shows the general shape of the frequency response of a tuned amplifier. It should be noted that the tuned-amplifier response of Fig. 13.41 is similar to that of the bandpass filter discussed in earlier sections.

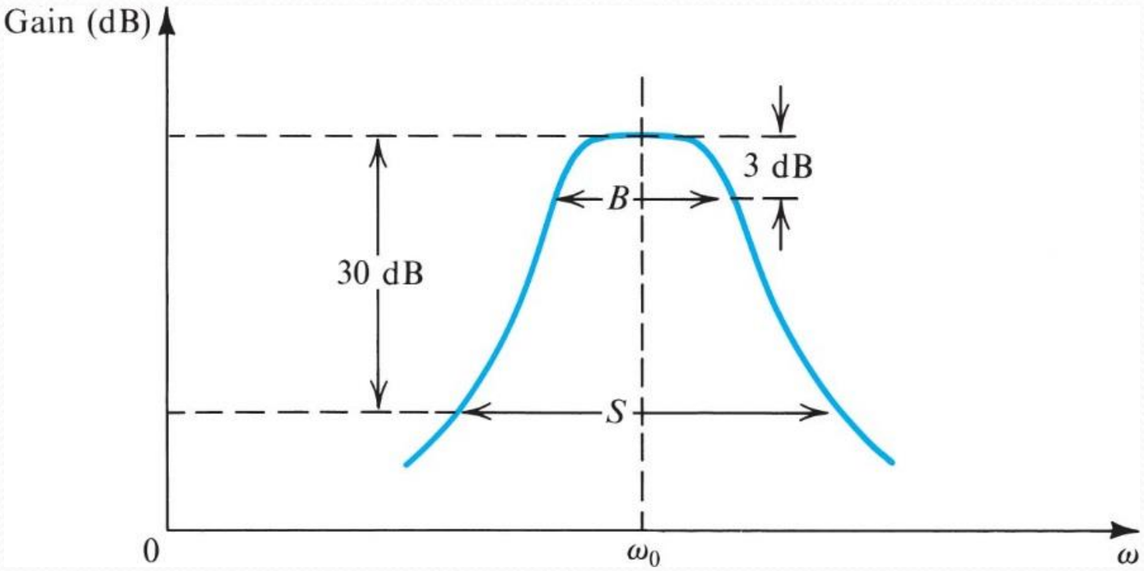


Figure 13.41 Frequency response of a tuned amplifier.

The Basic Principle

- A parallel LCR circuit as the load, or
- at the input, of a BJT or a FET amplifier.
- This is illustrated in Fig. below with a MOSFET amplifier having a tuned-circuit load.
- The amplifier equivalent circuit is shown in Fig. 13.42(b). From the equivalent circuit we can write

$$V_o = \frac{-g_m V_i}{Y_L} = \frac{-g_m V_i}{sC + 1/R + 1/sL}$$

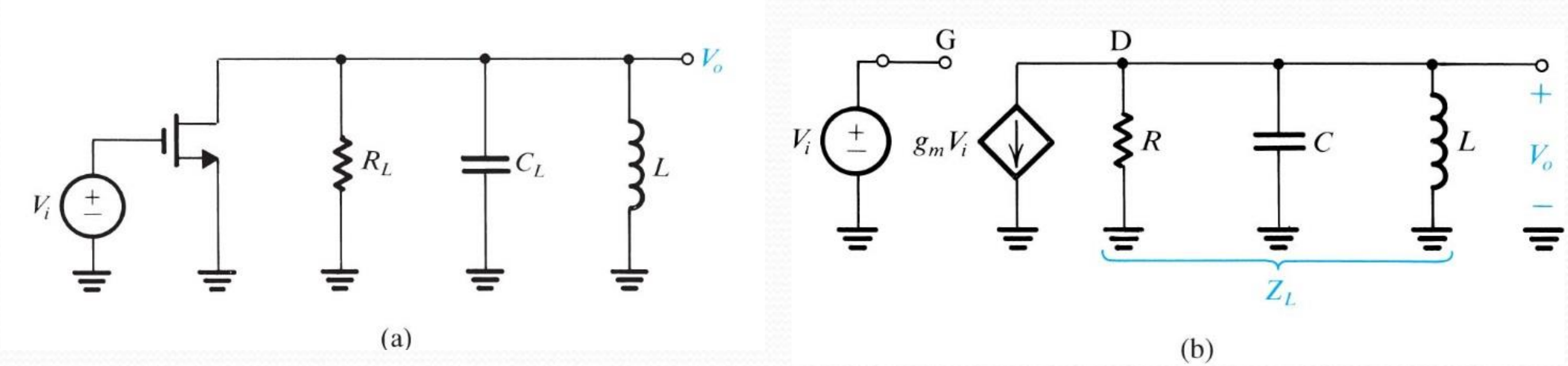
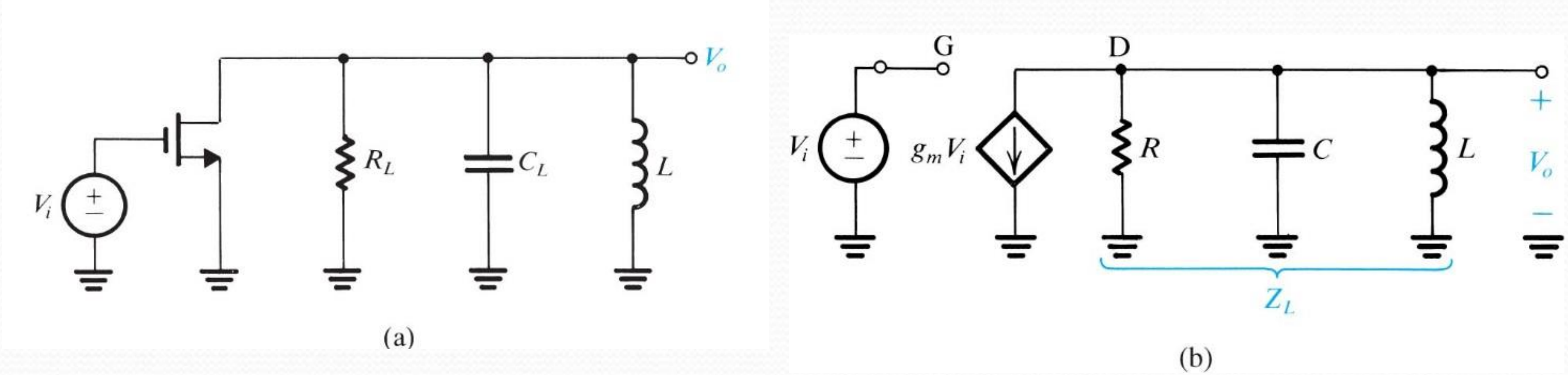


Figure 13.42 The basic principle of tuned amplifiers is illustrated using a MOSFET with a tuned-circuit load. Bias details are not shown.

The Basic Principle(Cont'd)

$$V_o = \frac{-g_m V_i}{Y_L} = \frac{-g_m V_i}{sC + 1/R + 1/sL}$$

- Thus the voltage gain can be expressed as
- $$\frac{V_o}{V_i} = -\frac{g_m}{C} \frac{s}{s^2 + s(1/CR) + 1/LC} \quad (13.114)$$
- The tuned amplifier has a center frequency of $\omega_0 = 1/\sqrt{LC}$ (13.115)
- a 3-dB bandwidth of $B = \frac{1}{CR}$ (13.116)
- a Q factor of $Q \equiv \omega_0/B = \omega_0 CR$ (13.117)
- and a center-frequency gain of $\frac{V_o(j\omega_0)}{V_i(j\omega_0)} = -g_m R$ (13.118)



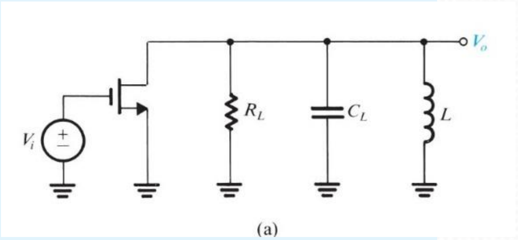
Example 13.5

It is required to design a tuned amplifier of the type shown in Fig. 17.42, having $f_0 = 1$ MHz, 3-dB bandwidth $= 10$ kHz, and center-frequency gain $= -10$ V/V. The FET available has at the bias point $g_m = 5$ mA/V and $r_o = 10$ kΩ. The output capacitance is negligibly small. Determine the values of R_L, C_L, and L.

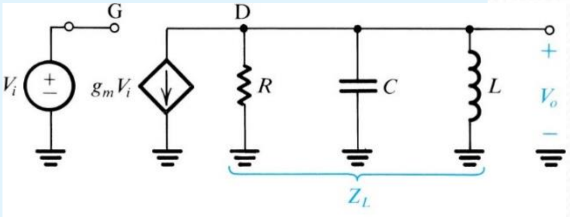
Solution

Center-frequency gain $= -10 = -5R$. Thus $R = 2$ kΩ. Since $R = R_L \parallel r_o$, then $R_L = 2.5$ kΩ.

$$B = 2\pi \times 10^4 = \frac{1}{CR}$$



(a)



(b)

Thus

$$C = \frac{1}{2\pi \times 10^4 \times 2 \times 10^3} = 7958 \text{ pF}$$

Since $\omega_0 = 2\pi \times 10^6 = 1/\sqrt{LC}$, we obtain

$$L = \frac{1}{4\pi^2 \times 10^{12} \times 7958 \times 10^{-12}} = 3.18\ \mu\text{H}$$

Inductor Losses

- The power loss in the inductor is usually represented by a series resistance r_s as shown in Fig. 13.43(a). However, rather than specifying the value of r_s, the usual practice is to specify the inductor Q factor at the frequency of interest,
  - $$Q_0 \equiv \frac{\omega_0 L}{r_s} \quad (13.119)$$
- Typically, **Q_0** is in the range of **50 to 200**.

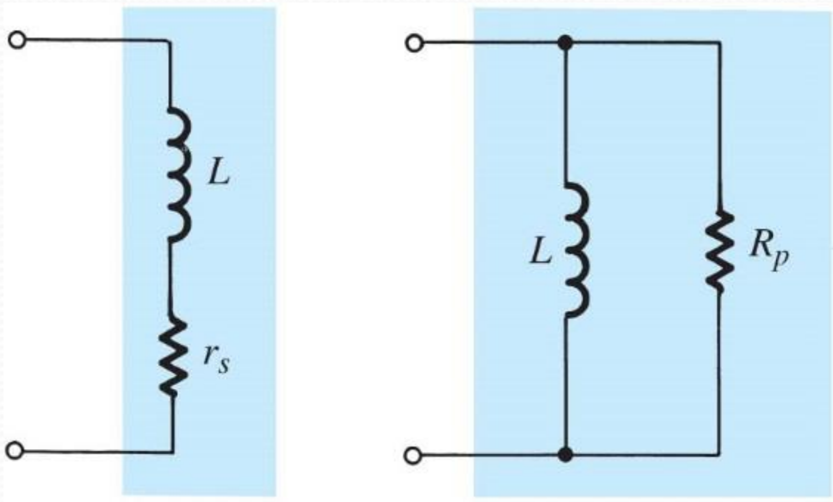


Figure 13.43 Inductor equivalent circuits

Inductor Losses (Cont'd)

- For the admittance of the circuit in Fig. (a)

$$Y(j\omega_0) = \frac{1}{r_s + j\omega_0 L}$$
$$= \frac{1}{j\omega_0 L}\frac{1}{1 - j(1/Q_0)} = \frac{1}{j\omega_0 L}\frac{1 + j(1/Q_0)}{1 + \left(1/Q_0^2\right)}$$

- For Q_0>>1 (13.120)

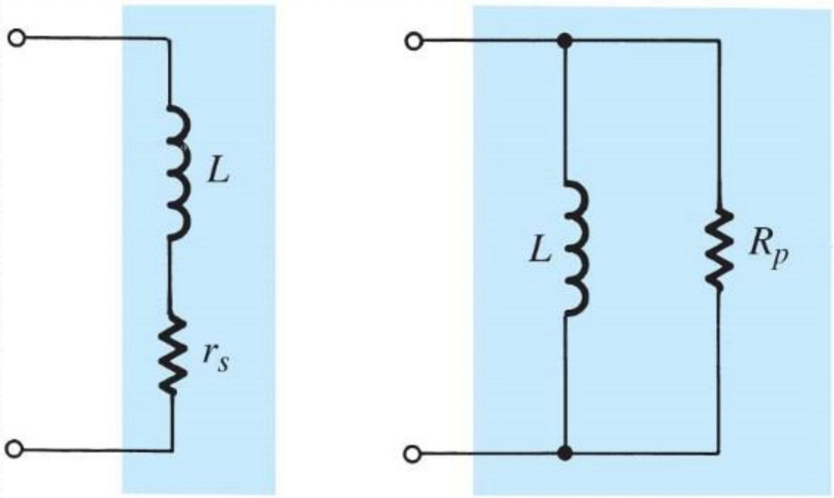
$$Y(j\omega_0) \simeq \frac{1}{j\omega_0 L}\left(1 + j\frac{1}{Q_0}\right)$$

- Equating this to the admittance of the circuit in Fig. 13.43(b) gives

  - $$Q_0 = \frac{R_p}{\omega_0 L}$$ (13.121)

- or, equivalently, (13.122)

$$R_p = \omega_0 L Q_0$$



Impedance Transformers

- In many cases it is found that the required value of inductance is not practical.
- A simple solution is to use a transformer to effect an impedance change. Alternatively, a tapped coil, known as an **autotransformer**, can be used, as shown in Fig. 13.44.
- The result is that the tuned circuit seen between terminals 1 and 1′ is equivalent to that in Fig. 13.42(b). For example, if a turns ratio n = 3 is used in the amplifier of Example 13.5, then a coil with inductance L' = 9 x3.18 = 28.6 μH and a capacitance C' = 7958/9= 884 pF will be required. Both these values are more practical than the original ones.

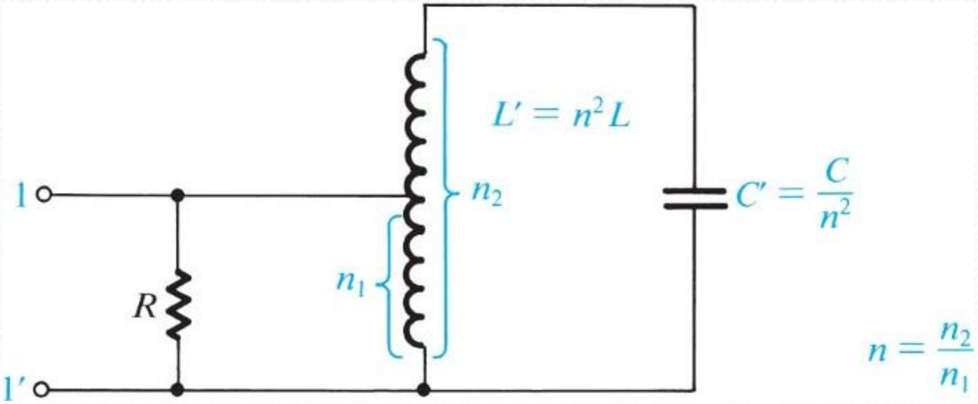


Figure 13.44 A tapped inductor is used as an impedance transformer to allow using a higher inductance, L', and a smaller capacitance, C'

Impedance Transformers (Cont'd)

- In applications that involve coupling the output of a tuned amplifier to the input of another amplifier,
- the tapped coil can be used to **raise the effective input resistance** of the latter amplifier stage.
  - In this way, one can avoid reduction of the overall Q. This point is illustrated in Fig. 13.45.

$$Q_0 \equiv \frac{\omega_0 L}{r_s}$$

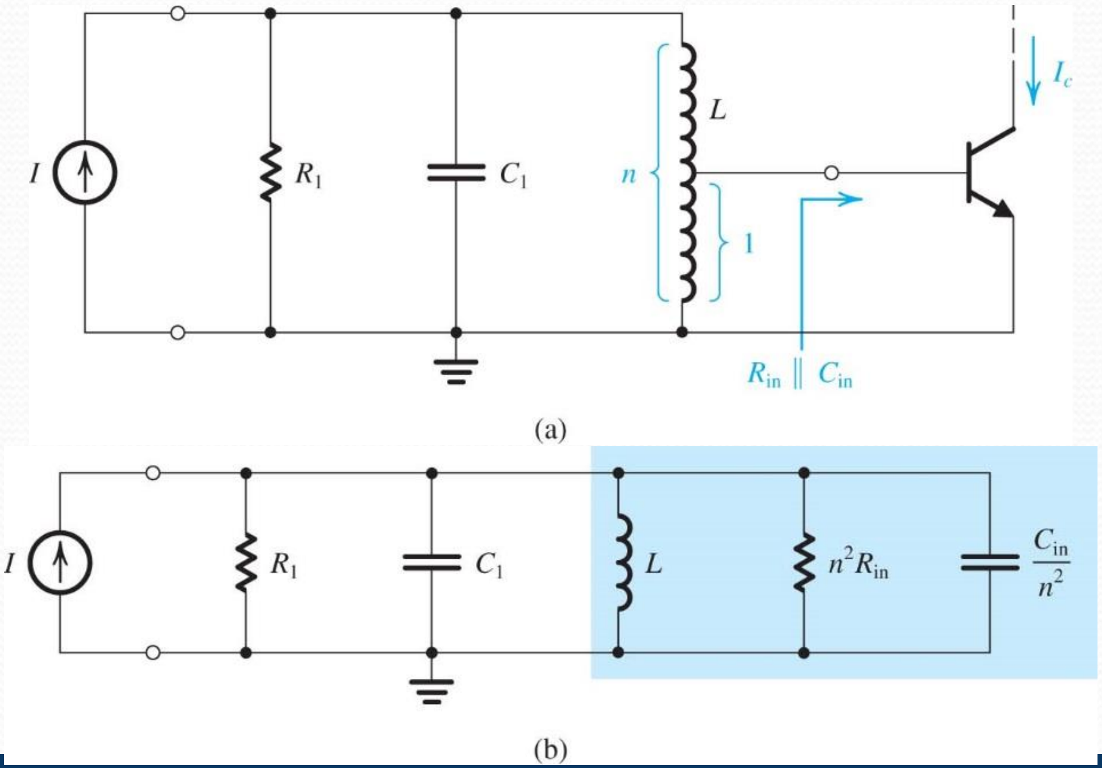


Figure 13.45 (a) The output of a tuned amplifier **is coupled to the input of another amplifier via a tapped coil.** **(b)** An equivalent circuit. Note that the use of a tapped coil increases the effective input impedance of the second amplifier stage.

Amplifiers with Multiple Tuned Circuits

- Figure 13.46 shows a BJT with tuned circuits at both the input and the output. However, to avoid the loading effect of the bias resistors R_{B1} and R_{B2} on the input tuned circuit, a **radio frequency choke** (RFC) is inserted in series with each resistor. Such chokes have high impedances at the frequencies of interest. The use of RFCs in biasing tuned RF amplifiers is

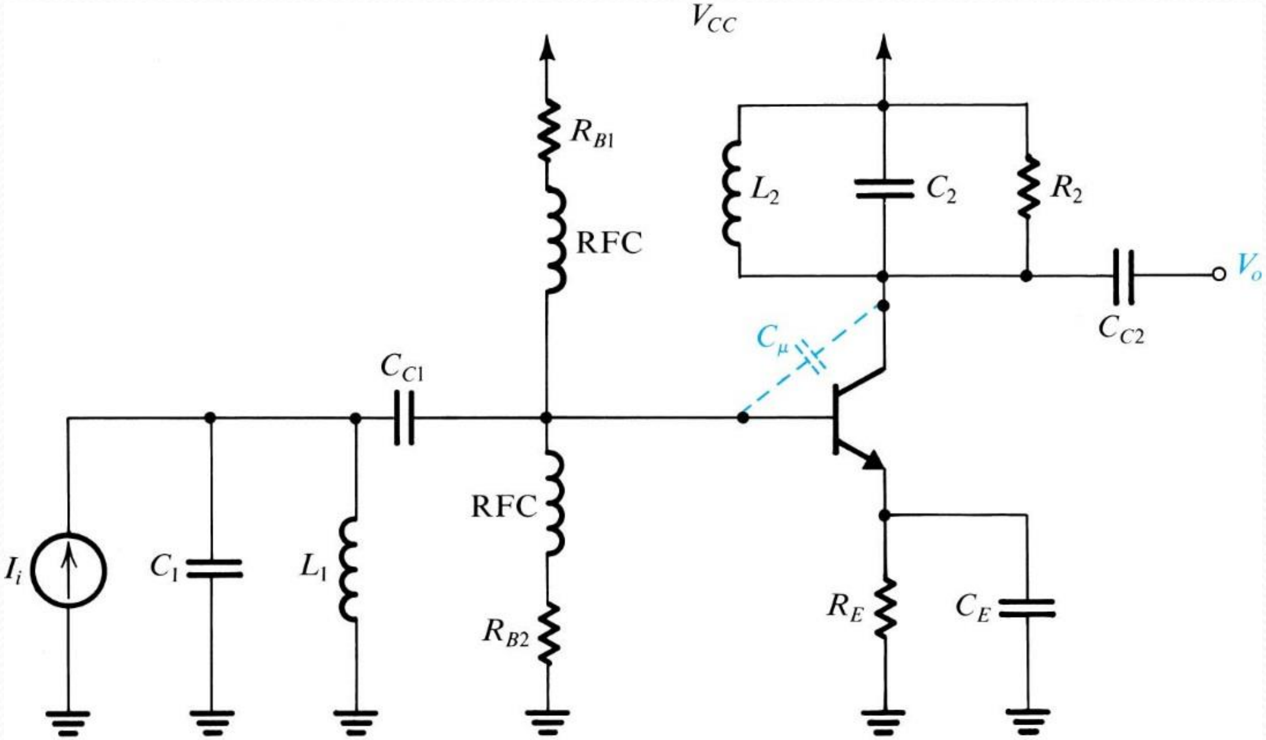


Figure 13.46 A BJT amplifier with tuned circuits at the input and the output.

The Cascode and the CC–CB Cascade

- Two amplifier configurations do not suffer from the Miller effect. These are
  - the cascode configuration and
  - the common-collector, common-base cascade.
  - Figure 13.47 shows tuned amplifiers based on these two configurations.

Synchronous and Stagger Tunings

- In the design of a tuned amplifier with multiple tuned circuits, the question of the frequency to which each circuit should be tuned arises. To investigate this question, we shall assume that the overall response is the product of the individual responses. This can easily be achieved using circuits such as those in Fig. 13.47.

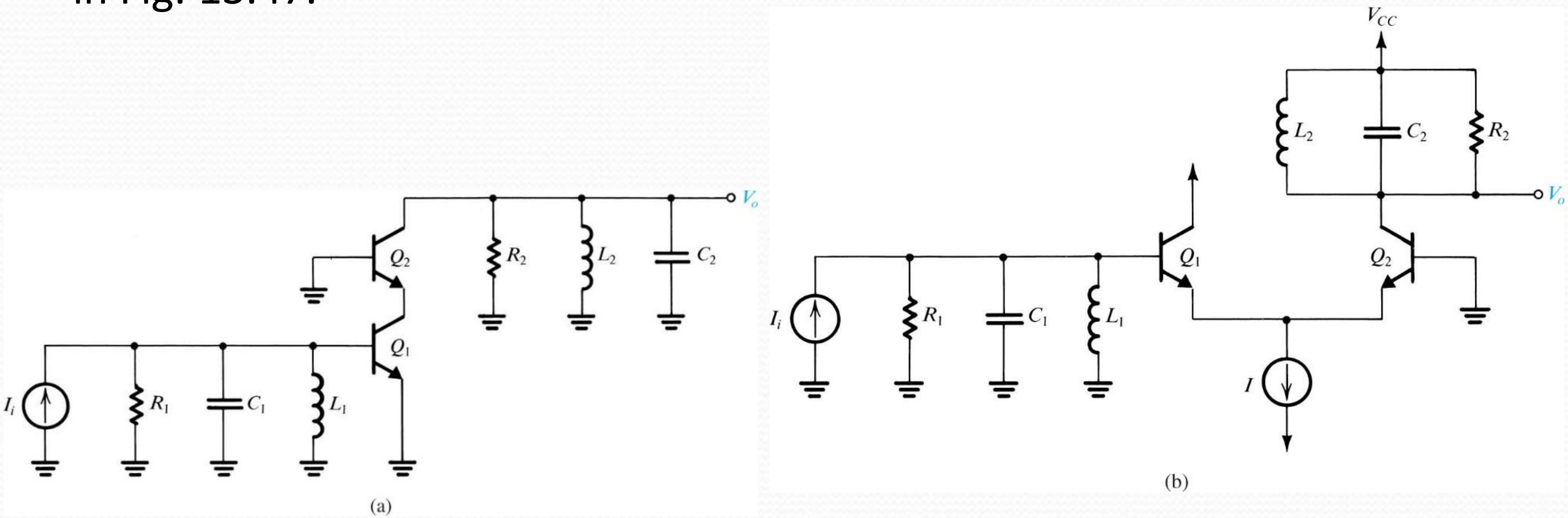


Figure 13.47 Two tuned-amplifier configurations that do not suffer from the Miller effect: **(a)** cascode and **(b)** common-collector, common-base cascade. (Note that bias details of the cascode circuit are not shown.)

Synchronous and Stagger Tunings

- Figure 13.48 shows the response of an individual stage and that of the cascade. The 3-dB bandwidth *B* of the overall amplifier is related to that of the individual tuned circuits, ω_0/Q, by

$$B = \frac{\omega_0}{Q}\sqrt{2^{1/N} - 1} \qquad (13.123)$$

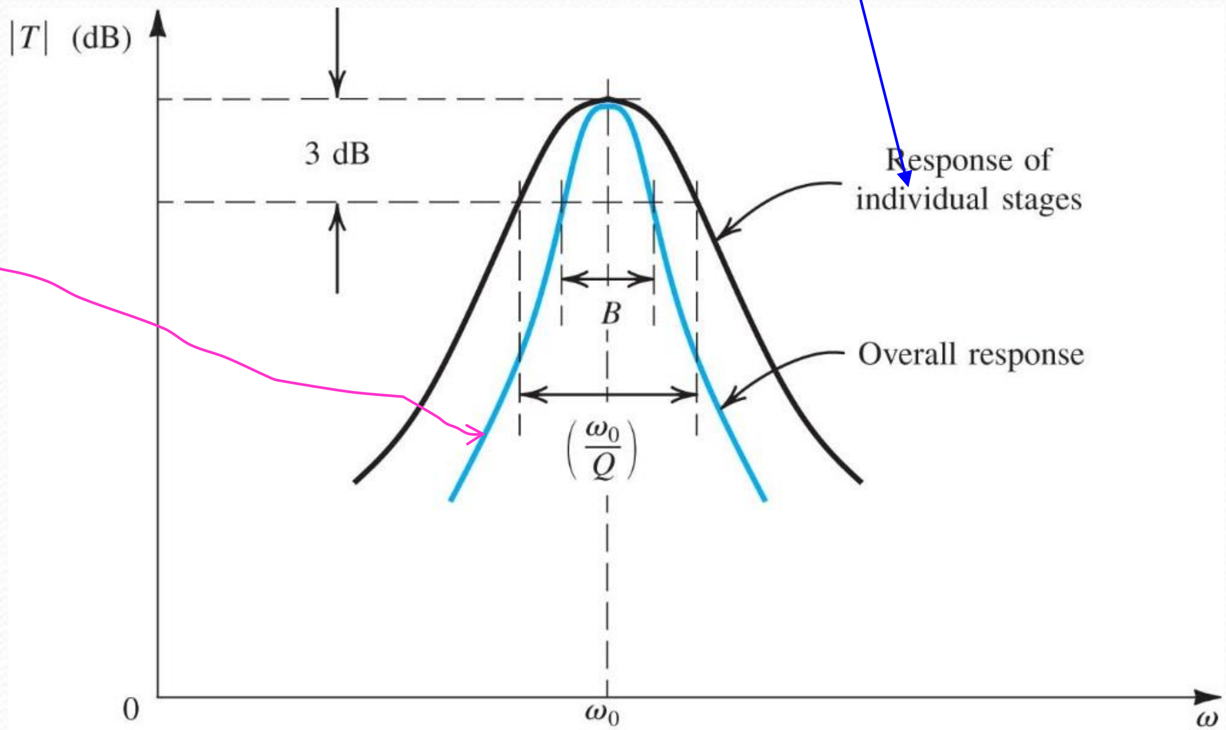


Figure 13.48 Frequency response of a synchronously tuned amplifier.

Synchronous and Stagger Tunings

- The factor $\sqrt{2^{1/N}-1}$ is known as the **bandwidth-shrinkage factor**. Given *B* and *N*, we can use Eq. (13.123) to determine the bandwidth required of the individual stages, ω_0/Q.
- A much better overall response is obtained by stagger-tuning the individual stages, as illustrated in Fig. 13.49.

$$\boxed{B = \frac{\omega_0}{Q}\sqrt{2^{1/N}-1}}$$

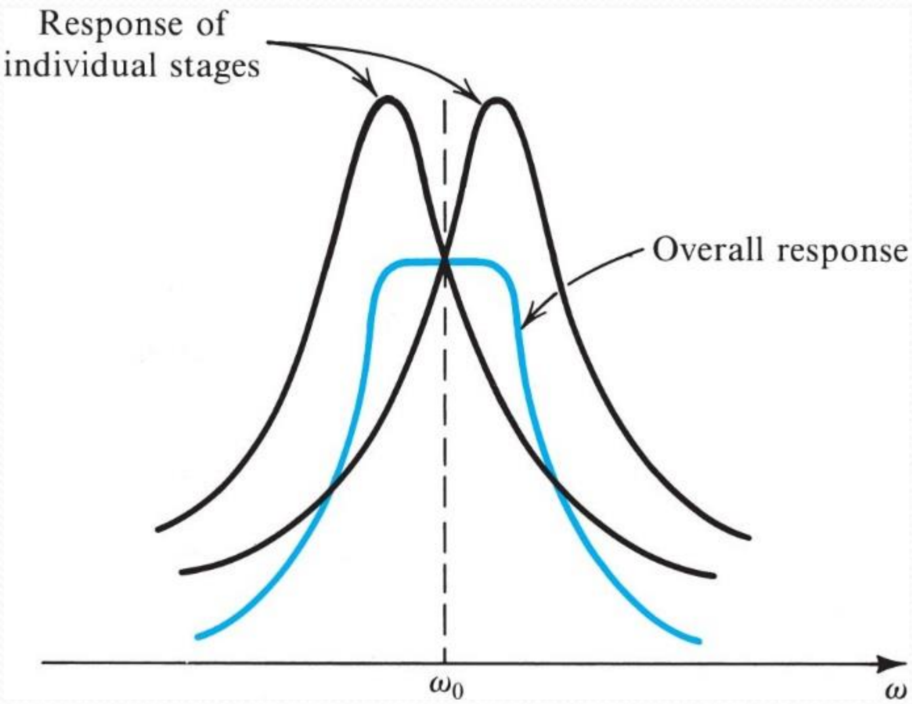


Figure 13.49 Stagger-tuning the individual resonant circuits can result in an overall response with a passband flatter than that obtained with synchronous tuning (Fig. 13.48)

HW

See WORD files in E3

