Continuous distributions

Name	Symbol	Parameter	Definition range	Density	Mean, variance, mode
Continuous Uniform	$Y \sim \mathrm{U}(a,b)$	$a,b \in \mathbb{R}$ $a < b$	$y \in [a,b]$	$p(y\|a,b) = \frac{1}{b-a}$	$\mathbb{E}(Y\|a,b) = \frac{a+b}{2}$ $\mathrm{Var}(Y\|a,b) = \frac{(b-a)^2}{12}$
Univariate Normal	$Y \sim \mathrm{N}(\mu,\sigma^2)$	$\mu \in \mathbb{R}$ $\sigma^2 > 0$	$y \in \mathbb{R}$	$p(y\|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\big(-\frac{1}{2}\frac{(y-\mu)^2}{\sigma^2}\big)$	$\mathbb{E}(Y\|\mu,\sigma^2) = \mu$ $\mathrm{Var}(Y\|\mu,\sigma^2) = \sigma^2$
Multivariate Normal	$Y \sim \mathrm{N}(\mu,\Sigma)$	$\mu \in \mathbb{R}^d$ $\Sigma \in \mathbb{R}^{d\times d}$ s.p.d.*	$y \in \mathbb{R}^d$	$p(y\|\mu,\Sigma) = (2\pi)^{-\frac{d}{2}} \det(\Sigma)^{-\frac{1}{2}} \cdot$ $\exp\big(-\frac{1}{2}(y-\mu)^T\Sigma^{-1}(y-\mu)\big)$	$\mathbb{E}(Y\|\mu,\Sigma) = \mu$ $\mathrm{Var}(Y\|\mu,\Sigma) = \Sigma$
Log-Normal	$Y \sim \mathrm{LogN}(\mu,\sigma^2)$	$\mu \in \mathbb{R}$ $\sigma^2 > 0$	$y > 0$	$p(y\|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}y} \exp\big(-\frac{(\log y-\mu)^2}{2\sigma^2}\big)$	$\mathbb{E}(Y\|\mu,\sigma^2) = \exp(\mu + \frac{\sigma^2}{2})$ $\mathrm{Var}(Y\|\mu,\sigma^2) = \exp(2\mu+\sigma^2)(\exp(\sigma^2)-1)$
Noncentral Student	$Y \sim t_\nu(\mu,\sigma)$	$\mu \in \mathbb{R}$ $\sigma^2, \nu > 0$	$y \in \mathbb{R}$	$p(y\|\mu,\sigma^2,\nu) =$ $\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\nu\pi}\sigma}\big(1+\frac{(y-\mu)^2}{\nu\sigma^2}\big)^{-\frac{\nu+1}{2}}$	$\mathbb{E}(Y\|\mu,\sigma^2,\nu) = \mu$ für $\nu > 1$ $\mathrm{Var}(Y\|\mu,\sigma^2,\nu) = \sigma^2\frac{\nu}{\nu-2}$ für $\nu > 2$
Beta	$Y \sim \mathrm{Be}(a,b)$	$a,b > 0$	$y \in [0,1]$	$p(y\|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}y^{a-1}(1-y)^{b-1}$	$\mathbb{E}(Y\|a,b) = \frac{a}{a+b}$ $\mathrm{Var}(Y\|a,b) = \frac{ab}{(a+b)^2(a+b+1)}$ $\mathrm{mod}(Y\|a,b) = \frac{a-1}{a+b-2}$ für $a,b > 1$
Gamma	$Y \sim \mathrm{Ga}(a,b)$	$a,b > 0$	$y > 0$	$p(y\|a,b) = \frac{b^a}{\Gamma(a)}y^{a-1}\exp(-by)$	$\mathbb{E}(Y\|a,b) = \frac{a}{b}$ $\mathrm{Var}(Y\|a,b) = \frac{a}{b^2}$ $\mathrm{mod}(Y\|a,b) = \frac{a-1}{b}$ für $a \geq 1$
Inverse Gamma	$Y \sim \mathrm{IG}(a,b)$	$a,b > 0$	$y > 0$	$p(y\|a,b) = \frac{b^a}{\Gamma(a)}y^{-a-1}\exp\big(-\frac{b}{y}\big)$	$\mathbb{E}(Y\|a,b) = \frac{b}{a-1}$ für $a > 1$ $\mathrm{Var}(Y\|a,b) = \frac{b^2}{(a-1)^2(a-2)}$ für $a > 2$ $\mathrm{mod}(Y\|a,b) = \frac{b}{a+1}$
Dirichlet	$Y \sim \mathrm{Dir}(\alpha_1,\dots,\alpha_k)$	$\alpha_j > 0$ $\alpha_0 \equiv \sum_{j=1}^k \alpha_j$ $\boldsymbol{y} = \{y_1,\dots,y_k\}$	$y \in \Delta_k$ †	$p(\boldsymbol{y}\|\alpha_1,\dots\alpha_k) = \frac{\Gamma(\alpha_1+\ldots+\alpha_k)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_k)}y_1^{\alpha_1-1}\cdots y_k^{\alpha_k-1}$	$\mathbb{E}(Y=y_j\|\alpha_1,\dots\alpha_k) = \frac{\alpha_j}{\alpha_0}$, $\mathrm{mod}(Y=y_j) = \frac{\alpha_j-1}{\alpha_0-k}$ $\mathrm{Var}(Y=y_j\|\alpha_1,\dots\alpha_k) = \frac{\alpha_j(\alpha_0-\alpha_j)}{\alpha_0^2(\alpha_0+1)}$ $\mathrm{Cov}(Y=y_i,Y=y_j\|\alpha_1,\dots\alpha_k) = -\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)}$
Exponential	$Y \sim \mathrm{Exp}(\lambda)$	$\lambda > 0$	$y \geq 0$	$p(y\|\lambda) = \lambda\exp(-\lambda y)$	$\mathbb{E}(Y\|\lambda) = \frac{1}{\lambda}$ $\mathrm{Var}(Y\|\lambda) = \frac{1}{\lambda^2}$

*s.p.d.: symmetric and positive definite

†$(k-1)$-dimensional simplex: $\Delta_k = \{y \in \mathbb{R}^k \| \sum_{j=1}^k y_j = 1, y_j \geq 0$ for $j = 1,2,\dots,k\}$

Discrete distributions

Name	Symbol	Parameter	Definition range	PMF	Mean, variance, mode
Discrete Uniform	$Y \sim \mathrm{U}(\{y_1, ..., y_k\})$	-	$y \in \{y_1, ..., y_k\}$	$\mathbb{P}(Y = y_i) = \frac{1}{k},\ i = 1, \ldots, k$	- For $y_i = i$: $\mathbb{E}(Y) = \frac{k+1}{2}, \mathrm{Var}(Y) = \frac{k^2-1}{12}$
Binomial	$Y \sim \mathrm{Bin}(n, \pi)$	$n \in \mathbb{N}$ $\pi \in [0, 1]$	$y \in \{0, \ldots, n\}$	$\mathbb{P}(Y = y\|\pi) = \binom{n}{y}\pi^y(1-\pi)^{n-y}$	$\mathbb{E}(Y\|\pi) = n\pi$ $\mathrm{Var}(Y\|\pi) = n\pi(1-\pi)$
Poisson	$Y \sim \mathrm{Poi}(\mu)$	$\mu > 0$	$y \in \mathbb{N}_0$	$\mathbb{P}(Y = y\|\mu) = \frac{\mu^y}{y!}\exp(-\mu)$	$\mathbb{E}(Y\|\mu) = \mu$ $\mathrm{Var}(Y\|\mu) = \mu$
Multinomial	$Y \sim \mathrm{Multin}(n, p_1, \ldots, p_k)$	$n \in \mathbb{N}$ $p_j \in [0, 1]$ $\sum_{j=1}^k p_j = 1$	$y \in \mathbb{N}_0$ $\sum_{j=1}^k y_j = n$	$\mathbb{P}(Y = y\|n, p_1, \ldots, p_k) = \frac{n!}{y_1! \cdots y_k!} p_1^{y_1} \cdots p_k^{y_k}$	$\mathbb{E}(Y = y_j\|n, p_1, \ldots, p_k) = np_j$ $\mathrm{Var}(Y = y_j\|n, p_1, \ldots, p_k) = np_j(1-p_j)$ $\mathrm{Cov}(Y = y_i, Y = y_j) = -np_ip_j$
Geometric	$Y \sim \mathrm{Geom}(\pi)$	$\pi \in [0, 1]$	$y \in \mathbb{N}_0$	$\mathbb{P}(Y = y\|\pi) = \pi(1-\pi)^{y-1}$	$\mathbb{E}(Y\|\pi) = \frac{1}{\pi}$ $\mathrm{Var}(Y\|\pi) = \frac{1-\pi}{\pi^2}$
Negative Binomial	$Y \sim \mathrm{NegBin}(\alpha, \beta)$	$\alpha, \beta > 0$	$y \in \mathbb{N}_0$	$\mathbb{P}(Y = y\|\alpha, \beta) = \binom{\alpha+y-1}{\alpha-1}\left(\frac{\beta}{\beta+1}\right)^\alpha\left(\frac{1}{\beta+1}\right)^y$	$\mathbb{E}(Y\|\alpha, \beta) = \frac{\alpha}{\beta}$ $\mathrm{Var}(Y\|\alpha, \beta) = \frac{\alpha}{\beta^2}(\beta+1)$

Relationships between different distributions

- $\log(Y) \sim N(\mu, \sigma^2) \Rightarrow Y \sim \mathrm{LogN}(\mu, \sigma^2)$
- $Y|\theta \sim \mathrm{N}(\mu, \frac{\sigma^2}{\theta}),\ \theta \sim \mathrm{Ga}(\frac{\nu}{2}, \frac{\nu}{2}) \Rightarrow Y \sim t_\nu(\mu, \sigma^2)$.
- $Y \sim \mathrm{IG}(a, b) \Leftrightarrow Y^{-1} \sim \mathrm{Ga}(a, b^{-1})$