Linear

April 23, 2019

1 Question 3

(a) Load the data regTrain.txt and use least squares regression to fit functions fk with the maximal frequency k = 1, 3, 5, . . . , 17. Plot the 9 resulting functions together with the training data into a single figure. Explain the effect of the increasing k on the fitting function fk

```
In [32]: import numpy as np
         from math import pi
         import matplotlib.pyplot as plt
         import numpy.linalg as lg
```

```
In [196]: train_data= np.loadtxt('regTrain.txt')
          test_data=np.loadtxt('regTest.txt')
          X=train_data[:,0]
          Y=train_data[:,1]
```

```
In [197]: def get_fi_Fourier(X,k):
              # print(X)
              lenX=len(X)
              fi=np.ones((lenX,1))
              for l in range(1,k+1):
                  tmp=np.cos(2*pi*l*X)/l
                  fi=np.append(fi,tmp.reshape(lenX,1),axis=1)
                  tmp=np.sin(2*pi*l*X)/l
                  fi=np.append(fi,tmp.reshape(lenX,1),axis=1)
              return fi
```

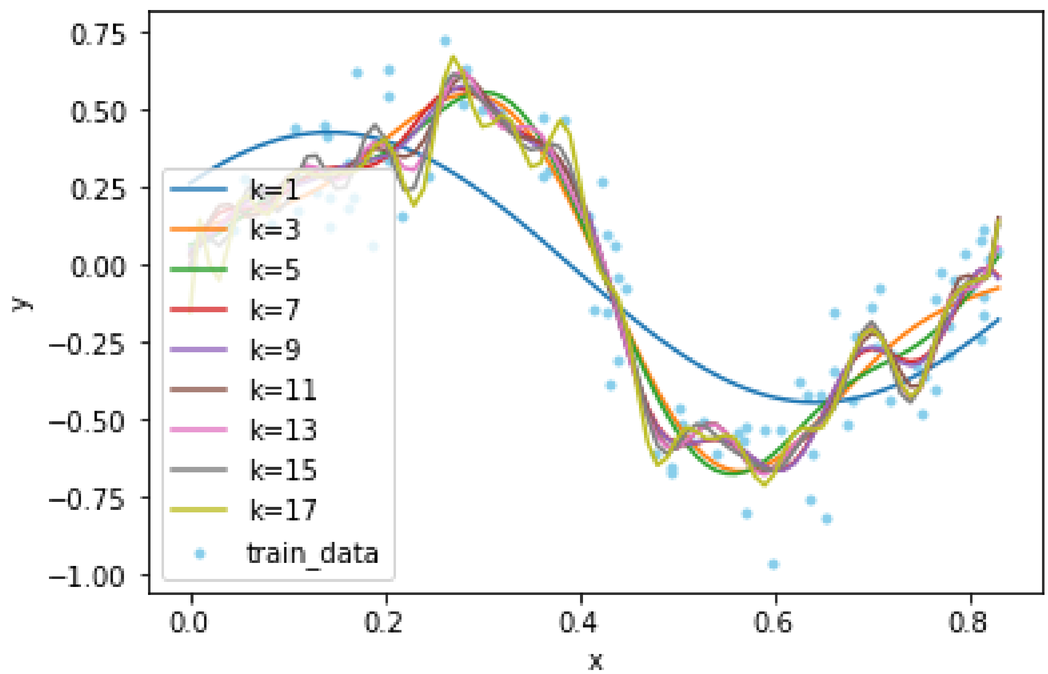
```
In [198]: def get_W(fi,Y):
              w=lg.inv(fi.T.dot(fi)).dot(fi.T).dot(Y)
              return w
```

```
In [199]: def get_W_regular(fi,Y,labmda):
              lenI=len(fi[0])
              w=lg.inv(fi.T.dot(fi)+np.eye(lenI)*labmda).dot(fi.T).dot(Y)
              return w
```

```
In [200]: def get_predict(X,w,k):
              predict_y=0
```

```
        fi=get_fi_Fourier(X,k)
#       print(fi)
        predict_y=fi.dot(w)
        return predict_y
```

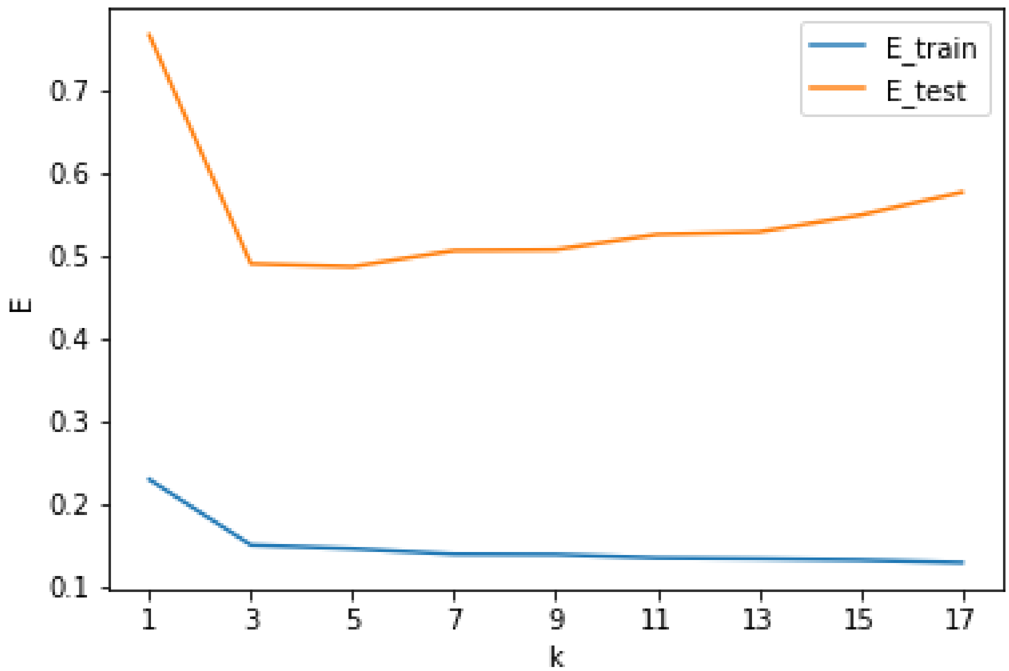
```
In [201]: X_test=np.arange(0,0.84,0.01)
          # print(X_test)
          for k in range(1,18,2):
              fi=get_fi_Fourier(X,k)
          #     print("fi1=",fi[1,:])
              w=get_W(fi,Y)
          #     print(w)
              predict_y=get_predict(X_test,w,k)
              plt.plot(X_test,predict_y,label="k=%s"%k)

          plt.scatter(X,Y,label="train_data",marker='.',c="#87CEEB")
          plt.xlabel("x")
          plt.ylabel("y")
          plt.legend()
          plt.show()
```



From the figure we can see that increasing k makes the model less smooth.

(b) For each k, compute the root mean square error ERMS = (2E(w)/N) on both, training and test data, as a function of maximum frequency k. Describe what you see and try to explain your observation in your own words.

In [203]:
```
E_train=np.zeros(9)
E_test=np.zeros(9)
for k in range(1,18,2):
    fi=get_fi_Fourier(X,k)
    w=get_W(fi,Y)
    predict_y=get_predict(train_data[:,0],w,k)
    E=np.sqrt(np.sum(np.square(predict_y-train_data[:,1]))/N)
    E_train[k//2]=E

    predict_y=get_predict(test_data[:,0],w,k)
    E=np.sqrt(np.sum(np.square(predict_y-test_data[:,1]))/N)
    E_test[k//2]=E

ks=np.arange(1,18,2)
# print(ks)
# print(E_train)
# print(E_test)
plt.plot(ks,E_train,label="E_train")
plt.plot(ks,E_test,label="E_test")
plt.xlabel("k")
plt.ylabel("E")
plt.xticks(ks)
plt.legend()
plt.show()
```



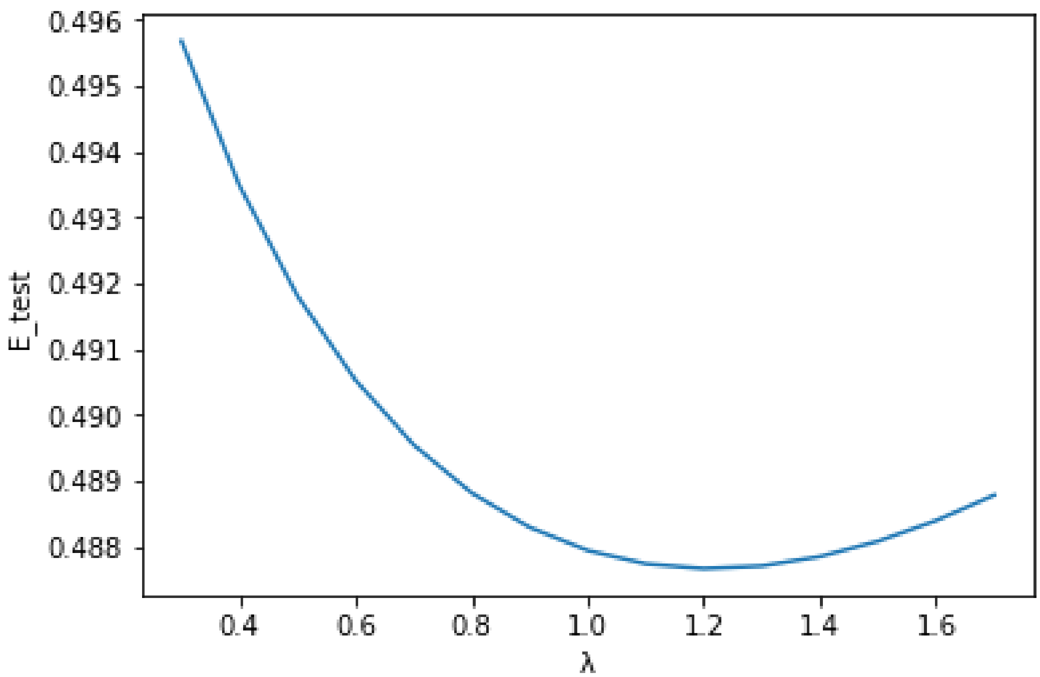
We can see that the error falls steeply from k=1 to k=3, because when k=1, the model doesn't have enough parameter. For k>3, the error of train data decreases while the error of test data increases. That's because bigger k brings too many parameters, which causes overfitting.

(c) Extend your code to incorporate a prior on the weights of the regression function to perform Ridge Regression. Repeat the above tasks with various regularization weights . Which value gives good results? Discuss the difference in terms of the fitting functions and the training and test errors.

```
In [326]: def ridge_regression(labmda):
              X_test=np.arange(0,0.85,0.01)
              # print(X_test)
              for k in range(1,18,2):
                  fi=get_fi_Fourier(X,k)
                  w=get_W_regular(fi,Y,labmda)
                  predict_y=get_predict(X_test,w,k)
          #         plt.plot(X_test,predict_y,label="k=%s"%k)

          #     plt.scatter(X,Y,label="train_data",marker='.',c="#87CEEB")
          #     plt.title("=%s"%labmda)
          #     plt.xlabel("x")
          #     plt.ylabel("y")
          #     plt.legend()
          #     plt.show()
              E_train=np.zeros(9)
              E_test=np.zeros(9)
              for k in range(1,18,2):
                  fi=get_fi_Fourier(X,k)
                  w=get_W_regular(fi,Y,labmda)
                  predict_y=get_predict(train_data[:,0],w,k)
                  E=np.sqrt(np.sum(np.square(predict_y-train_data[:,1]))/N)
                  E_train[k//2]=E

                  predict_y=get_predict(test_data[:,0],w,k)
                  E=np.sqrt(np.sum(np.square(predict_y-test_data[:,1]))/N)
                  E_test[k//2]=E

          #         E_train[k//2]=get_E_regular(train_data[:,0],train_data[:,1],k,labmda)
          #         E_test[k//2]=get_E_regular(test_data[:,0],test_data[:,1],k,labmda)

          #     ks=np.arange(1,18,2)

          #     plt.plot(ks,E_train,label="E_train")
          #     plt.plot(ks,E_test,label="E_test")
          #     plt.xlabel("k")
          #     plt.ylabel("E")
          #     plt.xticks(ks)
          #     plt.title("=%s"%labmda)
```

```
#     plt.legend()
#     plt.show()
#     print(E_test)
    return E_test[5]
```

In [339]:
```
labms=np.arange(0.3,1.8,0.1)
Nd=len(labms)
E_test=np.zeros(Nd)
for i in range(Nd):
    E_test[i]=ridge_regression(labms[i])

plt.plot(labms,E_test)
plt.xlabel("")
plt.ylabel("E_test")
plt.show()
```



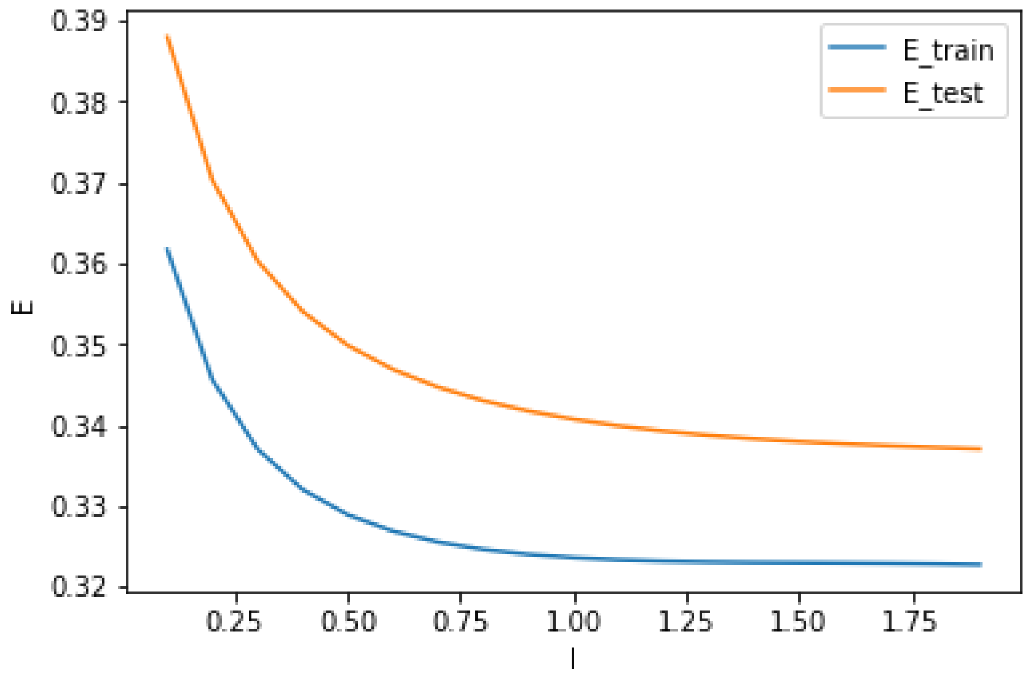
=1.2 gives the best result.
Compared with original model, small makes little difference. But too large leads to underfitting.

2 Question 4

(a) In the lecture it has been discussed that Ridge Regression can be combined with kernels. Extend the regression for the previous task such that it makes use of a kernel. For a first experiment use the squared exponential kernel:

```
In [176]: def kx_exp(X,newx,l):
              N=len(X)
              kx=np.zeros(N)
              for i in range(N):
                  kx[i]=np.exp(-np.square(X[i]-newx)/np.square(l))
              return kx

In [332]: def kernel_exp(l=0.1,labmda=1):
              ################train###############
              N=len(X)
              K=np.zeros((N,N))
              for i in range(N):
                  for j in range(N):
                      K[i,j]=np.exp(-np.square(X[i]-X[j])/np.square(l))
              # print(K)
              a=lg.inv(K+np.eye(N)*labmda).dot(Y)
              # print(a)
              #################test################
              Y_predict=np.zeros(N)
              for i in range(N):
                  Y_predict[i]=kx_exp(X,train_data[i,0],l).dot(a)
              E_train_exp=np.sqrt(np.sum(np.square(Y_predict-train_data[:,1]))/N)

              X_test=test_data[:,0]
              N=len(test_data)
              Y_predict=np.zeros(N)
              kk=np.zeros(N)
              for i in range(N):
                  Y_predict[i]=kx_exp(X,test_data[i,0],d).dot(a)
              E_test_exp=np.sqrt(np.sum(np.square(Y_predict-test_data[:,1]))/N)

          #     plt.scatter(X,Y)
          #     plt.plot(X_test,Y_predict)
          #     plt.show()
          #     E_test_exp=np.sqrt(np.sum(np.square(Y_predict-test_data[:,1]))/N)
              print(E_test_exp)
              return (E_train_exp,E_test_exp)
              # for i in range(len(Y_predict)):
              #     print(Y_predict[i],'---',test_data[:,1][i])

In [343]: ls=np.arange(0.1,2,0.1)
          Nd=len(ls)
          E_train_exp=np.zeros(Nd)
          E_test_exp=np.zeros(Nd)

          for i in range(0,Nd):
              (E_train_exp[i],E_test_exp[i])=kernel_poly(ls[i],labmda=1.2)
```

```
plt.plot(ls,E_train_exp,label="E_train")
plt.plot(ls,E_test_exp,label="E_test")
plt.legend()
plt.xlabel("l")
plt.ylabel("E")
# print(E_train_exp)
plt.show()
```



(b) Exchange the kernel for the polynomial kernel:

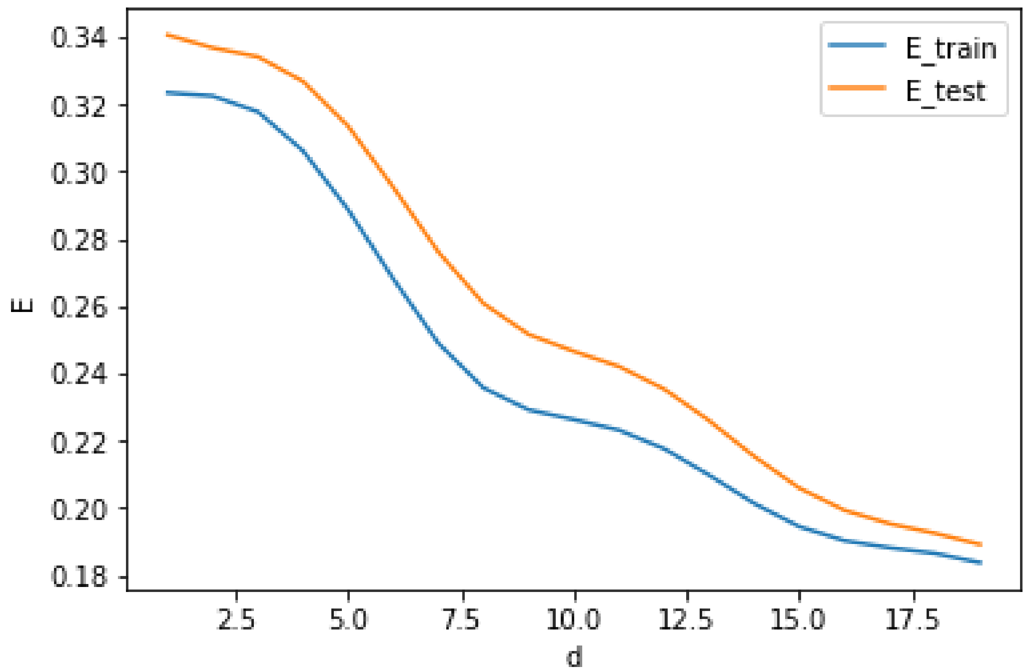
k(x, y) = (x y + 1)d
What is the effect of the parameter d?

```
In [264]: def kx_poly(X,newx,d):
              N=len(X)
              kx=np.zeros(N)
              for i in range(N):
                  kx[i]=(X[i]*newx+1)**d
              return kx
```

```
In [310]: def kernel_poly(d=10,labmda=1):
          ###############train###############
              N=len(X)
              K=np.zeros((N,N))
              #default l=1
```

```
    for i in range(N):
        for j in range(N):
            K[i,j]=(X[i]*X[j]+1)**d
    # print(K)
    a=lg.inv(K+np.eye(N)*labmda).dot(Y)
    # print(a)
    ##################test################
    Y_predict=np.zeros(N)
    for i in range(N):
        Y_predict[i]=kx_poly(X,train_data[i,0],d).dot(a)
    E_train_exp=np.sqrt(np.sum(np.square(Y_predict-train_data[:,1]))/N)

    X_test=test_data[:,0]
    N=len(test_data)
    Y_predict=np.zeros(N)
    kk=np.zeros(N)
    for i in range(N):
        Y_predict[i]=kx_poly(X,test_data[i,0],d).dot(a)
    E_test_exp=np.sqrt(np.sum(np.square(Y_predict-test_data[:,1]))/N)
#     plt.scatter(X,Y)
#     plt.plot(X_test,Y_predict)
#     plt.show()
#     print(E_test_exp)
#     print((E_train_exp,E_test_exp))
    return (E_train_exp,E_test_exp)
# for i in range(len(Y_predict)):
#     print(Y_predict[i],'---',test_data[:,1][i])
```

In [342]:
```
ds=np.arange(1,20)
Nd=len(ds)
E_train_exp=np.zeros(Nd)
E_test_exp=np.zeros(Nd)

for i in range(0,Nd):
    (E_train_exp[i],E_test_exp[i])=kernel_poly(ds[i],labmda=1.2)

plt.plot(ds,E_train_exp,label="E_train")
plt.plot(ds,E_test_exp,label="E_test")
plt.legend()
plt.xlabel("d")
plt.ylabel("E")
# print(E_train_exp)
plt.show()
```


0.34
0.32
0.30
0.28
0.26
0.24
0.22
0.20
0.18
E
2.5
5.0
7.5
10.0
12.5
15.0
17.5
d
E_train
E_test

```
In [ ]:
```