

CSC 474 Assignment 1

Nik Rados V00801209 & Max Kasprzik V00722495

QUESTION 1

Class

Class	Yes	No	Total
1st	203	122	325
2nd	118	167	285
3rd	178	528	706
Crew	212	673	885

1st

$$\begin{aligned} \text{info}([203, 122]) &= \text{entropy}\left(\frac{203}{325}, \frac{122}{325}\right) = \\ &= - \\ \text{frac}203325 \cdot \log\left(\frac{203}{325}\right) - \frac{122}{325} \cdot \log\left(\frac{122}{325}\right) &\approx 0.95472 \end{aligned}$$

2nd

$$\begin{aligned} \text{info}([118, 167]) &= \text{entropy}\left(\frac{118}{285}, \frac{167}{285}\right) = \\ &= -\frac{118}{285} \cdot \log\left(\frac{118}{285}\right) - \frac{167}{285} \cdot \log\left(\frac{167}{285}\right) \approx 0.97858 \end{aligned}$$

3rd

$$\begin{aligned} \text{info}([178, 528]) &= \text{entropy}\left(\frac{178}{706}, \frac{528}{706}\right) = \\ &= -\frac{178}{706} \cdot \log\left(\frac{178}{706}\right) - \frac{528}{706} \cdot \log\left(\frac{528}{706}\right) \approx 0.81463 \end{aligned}$$

Crew

$$\begin{aligned} \text{info}([212, 673]) &= \text{entropy}\left(\frac{212}{885}, \frac{673}{885}\right) = \\ &= -\frac{212}{885} \cdot \log\left(\frac{212}{885}\right) - \frac{673}{885} \cdot \log\left(\frac{673}{885}\right) \approx 0.79429 \end{aligned}$$

Average Weighted Entropy (Class)

$$\begin{aligned} &\text{info}([203, 122], [118, 167], [178, 528], [212, 673]) = \\ &= 0.95472 \cdot \left(\frac{325}{2201}\right) + 0.97858 \cdot \left(\frac{285}{2201}\right) + 0.81463 \cdot \left(\frac{706}{2201}\right) + 0.79429 \cdot \left(\frac{885}{2201}\right) \approx \end{aligned}$$

$$\approx 0.84837$$

Age

Age	Yes	No	Total
Adult	654	1438	2092
Child	57	52	109

Adult

$$\begin{aligned} \text{info}([654, 1438]) &= \text{entropy}\left(\frac{654}{2092}, \frac{1438}{2092}\right) = \\ &= -\frac{654}{2092} \cdot \log\left(\frac{654}{2092}\right) - \frac{1438}{2092} \cdot \log\left(\frac{1438}{2092}\right) \approx 0.89617 \end{aligned}$$

Child

$$\begin{aligned} \text{info}([57, 52]) &= \text{entropy}\left(\frac{57}{109}, \frac{52}{109}\right) = \\ &= -\frac{57}{109} \cdot \log\left(\frac{57}{109}\right) - \frac{52}{109} \cdot \log\left(\frac{52}{109}\right) \approx 0.79429 \end{aligned}$$

Average Weighted Entropy (Age)

$$\begin{aligned} \text{info}([654, 1438], [57, 52]) &= \\ &= 0.89617 \cdot \left(\frac{654}{2201}\right) + 0.79429 \cdot \left(\frac{1438}{2201}\right) \approx \\ &\approx 0.90124 \end{aligned}$$

Sex

Sex	Yes	No	Total
Male	367	1364	1731
Female	344	126	470

Male

$$\begin{aligned} \text{info}([367, 1364]) &= \text{entropy}\left(\frac{367}{1731}, \frac{1364}{1731}\right) = \\ &= -\frac{367}{1731} \cdot \log\left(\frac{367}{1731}\right) - \frac{1364}{1731} \cdot \log\left(\frac{1364}{1731}\right) \approx 0.74532 \end{aligned}$$

Female

$$\begin{aligned} \text{info}([344, 126]) &= \text{entropy}\left(\frac{344}{470}, \frac{126}{470}\right) = \\ &= -\frac{344}{470} \cdot \log\left(\frac{344}{470}\right) - \frac{126}{470} \cdot \log\left(\frac{126}{470}\right) \approx 0.83870 \end{aligned}$$

Average Weighted Entropy (Sex)

$$\begin{aligned}
info([367, 1364], [344, 126]) &= \\
&= 0.74532 \cdot \left(\frac{1731}{2201}\right) + 0.83870 \cdot \left(\frac{470}{2201}\right) \approx \\
&\approx 0.76526
\end{aligned}$$

Lowest Entropy = Sex = Root

Sex --> Male --> Class

Male - 1st Class

$$\begin{aligned}
info([62, 118]) &= entropy\left(\frac{62}{180}, \frac{118}{180}\right) = \\
&= -\frac{62}{180} \cdot \log\left(\frac{62}{180}\right) - \frac{118}{180} \cdot \log\left(\frac{118}{180}\right) \approx 0.92901
\end{aligned}$$

Male - 2nd Class

$$\begin{aligned}
info([25, 154]) &= entropy\left(\frac{25}{179}, \frac{154}{179}\right) = \\
&= -\frac{25}{179} \cdot \log\left(\frac{25}{179}\right) - \frac{154}{179} \cdot \log\left(\frac{154}{179}\right) \approx 0.76513
\end{aligned}$$

Male - 3rd Class

$$\begin{aligned}
info([88, 422]) &= entropy\left(\frac{88}{510}, \frac{422}{510}\right) = \\
&= -\frac{88}{510} \cdot \log\left(\frac{88}{510}\right) - \frac{422}{510} \cdot \log\left(\frac{422}{510}\right) \approx 0.66350
\end{aligned}$$

Male - Crew

$$\begin{aligned}
info([192, 670]) &= entropy\left(\frac{192}{862}, \frac{670}{862}\right) = \\
&= -\frac{192}{862} \cdot \log\left(\frac{192}{862}\right) - \frac{670}{862} \cdot \log\left(\frac{670}{862}\right) \approx 0.76513
\end{aligned}$$

Average Weighted Entropy (Male --> Class)

$$\begin{aligned}
info([62, 118], [25, 154], [88, 422], [192, 670]) &= \\
&= 0.92901 \cdot \left(\frac{180}{1731}\right) + 0.58336 \cdot \left(\frac{179}{1731}\right) + 0.66350 \cdot \left(\frac{510}{1731}\right) + 0.76513 \cdot \left(\frac{862}{1731}\right) \approx \\
&\approx 0.73349
\end{aligned}$$

Sex --> Male --> Age

Male - Adult

$$\begin{aligned}
info([338, 1329]) &= entropy\left(\frac{338}{1667}, \frac{1329}{1667}\right) = \\
&= -\frac{338}{1667} \cdot \log\left(\frac{338}{1667}\right) - \frac{1329}{1667} \cdot \log\left(\frac{1329}{1667}\right) \approx 0.72741
\end{aligned}$$

Male - Child

$$\begin{aligned} \text{info}([29, 35]) &= \text{entropy}\left(\frac{29}{64}, \frac{35}{64}\right) = \\ &= -\frac{29}{64} \cdot \log\left(\frac{29}{64}\right) - \frac{35}{64} \cdot \log\left(\frac{35}{64}\right) \approx 0.99365 \end{aligned}$$

Average Weighted Entropy (Male --> Age)

$$\begin{aligned} \text{info}([338, 1329], [29, 35]) &= \\ &= 0.72741 \cdot \left(\frac{1667}{1731}\right) + 0.99365 \cdot \left(\frac{64}{1731}\right) \approx \\ &\approx 0.73725 \end{aligned}$$

Lowest Entropy = Male --> Class

Sex --> Female --> Class

Female - 1st Class

$$\begin{aligned} \text{info}([141, 4]) &= \text{entropy}\left(\frac{141}{145}, \frac{4}{145}\right) = \\ &= -\frac{141}{145} \cdot \log\left(\frac{141}{145}\right) - \frac{4}{145} \cdot \log\left(\frac{4}{145}\right) \approx 0.18214 \end{aligned}$$

Female - 2nd Class

$$\begin{aligned} \text{info}([93, 13]) &= \text{entropy}\left(\frac{93}{106}, \frac{13}{106}\right) = \\ &= -\frac{93}{106} \cdot \log\left(\frac{93}{106}\right) - \frac{13}{106} \cdot \log\left(\frac{13}{106}\right) \approx 0.53691 \end{aligned}$$

Female - 3rd Class

$$\begin{aligned} \text{info}([90, 106]) &= \text{entropy}\left(\frac{90}{196}, \frac{106}{196}\right) = \\ &= -\frac{90}{196} \cdot \log\left(\frac{90}{196}\right) - \frac{106}{196} \cdot \log\left(\frac{106}{196}\right) \approx 0.99519 \end{aligned}$$

Female - Crew

$$\begin{aligned} \text{info}([20, 3]) &= \text{entropy}\left(\frac{20}{23}, \frac{3}{23}\right) = \\ &= -\frac{20}{23} \cdot \log\left(\frac{20}{23}\right) - \frac{3}{23} \cdot \log\left(\frac{3}{23}\right) \approx 0.55863 \end{aligned}$$

Average Weighted Entropy (Female --> Class)

$$\begin{aligned} \text{info}([141, 4], [93, 13], [90, 106], [20, 3]) &= \\ &= 0.18214 \cdot \left(\frac{145}{470}\right) + 0.53691 \cdot \left(\frac{106}{470}\right) + 0.99519 \cdot \left(\frac{196}{470}\right) + 0.55863 \cdot \left(\frac{23}{470}\right) \approx \\ &\approx 0.61964 \end{aligned}$$

Sex --> Female --> Age --> test

Female - Adult

$$\begin{aligned} info([316, 109]) &= entropy(\frac{316}{425}, \frac{109}{425}) = \\ &= -\frac{316}{425} \cdot \log(\frac{316}{425}) - \frac{109}{425} \cdot \log(\frac{109}{425}) \approx 0.82137 \end{aligned}$$

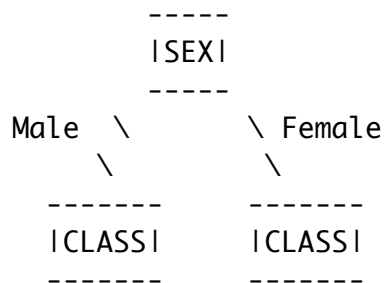
Female - Child

$$\begin{aligned} info([28, 17]) &= entropy(\frac{28}{45}, \frac{17}{45}) = \\ &= -\frac{28}{45} \cdot \log(\frac{28}{45}) - \frac{17}{45} \cdot \log(\frac{17}{45}) \approx 0.95646 \end{aligned}$$

Average Weighted Entropy (Female --> Age)

$$\begin{aligned} info([316, 109], [28, 17]) &= \\ &= 0.82137 \cdot (\frac{425}{470}) + 0.95646 \cdot (\frac{45}{470}) \approx \\ &\approx 0.83430 \end{aligned}$$

Lowest Entropy = Female --> Class



QUESTION 2

We are looking for **If ? then play = yes** where:

$$\frac{P}{t} = 1$$

Rule 1

Dataset

Classes	Attribute	Play = Yes / Total
Outlook	Overcast	4/4
	Sunny	2/5
	Rainy	3/5
Temperature	Hot	2/4
	Mild	4/6

Classes	Attribute	Play = Yes / Total
	Cool	3/4
Humidity	High	3/7
	Normal	6/7
Windy	True	3/6
	False	6/8

Rule 1 => If outlook = overcast then play = yes

Rule 2

Dataset

Classes	Attribute	Play = Yes / Total
Outlook	Sunny	2/5
	Rainy	3/5
Temperature	Hot	0/2
	Mild	3/5
	Cool	2/3
Humidity	High	1/5
	Normal	4/5
Windy	True	1/4
	False	4/6

So far we have, **If humidity = normal and ? then play = yes**

Narrow Dataset to Humidity = Normal

Classes	Attribute	Play = Yes / Total
Outlook	Sunny	2/2
	Rainy	2/3
Temperature	Hot	1/1
	Mild	2/2
	Cool	3/4
Windy	True	2/3

Classes	Attribute	Play = Yes / Total
	False	4/4

Rule 2 = If humidity = normal and windy = false then play = yes

QUESTION 3

Evidence = E = 2nd, child, male

$$\begin{aligned}
 P(\text{survive} = \text{yes} | E) &= \frac{(P(2nd | \text{yes}) \cdot P(\text{child} | \text{yes}) \cdot P(\text{male} | \text{yes}) \cdot P(\text{yes}))}{P(E)} \\
 &= \frac{(\frac{118}{711} \cdot \frac{57}{711} \cdot \frac{367}{711} \cdot \frac{711}{2201})}{\frac{P(E)}{0.0022}} \\
 &= \frac{0.0022}{P(E)}
 \end{aligned}$$

$$\begin{aligned}
 P(\text{survive} = \text{no} | E) &= \frac{(P(2nd | \text{no}) \cdot P(\text{child} | \text{no}) \cdot P(\text{male} | \text{no}) \cdot P(\text{no}))}{P(E)} \\
 &= \frac{(\frac{167}{1490} \cdot \frac{52}{1490} \cdot \frac{1364}{1490} \cdot \frac{1490}{2201})}{\frac{P(E)}{0.0024}} \\
 &= \frac{0.0024}{P(E)}
 \end{aligned}$$

Normalization Constant

$$\begin{aligned}
 P(\text{survive} = \text{yes} | E) + P(\text{survive} = \text{no} | E) &= 1 \\
 &= \frac{0.0022}{P(E)} + \frac{0.0024}{P(E)} = 1 \\
 P(E) &= (0.0022 + 0.0024)
 \end{aligned}$$

Therefore,

$$P(\text{survive} = \text{yes} | E) = \frac{0.0022}{(0.0022 + 0.0024)} = 0.478 = 47.8\%$$

$$P(\text{survive} = \text{no} | E) = \frac{0.0024}{(0.0022 + 0.0024)} = 0.522 = 52.2\%$$