1/A

$$\frac{\partial E}{\partial W_2} = \underbrace{\frac{\partial E}{dY_2}\frac{dY_2}{dZ_2}}_{\delta_2}\underbrace{\frac{dZ_2}{dW_2}}_{Y_1} = \underbrace{(Y_2-\hat{Y})\circ\sigma'(Z_2)}_{\delta_2}\circ Y_1 = \text{ANALOGICZNIE}$$

$$= \delta_2 \circ Y_1$$

$$\frac{\partial E}{\partial W_1} = \underbrace{\frac{dE}{dY_2}\frac{dY_2}{dZ_2}}_{\delta_2}\underbrace{\frac{dZ_2}{dY_1}}_{W_2}\underbrace{\frac{dY_1}{dZ_1}}_{\sigma'(Z_1)}\underbrace{\frac{dZ_1}{dW_1}}_{Y_0} = \underbrace{(\delta_2 W_2^T)\circ\sigma'(Z_1)}_{\delta_1}\circ Y_0 =$$

$$= \delta_1 \circ Y_0$$

$$\frac{\partial E}{\partial B_2} = \underbrace{\frac{\partial E}{dY_2}\frac{dY_2}{dZ_2}}_{\delta_2}\underbrace{\frac{dZ_2}{db_2}}_{1} = \delta_2$$

$$\frac{\partial E}{\partial B_1} = \underbrace{\frac{dE}{dY_2}\frac{dY_2}{dZ_2}\frac{dZ_2}{dY_1}\frac{dY_1}{dZ_1}}_{\delta_1}\underbrace{\frac{dZ_1}{db_1}}_{1} = \delta_1$$

$$E = \frac{1}{2}(Y_2-\hat{Y})^2 \quad \left|\quad \frac{dE}{dY_2} = Y_2-\hat{Y}\right.$$

$$Y_i = \sigma(Z_i) = \begin{cases} 0, \text{ if } Z_i<0 \\ x_i \text{ else} \end{cases} \quad \left|\quad \frac{dY_i}{dZ_i} = \sigma'(Z_i)\right.$$

$$\sigma'(Z_i) = \begin{cases} 0, \text{ if } Z_i<0 \\ 1, \text{ else} \end{cases}$$

$$Z_i = Y_{i-1}W_i + B_i \quad \left|\quad \frac{dZ_i}{dW_i} = Y_{i-1}\right.$$

$$\left|\quad \frac{dZ_i}{dY_{i-1}} = W_i\right.$$

$$\left|\quad \frac{dZ_i}{db_i} = 1\right.$$

$$\delta_{MAX} = (Y_{MAX}-\hat{Y})\circ\sigma'(Z_{MAX})$$

$$\delta_i = (\delta_{i+1}W_{i+1}^T)\circ\sigma'(Z_i)$$

$$\frac{\partial E}{\partial W_i} = \delta_i \circ Y_{i-1}, \quad \frac{\partial E}{\partial B_i} = \delta_i$$

2/A (TRUCHAS BEZ SENSU, BO I TAK JUŻ WYPROWADZIŁEM WEKTOROWO)

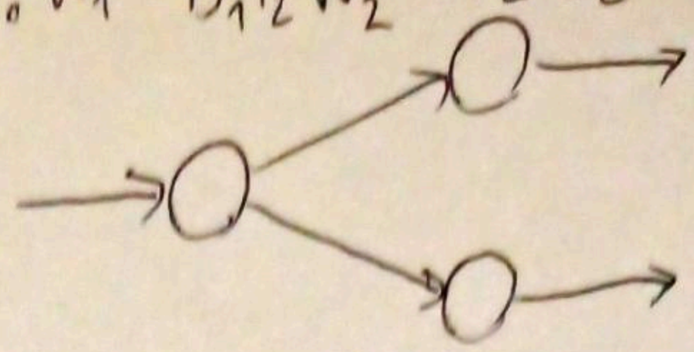
$$\frac{\partial E}{\partial w_{11}^{(2)}} = \frac{dE}{dy_1^{(2)}}\frac{dy_1^{(2)}}{dz_1^{(2)}}\frac{dz_1^{(2)}}{dw_1^{(2)}} = \delta_1^{(2)} y^{(1)} \quad \text{i} \quad \frac{\partial E}{\partial w_{12}^{2}} = \frac{dE}{dy_2^2}\frac{dy_2^2}{dz_2^2}\frac{dz_2^2}{dw_2^2} = \delta_2^{(2)} y^{(1)}$$

$$\frac{\partial E}{\partial w_{11}^{(1)}} = \frac{dE}{dy_1^2}\frac{dy_1^2}{dz_1^2}\frac{dz_1^2}{dy^1}\frac{dy^1}{dz^1}\frac{dz^1}{dw^1} + \frac{dE}{dy_2^2}\frac{dy_2^2}{dz_2^2}\frac{dz_2^2}{dy^1}\frac{dy^1}{dz^1}\frac{dz^1}{dw_{11}^1} = \underbrace{(\delta_1^{(2)}w_1^{(2)} + \delta_2^{(2)}w_2^{(2)})}_{\delta^{(2)}W^{(2)T}}\sigma'(z^{(1)})\,y^{(0)} = \delta^{(1)}y^{(0)}$$

$$\frac{\partial E}{\partial b^{(1)}} = \text{I.O. SAMO, TYLKO ZAMIAST } dw^1 \to db^1 = \delta^{(1)}.$$

2/B

W NATACJI: WEKTOROWO :) $Y_0 W_1 \; B_1 Y_2 W_2 \; B_2 Y_2$



FORWARD

$Y_0 = \begin{bmatrix} 0 \\ 4 \end{bmatrix} \quad \hat{Y} = \begin{pmatrix} 0 & 1 \\ 2 & 5 \end{pmatrix}$

$Z_1 = Y_0 W_1 + B_1 = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \circ (0,2) + (0,5) = \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix}$

$Y_1 = \sigma(Z_1) = \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix}$

$Z_2 = Y_1 W_2 + B_2 = \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix} \circ (0,5 \;\; 0) + (-0,5 \;\; 0,5) = \begin{pmatrix} -0,25 & 0,5 \\ 0,15 & 0,5 \end{pmatrix}$

$Y_2 = \sigma(Z_2) = \begin{pmatrix} 0 & 0,5 \\ 0,15 & 0,5 \end{pmatrix}$

BACKWARD

REDUKCJA - ŚREDNIA PO KOLUMNACH

$\delta_2 = (Y_2 - \hat{Y}) \circ \sigma'(Z_2) = \begin{pmatrix} 0 & -0,5 \\ -1,85 & -4,5 \end{pmatrix} \circ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -0,5 \\ -1,85 & -4,5 \end{pmatrix}$ $\quad W_1^{(2)} \;\; W_2^{(2)}$

$W_2' = W_2 - \mu \, Y_1 \circ \delta_2 = (0,5 \;\; 0) - 0,1 \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix} \begin{pmatrix} 0 & -0,5 \\ -1,85 & -4,5 \end{pmatrix} = (0,162 \;\; 0,305)$ $\quad b_1^{(2)} \;\; b_2^{(2)}$

$B_2' = B_2 - \mu \bar{\delta}_2 = (-0,5 \;\; 0,5) - 0,1 \begin{pmatrix} 0 & -0,5 \\ -1,85 & -4,5 \end{pmatrix} = (-0,4075 \;\; 0,75)$

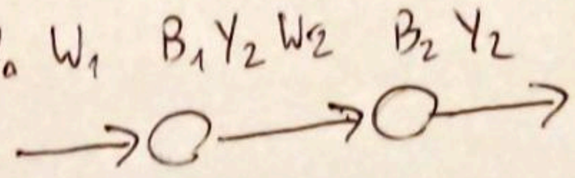
$\delta_1 = (\delta_2 W_2^T) \circ \sigma'(Z_1) = \left(\begin{pmatrix} 0 & -0,5 \\ -1,85 & -4,5 \end{pmatrix} (0,5 \;\; 0)^T \right) \circ \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -0,925 \end{pmatrix}$

$W_1' = W_1 - \mu \, Y_0 \circ \delta_1 = (0,2) - 0,1 \begin{pmatrix} 0 \\ 4 \end{pmatrix} \circ \begin{pmatrix} 0 \\ -0,925 \end{pmatrix} = (0,385)$

$B_1' = B_1 - \mu \bar{\delta}_1 = (0,5) - 0,1 \begin{pmatrix} 0 \\ -0,925 \end{pmatrix} = (0,54625)$

1/B

$Y_0 \; W_1 \; B_1 Y_2 W_2 \; B_2 Y_2$



(NIE POWTARZAM JUŻ TU WZORÓW Z ZADANIA 2/B)

FORWARD

$Y_0 = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad \hat{Y} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$Z_1 = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \circ (0,2) + (0,5) = \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix}$; $Y_1 = \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix}$

$Z_2 = \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix} \circ (0,5) - (0,5) = \begin{pmatrix} -0,25 \\ 0,15 \end{pmatrix}$; $Y_2 = \begin{pmatrix} 0 \\ 0,15 \end{pmatrix}$

BACKWARD

$\delta_2 = \begin{pmatrix} 0 \\ -1,85 \end{pmatrix} \circ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1,85 \end{pmatrix}$; $W_2' = (0,5) - 0,1 \cdot \begin{pmatrix} 0,5 \\ 1,3 \end{pmatrix} \begin{pmatrix} 0 \\ -1,85 \end{pmatrix} = (0,620)$; $B_2' = (-0,5) - 0,1 \cdot (-0,925) = (-0,4075)$

$\delta_1 = \left(\begin{pmatrix} 0 \\ -1,85 \end{pmatrix} (0,5)^T \right) \circ \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -0,925 \end{pmatrix}$

$W_1' = (0,2) - 0,1 \begin{pmatrix} 0 \\ 4 \end{pmatrix} \begin{pmatrix} 0 \\ -0,925 \end{pmatrix} = (0,385)$

$B_1' = (0,5) - 0,1 \cdot (-0,4625) = (0,54625)$