Harmonic Distortion

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Fourier Series approach a certain function as the number of terms approach ∞. Let

$$S_K(t) = \sum_{-K}^{K} C_k e^{j\frac{2\pi k t}{T}}$$ be the Fourier

Series of $s(t)$ up to K^{th} harmonic.

what is the error of the K^{th} harmonic?

$$\epsilon_K = \sum_{|k|=K+1}^{\infty} C_k e^{j\frac{2\pi k t}{T}}$$

Let's find the rms^2 error. What is RMS?

For a function $f(t)$, $rms\{f(t)\} \equiv$

$$\lim_{T\to\infty} \sqrt{\frac{1}{T}\int_0^T |f(t)|^2 dt}$$

If $f(t) \in \mathbb{C}$, then

$$rms^2\{f(t)\} = \lim_{T\to\infty} \frac{1}{T}\int_0^T f^*(t) f(t)\, dt$$

So, what is $rms^2(\epsilon_K)$?

$$rms^2(\epsilon_K) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(\sum_{|k_1|=K+1}^{\infty} C_k e^{j\frac{2\pi k_1 t}{T}}\right)\left(\sum_{|k_2|=K+1}^{\infty} C_k^* e^{-j\frac{2\pi k_2 t}{T}}\right) dt$$

$$= \sum^{\infty} \sum^{\infty} \lim_{T\to\infty} \frac{1}{T}\int^{T} e^{j\frac{2\pi k_1 t}{T}} e^{-j\frac{2\pi k_2 t}{T}} |C_k|^2 \, dt$$

$$= \sum_{|k_1|=K+1} \sum_{|k_2|=K+1}^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_0^T e^{j\frac{2\pi k_1 t}{T}} e^{-j\frac{2\pi k_2 t}{T}} |C_k|^2 \, dt$$

$$= \sum_{|k|=K+1}^{\infty} |C_k|^2 = 2 \sum_{k=K+1}^{\infty} |C_k|^2$$

We define total harmonic distortion as

$$\frac{\mathrm{rms}^2(\epsilon_K)}{\mathrm{rms}^2(S_K)} \equiv THD_{RMS}$$

A note about $=$ ...

A Fourier Series may never be point-wise equal to the function it represents, but it will always be mean-square equal. This means that

$$\mathrm{rms}(S_K(t) - S(t)) \to 0 \text{ as } K \to \infty$$

$$\lim_{K \to \infty} \epsilon_K = 0$$