Suppose we have points (Xn, Yn) for nE[1,..., N]. How can we sit a polynomial to this set of points?

Fitting a Line y= mx+b, one way to select ideal coefficients m and b is to minimize the square error btw mx+b and the data.

we seek in and b as follows:

min
$$G^{2}(m,b) = min \sum_{n=1}^{N} [y_{n} - (mx_{n}+b)]^{2}$$

m,b

 m,b

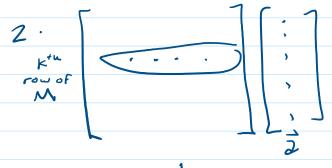
we can best organize these ideas with matrices.

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix}$$

y - X a is a vector of unsquared errors.

(y-Xi) T(y-Xi) squares and suns these errors using the dol product.

This quantity is minimized when



I looks line

End note.

$$\nabla_{\vec{a}} \left(\vec{a}^{\mathsf{T}} \times {}^{\mathsf{T}} \times \vec{a} \right) = 2 \times {}^{\mathsf{T}} \times \vec{a}$$

So we arrive at $\nabla_{\vec{a}} \left(\|\vec{y} - \vec{X}\vec{a}\|^2 \right)$ = $-2 \times \vec{Y} \vec{y} + 2 \times \vec{X} \times \vec{a} = 0$ 25 our applicipation equation.

XTX is invertible if we have 2 or more unique

XTX is invertible if we have 2 or more unique points in our data.

Fitting a Polynomial

$$\begin{bmatrix} 1 & X_1 & X_1^2 \\ 1 & X_2 & X_2^2 \\ \vdots & \vdots & \vdots \\ 1 & X_N & X_N^2 \end{bmatrix} \begin{bmatrix} C \\ b \\ a \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix}$$

we can arbitrarily generalize to any degree. But what degree is appropriate? The one that minimizes the "minimum description length criterion".

Fifting 2 Sinusoid

How do we fit S[n] of length N to a sinusoid

y = A cos (2 \pi f. n + \P)?

$$a_1 = A\cos(\phi)$$
 $a_2 = A\sin(\phi)$

$$A = \sqrt{\partial_{-}^{2} + \partial_{-}^{2}}$$
 $\phi = \angle \partial_{-}^{-1} \left(\frac{\partial_{2}}{\partial_{-}^{2}}\right)$

A:
$$\sqrt{a_1^2} \cdot a_2^2$$
 $\phi = \pm a_1^{-1} \left(\frac{a_2}{a_1}\right)$

$$\overrightarrow{a} = \begin{bmatrix} a_1 \\ 2a \end{bmatrix}$$

$$\begin{bmatrix} cos(2\pi f_1) & sin(2\pi f_2) \\ cos(4\pi f_1) & sin(4\pi f_2) \\ \vdots & \vdots & \vdots \\ cos(2\pi f_1(N-1)) & sin(4\pi f_2(N-1)) \end{bmatrix}$$

$$\overrightarrow{a} = \begin{bmatrix} x \\ x \\ x \end{bmatrix}$$

$$\begin{bmatrix} cos(2\pi f_2(N-1)) & sin(4\pi f_2(N-1)) \\ \vdots & \vdots & \vdots \\ cos(2\pi f_2(N-1)) & sin(2\pi f_2(N-1)) \end{bmatrix}$$

$$\begin{bmatrix} cos(2\pi f_2) & \cdots & cos(2\pi f_2(N-1)) \\ \vdots & \vdots & \vdots \\ cos(2\pi f_2(N-1)) & sin(2\pi f_2(N-1)) \end{bmatrix}$$

$$\begin{bmatrix} cos(2\pi f_2(N-1)) & sin(2\pi f_2(N-1)) \\ \vdots & \vdots & \vdots \\ cos(2\pi f_2(N-1)) & sin(2\pi f_2(N-1)) \end{bmatrix}$$

$$= x^T x \cdot \begin{bmatrix} \vec{c} \\ \vec{c} \\ \vec{c} \end{bmatrix}$$

$$\begin{bmatrix} \vec{c} \\ \vec{c} \end{bmatrix} = x^T x \cdot \begin{bmatrix} \vec{c} \\ \vec{c} \end{bmatrix}$$