January 29, 2018

To the Editorial Board of the Entropy Journal,

I have the pleasure of sending you the revision of our manuscript entitled "Complexity of simple, switched and skipped chaotic maps in finite precision" authored by M. Antonelli, L. De Micco, O. A. Rosso and H. A. Larrondo to be considered for publication as a research article in your prestigious journal.

As I was asked in point 4 in the body of the email sent to me with the revisions, I explain point-by-point the details of the revisions in the manuscript and the responses to the reviewers' comments.

REVIEWER #1

The title is not suitable as the authors report results for the simple logistic map. This is also made clear in the first sentence of conclusion.

The title was modified

p. 2, line 30: These are not the unique arithmetic available. Rethink this sentence.

In these new devices, floating- and fixed-point are the available arithmetic.

The reviewer is right. The sentence was corrected.

p.8, line: 246: Regarding the sentence "Five pseudo chaotic maps". The authors should decide if they want to study the chaotic maps, as represented in Eq. (10) or their pseudo representation. It seems that the later is more appropriated. If they want to study their pseudo representation they shoulr replace the common arithmetic representation by its counterpart in finite precision, which has not been done. So I believe that they investigate the chaotic maps using their pseudo orbits.

In all cases we used the pseudo chaotic maps. To clarify this point we add a sentence on p.9, line 253.

p. 8, line 248: the description of number system representation should be improved. In Eq. (10) there are integers which are not covered by the concise description of fixed point representation. Is that covered by any standard, as it has been mentioned to floating point? Please, for the sake of reproducibility the authors should also describe the software and hardware in which their results have been implemented.

Thank you very much for this observation. We add a paragraph on p.9, line 259 to clarify this point.

p. 9, Eq. (11): it seems somehting missing in the details of this equation. A multiplicaption operations requires double of the number of bits, although the round operation is not indicated for the operation x_n(1-x_n). This should be clarified as well as the mention that the distributive propertie remains in such system. The author should indicate a reference, or even better, gives the reader some examples.

As in the previous observation, the references to the used method can be found on p.9, line 259. There, the rounding method is widely explained and validated.

To check the results precisely a better description of the implementation of the algorithm should be provided.

The method used in this work is presented in reference [13]. There, they used the same pseudo chaotic maps. We have added on p.11, line 297 a sentence clarifying this point.

REVIEWER #2

1. In Eq. (9) in page 5 and in the line 174, two issues:
 Correct \bar{X}_j^D . Now it is \bar{X_j^D}

 And erase the expression x_{j+k-1} and the following "-" sign
 in line 174. \bar{X}_j^D must be the arithmetic mean.

Done

2. I do not agree of the used Tent map. It has bad statistical
and numerical properties when it is implemented.
 The general equation for Tent map is:

```
         | u x(n), for 0 <= x(n) <= 1/u,
x(n+1) = |
         | (u/(u-1))(1-x(n)), for 1/u < x(n) <= 1.0  .
```

The problem is when u=2, as authors do in the document.
Also the initial value must be in the interval (0,1),
without taking the values of the bounds.

A good value for u could be 1.96 . Please check the reference
[2] in the next point.

Thank you very much for this contribution. The information requested by the reviewer greatly enriched our work. The equation of the tent map was corrected and the results section of this map was completely rewritten.
In this section the results are now shown for u = 1.96 and not for u = 2.
A very interesting aspect that emerges with this new information is that for SWITCH, EVEN and ODD maps the results using u = 1.96 and u = 2 are equivalent, so we chose u = 2 in those schemes because their implementation and simulation are easier.
To explain the latter we add a paragraph on p.17, line 379.

3. To use the value of the entropy to choose the number
 of bits in the fractional part, when maps are implemented in
 integer arithmetic, has been proposed in :

[1] VHDL Descriptions for the FPGA Implementation of PWL-Function-Based Multi-Scroll Chaotic Oscillators,
 PLOS ONE 11, pp.\ 1-32, http://dx.doi.org/10.1371/journal.pone.0168300, 2016.

[2] Hardware Implementation of pseudo-random number generators based on chaotic maps,Nonlinear Dynamics (2017)
Volume 90, Issue 3, pages 1661–-1670, https://doi.org/10.1007/s11071-017-3755-z

Please note in reference [2] Fig. 3, in the implementation of Bernoulli map with integer arithmetic, that although 37 bits of the fractional part keep entropy statistics stable, the generated sequences do not pass the NIST suite test, and 50 bits were necessary to use in order to generate binary

sequences that pass all NIST test.This result shows that the entropy alone does not capture the complexity property.
If authors redo calculations with Tent map with u=1.96 and their proposed analysis shows that more bits are necessary to capture the complex behavior, then the paper could be published.

As we said in point 2, the proposed papers enriched our publication and open up a whole field to explore. Without the two randomization processes used in [2] (eq 7 and the bit counting redundancy reduction technique), the convergence of the quantifiers remains for B = 34.

On the other hand, in reference [4] of our work is shown that chaotic maps can not pass the NIST or DieHard tests, so a randomization process is required, such as the one used in the references suggested above.

REVIEWER #3

This paper explored the statistical degradation of simple, switched and skipped chaotic maps due to the inherent error of a based-2 systems. It evaluated mixing and amplitude distributions from a statistical point of view. The main contribution of this paper is the study of how different statistical quantifiers detect the evolution of stochasticity and mixing of the chaotic maps according as the numerical precision varies.To illustrate this sequences generated by well known maps were used, and also sequences obtained by randomization methods like skipping and switching. The idea is new and the work is useful. The following suggestions are listed as follows:

1) The English of the paper should be improved with much effort.

Thank you very much for this observation. The manuscript was checked by collaborators to improve the English of the paper.

2) The Section 4 Conclusions is too long, it should be simplified and extracted.

The conclusions were summarized in this new version.

3) In simulation examples, only discrete chaotic maps are used, but continue 3 Dimensional chaos system should be considered such as Lorenz System.

This point is very interesting, in this particular work we are interested in analysing the evolution of pseudo chaotic maps with a switching scheme. In reference [4] of our work we made an analysis for continuous three-dimensional chaotic maps, but, in that case, switching process is very difficult because the convergence interval varies greatly between maps. This is why we chose using maps with convergence regions in [0, 1].

We hope that it will interest your audience.

Kind regards,

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