Article

Complexity of switching chaotic maps in finite precision

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1   **Abstract:** In this paper we investigate the degradation of the statistic properties of chaotic maps
2   as consequence of their implementation in a digital media such as Digital Signal Processors (DSP),
3   Field Programmable Gate Arrays (FPGA) or Application-Specific Integrated Circuits (ASIC). In these
4   systems, binary floating- and fixed-point are the numerical representations available. Fixed-point
5   representation is preferred over floating-point when speed, low power and/or small circuit area are
6   necessary. Then, in this paper we compare the degradation of fixed-point binary precision version of
7   chaotic maps with the floating point IEEE754 to evaluate the feasibility of their FPGA implementation.
8   The specific period that every fixed-point precision produces was investigated in previous reports,
9   using as example the tent map and the logistic map. Statistical characteristics are also relevant. It has
10 been recently shown that it is convenient to describe the statistical characteristic using both, causal
11 and non-causal quantifiers. In this paper we complement the period analysis by characterizing the
12 behavior of these maps from an statistical point of view using causal and non-causal entropies and
13 complexities. Here we do not look for the system to be similar to the implemented in real numbers,
14 but that certain conditions related to the statistics of systems are met.

15 **Keywords:** chaos; finite precision; hardware implementaion; switching maps

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Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
TLA	Three letter acronym
LD	linear dichroism

Appendix A

Appendix A.1

The appendix is an optional section that can contain details and data supplemental to the main text. For example, explanations of experimental details that would disrupt the flow of the main text, but nonetheless remain crucial to understanding and reproducing the research shown; figures of replicates for experiments of which representative data is shown in the main text can be added here if brief, or as Supplementary data. Mathematical proofs of results not central to the paper can be added as an appendix.

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References

1. Author1, T. The title of the cited article. *Journal Abbreviation* **2008**, *10*, 142-149, DOI.
2. Author2, L. The title of the cited contribution. In *The Book Title*; Editor1, F., Editor2, A., Eds.; Publishing House: City, Country, 2007; pp. 32-58, ISBN.

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