Sheet10

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a) $$\dot{Y}_p = -2\frac{Y_p^2}{2}\rho N_A \langle\sigma v\rangle_{pp} - Y_p Y_d \rho N_A \langle\sigma v\rangle_{pd} + 2\frac{Y_{He^3}^2}{2}\rho N_A \langle\sigma v\rangle_{^3He^3He}$$

$$\dot{Y}_d = +\frac{Y_p^2}{2}\rho N_A \langle\sigma v\rangle_{pp} - Y_p Y_d \rho N_A \langle\sigma v\rangle_{pd}$$

$$\dot{Y}_{^3He} = +Y_p Y_d \rho N_A \langle\sigma v\rangle_{pd} - 2\frac{Y_{^3He}^2}{2}\rho N_A \langle\sigma v\rangle_{^3He^3He}$$

$$\dot{Y}_{^4He} = +\frac{Y_{^3He}^2}{2}\rho N_A \langle\sigma v\rangle_{^3He^3He}$$

b) $$\frac{Y_i^{n+1} - Y_i^n}{\Delta t} - f(\vec{Y}^{n+1}) \equiv C_i(\vec{Y}^{n+1}, Y_i^n, \Delta t) = 0$$

$$\Rightarrow C_p = \frac{Y_p^{n+1} - Y_p^n}{\Delta t} + 2\frac{Y_p^{2\,n+1}}{2}\lambda_{pp} + Y_p^{n+1}Y_d^{n+1}\lambda_{pd} - 2\frac{Y_{^3He}^{2\,n+1}}{2}\lambda_{^3He^3He} = 0$$

$$C_d = \frac{Y_d^{n+1} - Y_d^n}{\Delta t} - \frac{Y_p^{2\,n+1}}{2}\lambda_{pp} + Y_p^{n+1}Y_d^{n+1}\lambda_{pd} = 0$$

$$C_{^3He} = \frac{Y_{^3He}^{n+1} - Y_{^3He}^n}{\Delta t} - Y_p^{n+1}Y_d^{n+1}\lambda_{pd} + 2\frac{Y_{^3He}^{2\,n+1}}{2}\lambda_{^3He^3He} = 0$$

$$C_{^4He} = \frac{Y_{^4He}^{n+1} - Y_{^4He}^n}{\Delta t} - \frac{Y_{^3He}^{2\,n+1}}{2}\lambda_{^3He^3He} = 0$$

c) $$J_{ij} \equiv \frac{\partial C_i}{\partial Y_j^{n+1}}$$

$$J_{pp} = \frac{1}{\Delta t} + 2Y_p^{n+1}\lambda_{pp} + Y_d^{n+1}\lambda_{pd}$$

$$J_{pd} = Y_p^{n+1}\lambda_{pd}$$

$$J_{p^3He} = -2Y_{^3He}^{n+1}\lambda_{^3He^3He}$$

$$J_{p^4He} = 0$$

$$J_{dp} = -Y_p^{n+1}\lambda_{pp} + Y_d^{n+1}\lambda_{pd}$$

$$J_{dd} = \frac{1}{\Delta t} + Y_p^{n+1}\lambda_{pd}$$

$$J_{dd} = \frac{1}{\Delta t} + Y_p^{n+1} \lambda_{pd}$$

$$J_{d\,^3He} = J_{d\,^4He} = 0$$

$$J_{^3He\,p} = - Y_d^{n+1} \lambda_{pd}$$

$$J_{^3He\,d} = - Y_p^{n+1} \lambda_{pd}$$

$$J_{^3He\,^3He} = \frac{1}{\Delta t} + 2 Y_{^3He}^{n+1} \lambda_{^3He\,^3He}$$

$$J_{^3He\,^4He} = 0$$

$$J_{^4He\,p} = J_{^4He\,d} = 0$$

$$J_{^4He\,^3He} = - Y_{^3He}^{n+1} \lambda_{^3He\,^3He}$$

$$J_{^4He\,^4He} = \frac{1}{\Delta t}$$