Exercises in "Numerical Methods in Plasma Astrophysics"

Winter 2022/23 Exercise 11

Exercise 11: Vlasov equation

The Lattice-Boltzmann simulation on one of the last exercise sheets was based on the slightly utopian idea that fluid elements are always either at utopian notion that fluid elements are always either at rest or moving or moving at the speed of sound. On the one hand, this is wrong, and for collision-free plasmas, in the absence of the speed of sound. speed of sound.
Instead, the electrostatic Vlasov-Poisson system (for each particle species α) is to be simulated directly:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \frac{\partial f_\alpha}{\partial \vec{x}} + \frac{q_\alpha \vec{E}}{m_\alpha} \cdot \frac{\partial f_\alpha}{\partial \vec{v}} = 0 \tag{1}$$

$$\nabla \cdot \vec{E} = 4\pi \varrho \tag{2}$$

Use an upwind method, where first the updates in the spatial direction, then solving the Poisson equation and finally the updates in momentum direction.
Density

$$\varrho(\vec{x}) = \sum_\alpha \int q_\alpha f_\alpha(\vec{x}, \vec{v}) \, \mathrm{d}\vec{v} \tag{3}$$

is the spatial charge density.

The physical scenario is again the same as in the PiC simulation of the Pierce diode: Into a spatially homogeneous background plasma (with thermal electron and ion population) an electron beam (with $v_{\mathrm{Beam}} = 5 \cdot v_{\mathrm{th}}$) is injected.

For the sake of simplicity, all boundary conditions and the actual shot of the electrons into the simulation, and simply a homogeneous, 1-dimensional periodic space is assumed, which is prefixed with the electron beam population. Pay attention to total charge equilibrium!