Computer Graphics - Exercise 04

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Winter Semester 2022/23

4.1 Bidirectional Reflectance Distribution Function

a) In the context of lighting models, BRDFs are highly relevant. Name three parameters of BRDFs.

- incident light angle
- exiting light angle
- point x at which the light is reflected

b) What does the value of BRDF for a vertain parameter set tell us?
The bidirectional reflectance distribution function (BRDF) tells us the proportion of the exiting light (at a given angle) to the incident light (at a given angle).

c) How can we obtain BRDFs? Name two different approaches.

- Measurement of real samples
- Phenomenologically or physically motivated models

4.2 Analytic Geometry

a) Start by calculating the discriminant

$$b^2 - (4ac)$$

where $a = \mathbf{d} \cdot \mathbf{d}$, $b = 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{M})$, and $c = (\mathbf{e} - \mathbf{M}) \cdot (\mathbf{e} - \mathbf{M}) - r^2 = \|\mathbf{e} - \mathbf{M}\|^2 - r^2$.

(i) $\mathbf{e} - \mathbf{M} = (0, 0, 1)^T - (5, 5, 2)^T = (-5, -5, -1)^T$

(ii) $\|\mathbf{e} - \mathbf{M}\|^2 = (-5) \cdot (-5) + (-5) \cdot (-5) + (-1) \cdot (-1) = 51$

(iii) $a = 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 2$

(iv) $b = 2((1, 1, 0)^T \cdot (-5, -5, -1)^T) = 2 \cdot (-10) = -20$

(v) $c = 51 - 3^2 = 42$

Hence the discriminant is

$$(-20)^2 - (4 \cdot 2 \cdot 42) = 400 - 336 = 64 > 0.$$

A positive discriminant confirms that the ray does intersect with the given sphere.

To calculate the intersection points, in regard to the given ray $r(t) = \mathbf{d} + t\mathbf{d}$, one needs to solve the quadratic equation:

$$t_{1/2} = \frac{-b \pm \sqrt{b^2 - (4ac)}}{2a}$$

Since we already calculated a,b, and c above, we quickly arrive at the result

$$t_{1/2} = \frac{20 \pm \sqrt{64}}{4} = \frac{20 \pm 8}{4} = 5 \pm 2 \implies t_1 = 7, t_2 = 3.$$

Because $\|3\mathbf{d}\| < \|7\mathbf{d}\|$: $t_2 = 3$ is closer to $\mathbf{e}$ and therefore the visible intersection point. The Intersection point is

$$\mathbf{S} = \mathbf{e} + t_2\mathbf{d} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$$

b) First we compute the Hessian normal form of the surface

$$\mathbf{v}_1 = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -6 \\ 6 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2 = \mathbf{C} - \mathbf{A} = \begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix} \quad \Rightarrow \quad \mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{pmatrix} 36 \\ 36 \\ 36 \end{pmatrix}$$

The normal vector of the surface can be normalized and finally used to obtain the HNF by

$$\mathbf{n} = \frac{\mathbf{n}}{\sqrt{\mathbf{n} \cdot \mathbf{n}}} = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \quad \Rightarrow \quad a = b = c = \frac{1}{\sqrt{3}}, \quad d = -\mathbf{A} \cdot \mathbf{n} = 2\sqrt{3}$$

The parameter of the intersection can be computed according to the lecture with

$$t = \frac{d - \mathbf{e} \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}} = \frac{2\sqrt{3} - 1/\sqrt{3}}{2/\sqrt{3}} = \frac{5}{2}$$

The intersection is therefore

$$\mathbf{S} = \mathbf{r}(t = 5/2) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \frac{5}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5/2 \\ 5/2 \\ 1 \end{pmatrix}$$

The triangle can now be projected on a main plain with maximal coordinates. Since every projection of the triangle leads to similar coordinates, we chose the x-y-plane. The new, projected points are $\mathbf{P} = (6, 0)^T$, $\mathbf{Q} = (0, 6)^T$ and the origin $\mathbf{R} = (0, 0)^T$. The projected intersection is $\mathbf{S} = (5/2, 5/2)^T$. From three points, the area of a triangle can be calculated with help of the determinant.

$$A_\Delta(P, Q, R) = \frac{1}{2}\left|\det\begin{pmatrix} (Q_x - P_x) & (R_x - P_x) \\ (Q_y - P_y) & (R_y - P_y) \end{pmatrix}\right| = 18$$

The other areas with the intersection point are calculated analogously

$$A_\Delta(S, Q, R) = \frac{15}{2}$$
$$A_\Delta(P, S, R) = \frac{15}{2}$$
$$A_\Delta(P, Q, S) = 3$$

Since all areas are positive, all barycentric coordinates λ_i are positive, hence the intersection lays inside of the triangle.