Computer Graphics Exercise 07

Ex 1: Active & Passive Rotation

a) Generic trf. matrix

$$R(\phi) = \begin{pmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{pmatrix}, \quad \psi = \frac{2\pi\phi}{360^\circ}$$

Apply for $\phi = 30^\circ$ to $\vec{p} = \begin{pmatrix} 0.5 \\ 2 \end{pmatrix}$

$$\vec{p}\,' = R(\phi)\,\vec{p} = \begin{pmatrix} \cos\left(\frac{2\pi 30^\circ}{360^\circ}\right) & -\sin\left(\frac{2\pi 30^\circ}{360^\circ}\right) \\ \sin\left(\frac{2\pi 30^\circ}{360^\circ}\right) & \sin\left(\frac{2\pi 30^\circ}{360}\right) \end{pmatrix} \begin{pmatrix} 0.5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{3}/4 - 1 \\ 1/2 + \sqrt{3} \end{pmatrix} \approx \begin{pmatrix} -0.57 \\ 1.98 \end{pmatrix}$$

b) $B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$, $B' = \left\{ \begin{pmatrix} \sqrt{3}/2 \\ -1/2 \end{pmatrix}, \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix} \right\}$

c) $$R_{B,B'}(\phi) = \vec{R}^{-1}(\phi) = \vec{R}^{T}(\phi) = \begin{pmatrix} \cos(\psi) & \sin(\psi) \\ -\sin(\psi) & \cos(\psi) \end{pmatrix}$$

$$\vec{b}_1' = R_{B,B'}(\phi)\,\vec{b}_1 = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 \\ -1/2 \end{pmatrix}$$

$$\vec{b}_2' = R_{B,B'}(\phi)\,\vec{b}_2 = \text{— " —} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix}$$

d) $$\vec{p}_{B'}' = R^{-1}_{B,B'}(\phi)\,\vec{p}_B = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} 0.5 \\ 2 \end{pmatrix} \approx \begin{pmatrix} -0.57 \\ 1.98 \end{pmatrix}$$

e) $\Rightarrow$ Active and passive rotation yield the same result in their respective Basis

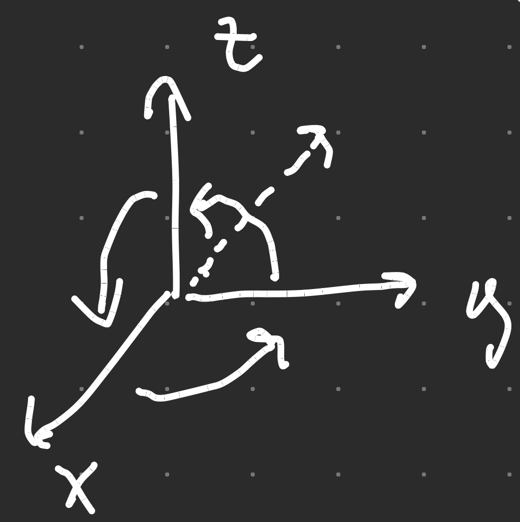
$\Rightarrow$ Choice of basis is not important

→ Choice of basis is not important.

Ex 2: Transformations & Homogeneous coordinates

a) Y-Axis has "opposite" signs because:

- Rotation in mathematics is defined counter-clock wise
- Mathematics uses right system



b) Let M be a linear trf. Normal vectors are then transformed by multiplication with inverse transposed

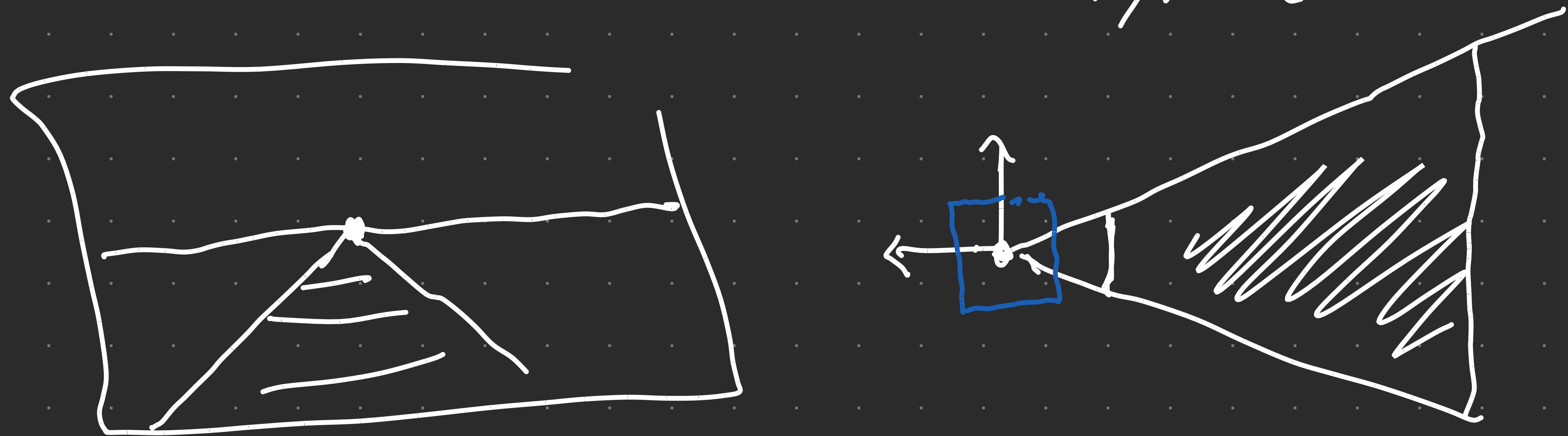
Inverse Transposed

$$\vec{n}' = (M^{-1})^T \vec{n}$$

c)

Coord.	Pros	Cons
world	- Parameter t is already distance	- Might be difficult to compute because of e.g. shear trf.
Object	- No complicated transformations	- A lot of matrix inversions - Distance t is not valid anymore

d)



In perspective projection, a 3D point in truncated view frustrum (eye coords.) is mapped to a cube

Range of x-coord: $[l, r]$ to $[-1, 1]$
" " y- " $[b, t]$ to $[-1, 1]$
" " z- " $[-n, -f]$ to $[-1, 1]$

Homogeneous perspective projection matrix

$$M = \begin{pmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$\Rightarrow$ Projected & norm. coords. calculated by dividing w-component

$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ w_c \end{pmatrix} = M \cdot \begin{pmatrix} x_c \\ y_e \\ z_e \\ w_e \end{pmatrix} \Rightarrow \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x_c/w_c \\ y_c/w_c \\ z_c/w_c \end{pmatrix}$$