Computer Graphics - Exercise 03

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1 Basic Concepts

Explain the following concepts and, if applicable, their difference in a few sentences.

a) **Rasterization and Raytracing**

Rasterization projects 3D objects onto a 2D image. For each object, all pixels that are affected by the object are searched and the pixel values are calculated accordingly.

Ray tracing works the other way around by casting a ray for each pixel a ray. Then all objects are tested against each ray to identify the nearest object hit by each ray. The ray may then be absorbed or (partially) reflected which may require further rays to be cast. Each object a ray hits influences it's pixel's color value until the ray is either fully absorbed or does not hit any further object. This can be interpreted as tracing the light rays that pass through a pinhole camera.

b) **Image-based and object-based rendering**

In object-based rendering, objects are rendered sequentially, whereas in image-order rendering, the pixels are considered sequentially.

c) **Implicit and parametric representation**

In implicit representation, points are described by a equation / condition that has to be met.

In parametric representation, on the other hand, the points are described by a function with arbitrary parameters. All considered points can be calculated by plugging all valid parameter values into the function.

d) Barycentric coordinates: How are they computed? How can they be used to determine whether a point is located within a triangle? What properties do they have?

Barycentric coordinates of a point Q respective to the Points $P_1, ..., P_k$ are expressed by

$$Q = \lambda_1 P_1 + ... + \lambda_k P_k. \tag{1}$$

If a point Q lies in a triangle the with vertices P_1, P_2 and P_3, the following applies

$$Q = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3 \tag{2}$$

with $\lambda_1, \lambda_2, \lambda_3$ are positive and $\lambda_1 + \lambda_2 + \lambda_3 = 1$. The barycentric coordinates of a point Q for a triangle can be calculated by subdividing the triangle

$$\lambda_1 = \frac{A_\Delta(Q, P_2, P_3)}{A_\Delta(P_1, P_2, P_3)}$$
$$\lambda_2 = \frac{A_\Delta(P_1, Q, P_3)}{A_\Delta(P_1, P_2, P_3)}$$
$$\lambda_3 = \frac{A_\Delta(P_1, P_2, Q)}{A_\Delta(P_1, P_2, P_3)}$$

The barycentric coordinates can than be used to interpolate quantities on the vertices to the inside of the triangle.

2 Barycentric Coordinates

a) How many barycentric coordinates does one need for an n-simplex?

For an n-simplex one needs $n + 1$ barycentric coordinates.

b) Imagine splitting the square into two triangles along the diagonal between P1 and P3, and interpolate the color values using barycentric interpolation. Briefly (!) describe the color variation along P0P2 (see c) below).

The color values along $\overline{P_0P_2}$ in the lower left triangle would vary from blue at P_0 to red at the intersection with $\overline{P_1P_3}$. In the upper right triangle the color would shift in the opposite direction along $\overline{P_1P_2}$ from blue at P_2 to red at the intersection.

c) How does the color variation along P0P2 change if you split along the other diagonal instead?

If one would split the square and interpolate the colors along $\overline{P_1P_3}$ it would be represent a shift from blue to green (P_0 to P_2) without containing any red share.

d) Come up with a way to directly calculate barycentric coordinates for a rectangle without prior subdivision into triangles. How many barycentric coordinates do you need and what are their values at the vertices? Is it possible to apply this method to arbitrary quadrangles?

- One would need a set of four barycentric coordinates to describe the distance to the vertices and also the resulting share of color.
- Idea: Use bilinear interpolation.
- This does not work for arbitrary quadrangles: Use Newton method to find transformation of quadrangle to rectangle.

3 Ray Tracing

See code.