Computer Graphics

Tutorial for Exercise Sheet 03

Exercise 1: Basic Concepts

a) Rasterization vs Ray-tracing

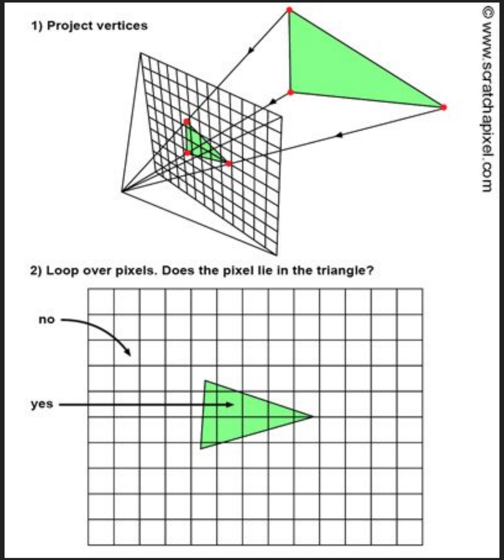
Rasterization:

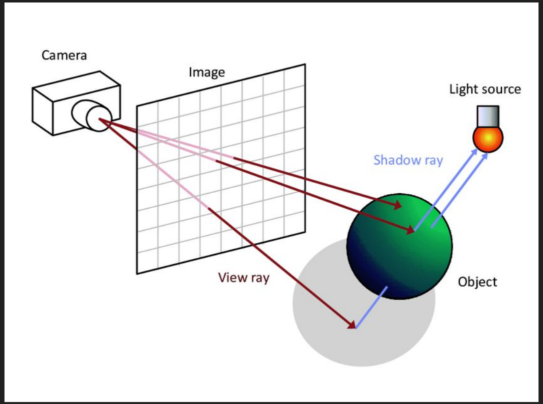
- Projects 3D objects onto a 2D Image
- All pixels that are affected by the object are searched
- Pixel values are calculated accordingly
- Fast but inaccurate

Ray-Tracing:

- Other way around: Casting a ray for each pixel
- All object are tested against each ray to identify nearest object hit
- Ray may be absorbed or (partially) reflected
  - may require further rays to be cast
- Each object a ray hits influences its pixels color value
  - until ray is either full absorbed or ends in light source
- Slow but photorealistic

https://www.youtube.com/watch?v=bUX3u1iD0jM





Exercise 1: Basic Concepts

a) Rasterization vs Ray-tracing

Exercise 1: Basic Concepts

c) Image-Based vs Object-Based

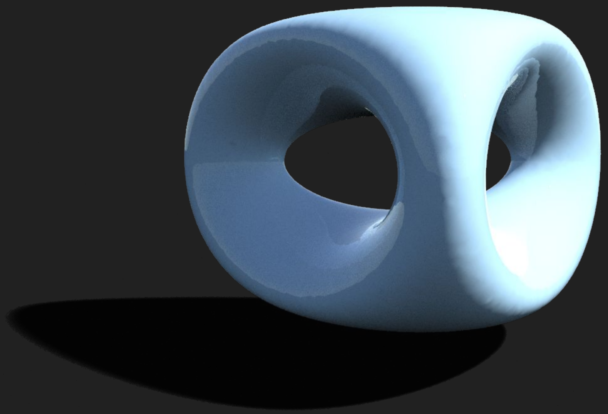
Image-Based:

- E.g.: Raster Graphics
- Consider pixels sequentially
- Determine which object is visible at the position
- Determine pixel color
-

Object-Based:

- Eg.: Vector Graphics
- Consider objects/surfaces sequentially
- Determine which pixels are covered by the object
- Determine pixel color

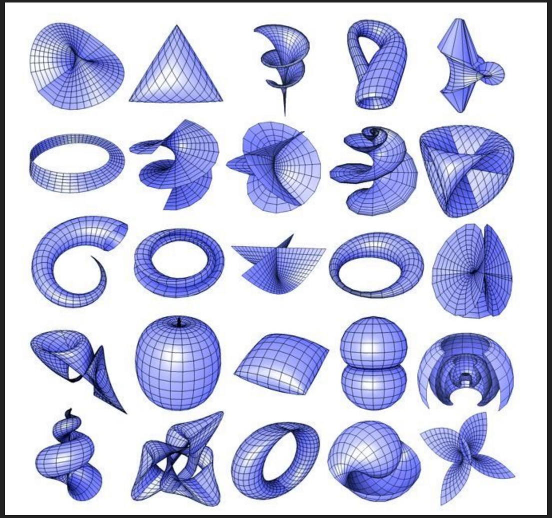
Exercise 1: Basic Concepts



b) Implicit vs Parametric Representation

Implicit Representation:

- Of the form F(x)=0
- Points are described by an equation / condition that has to be met
- Only solvable in certain areas around points

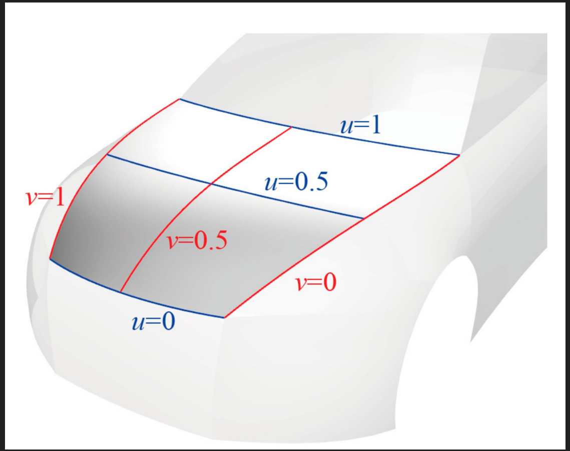


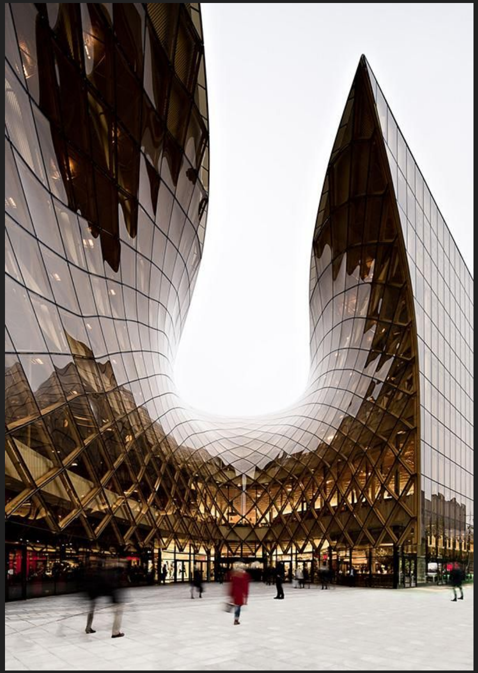
Parametric Representation

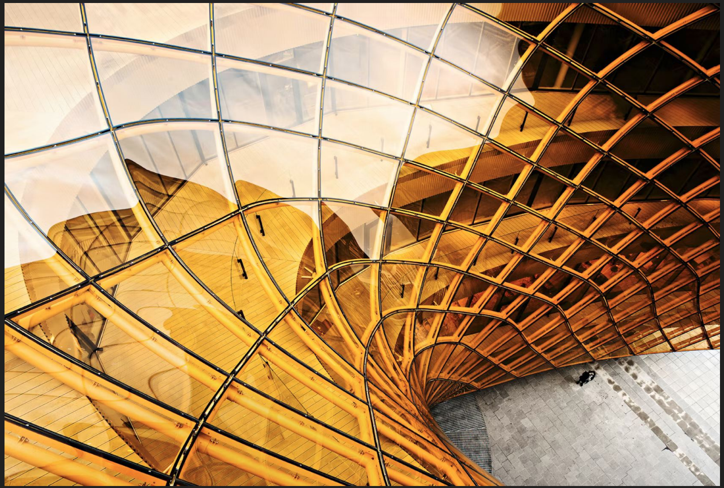
- Points are described by function with arbitrary parameters
- All points can be calculated by evaluating function for all valid parameter values
- Complicated surfaces must be glued together by patches

Exercise 1: Basic Concepts

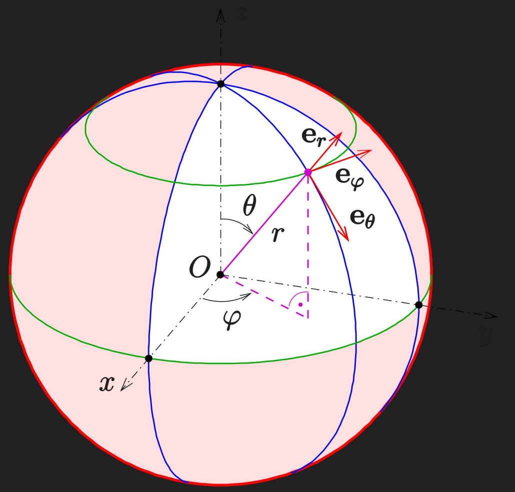
b) Implicit vs Parametric Representation







Exercise 1: Basic Concepts



b) Implicit vs Parametric Representation

Example: Sphere of radius r

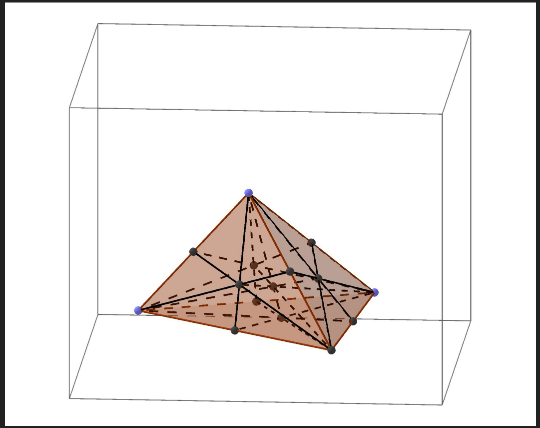
Implicit:

$$F(x, y, z) = x^2 + y^2 + z^2 - r^2 = 0$$

Parametric:

$$x(r, \theta, \varphi) = \begin{pmatrix} r \sin\theta \cos\varphi \\ r \sin\theta \sin\varphi \\ r \cos\theta \end{pmatrix} \qquad r \in \mathrm{R}, \theta \in [0, \pi], \varphi \in [0, 2\pi)$$

Exercise 1: Basic Concepts



c) Barycentric Coordinates

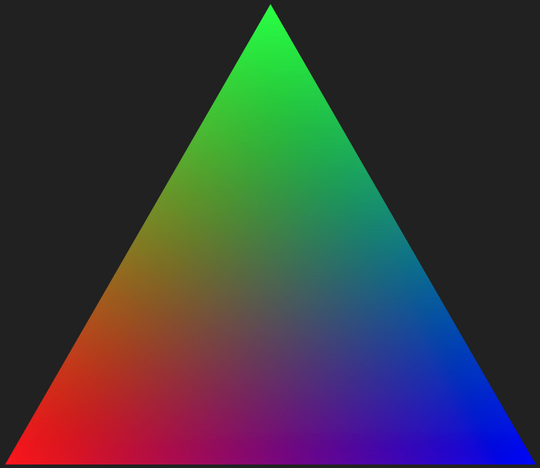
- Barycentric coordinates of a point Q respective to the Points $P_1, \ldots, P_k$ are expressed by

$$Q = \lambda_1 P_1 + \ldots + \lambda_k P_k \qquad \sum_{i=1}^{k} \lambda_i = 1$$

- A point Q lies in a triangle with vertices $P_1, \ldots, P_k$

if $\lambda_i \geq 0 \quad i \in \{1, \ldots, k\}$

and $\sum_{i=1}^{k} \lambda_i = 1$

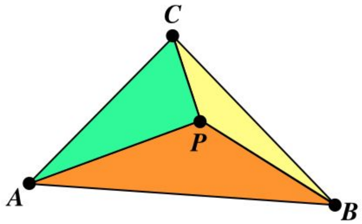


Exercise 1: Basic Concepts

c) Barycentric Coordinates: Computation for Triangle

Barycentric Coordinates

- **A coordinate system in which the location of a point of a simplex**



$$P = w_A \times A + w_B \times B + w_C \times C$$

$$w_A = \frac{\Delta PBC}{\Delta ABC}$$

$$w_B = \frac{\Delta PCA}{\Delta ABC}$$

$$w_C = \frac{\Delta PAB}{\Delta ABC}$$

inside condition

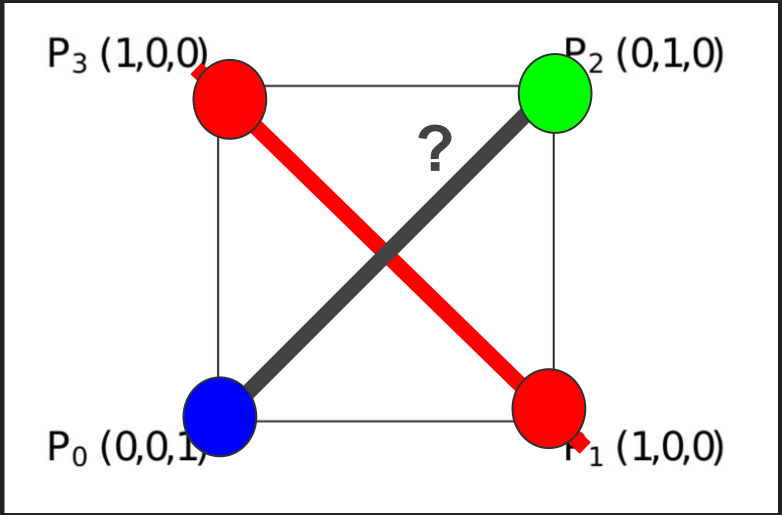
$$0 \le w_A, w_B, w_C \le 1 \qquad w_A + w_B + w_C = 1$$

Exercise 2: Barycentric Coordinates

a) **How many barycentric coordinates does one need for an n-simplex?**

For an n-simplex one needs n+1 barycentric coordinates.

b) **Diagonal between P1 and P3**

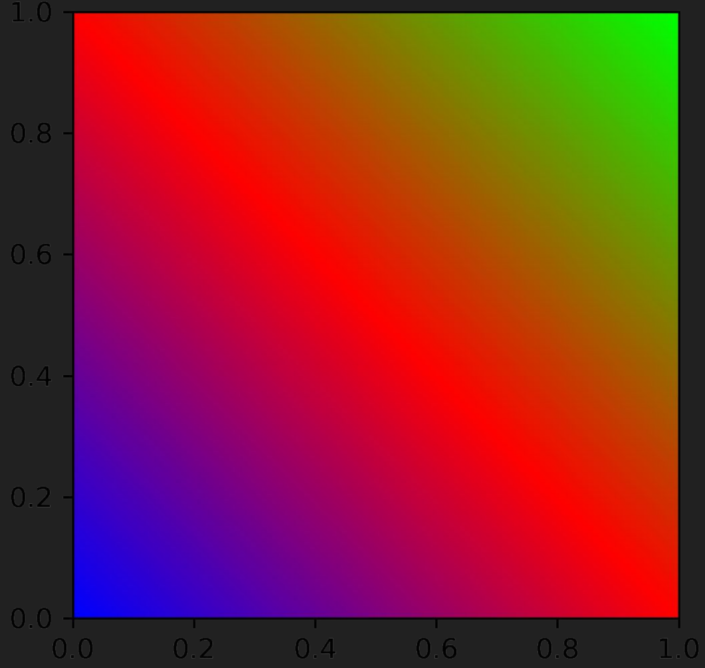


From P0 to Intersection: Blue to Red

From P2 to intersection: Green to Red

From P0 to P2: Blue to Red to Green

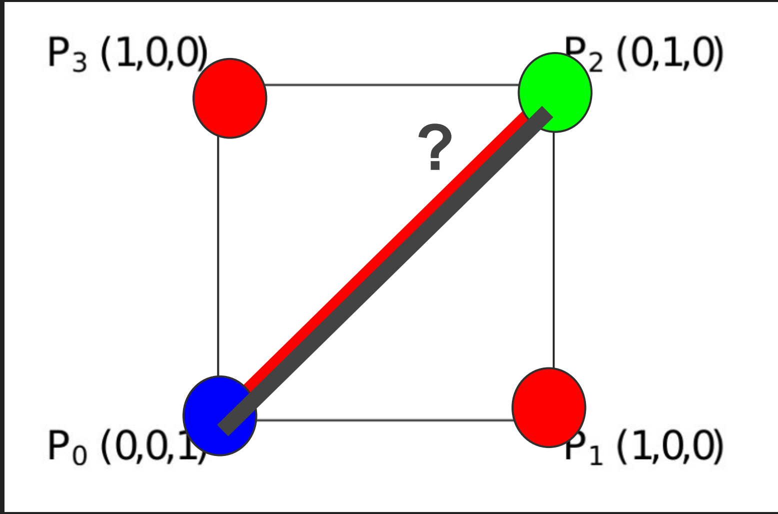
Exercise 2: Barycentric Coordinates



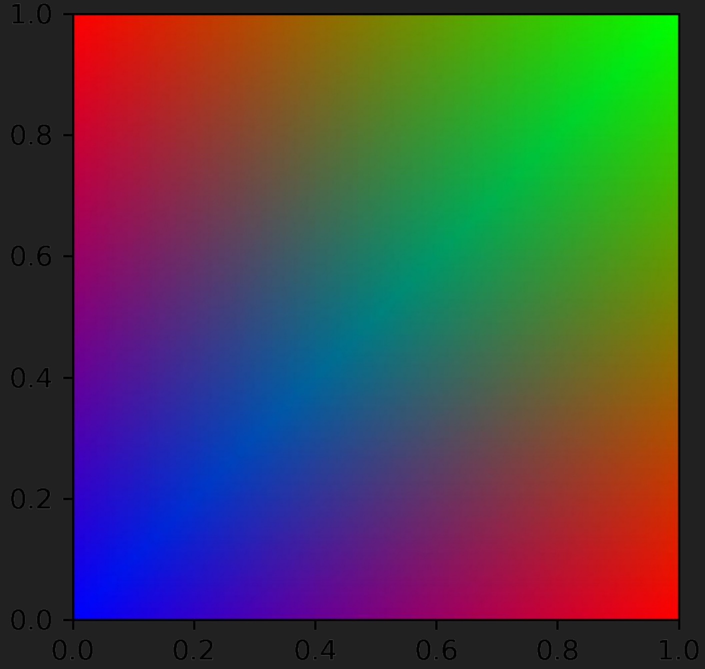
Exercise 2: Barycentric Coordinates

c) Diagonal between P0 and P2

Interpolation between P0 and P2 would then be shift from blue to green without containing any red



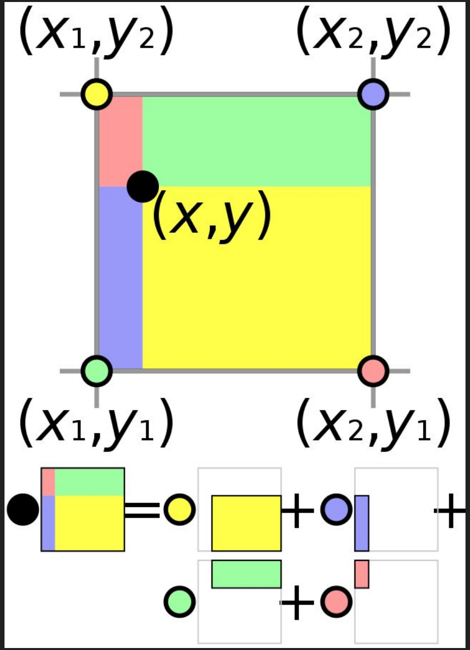
Exercise 2: Barycentric Coordinates

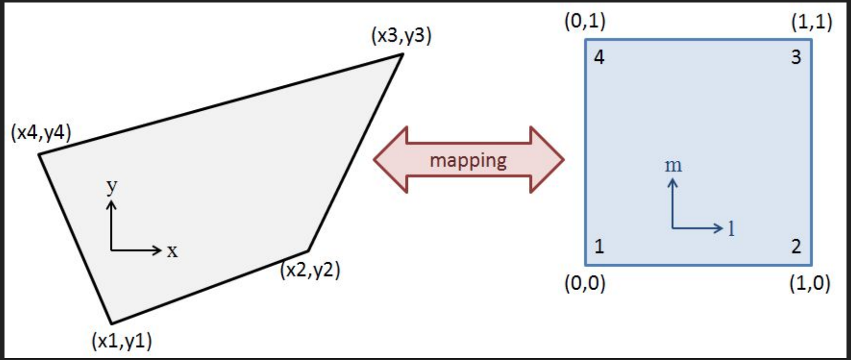


Exercise 2: Barycentric Coordinates

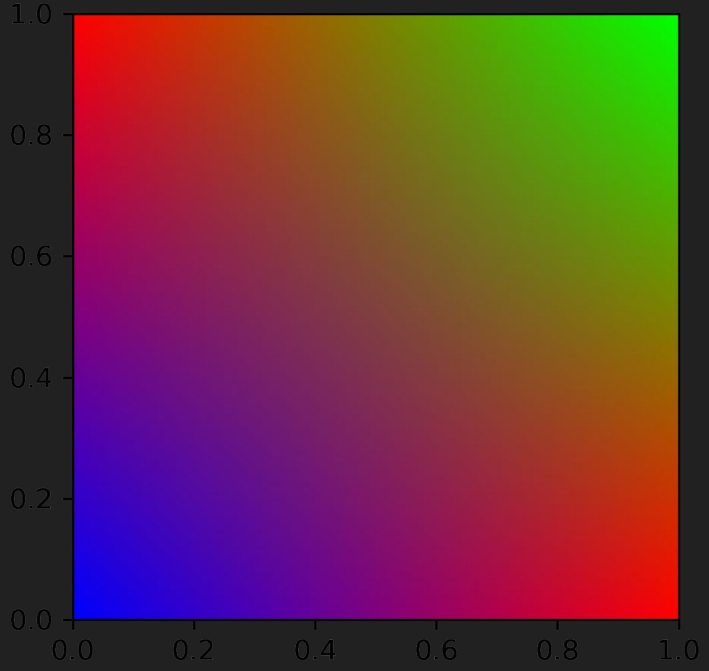
d) Direct calculation of barycentric coordinates for a rectangle

- Idea: Use bilinear interpolation
  - Calculate area of the small rectangles and normalize with full area
  - Problem: Only rectangles
- For arbitrary quadrangles:
  - Newton Method

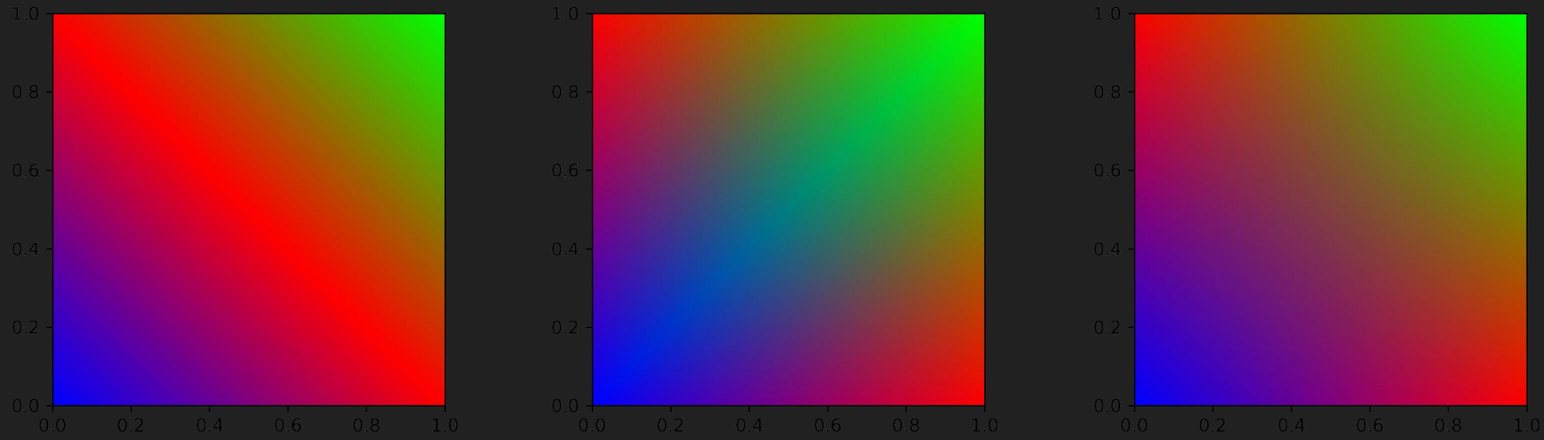




Exercise 2: Barycentric Coordinates



Exercise 2: Barycentric Coordinates

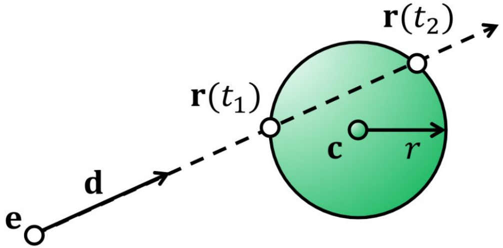


Exercise 3: Ray Tracing

Determination of Intersection

Ray-sphere intersection

- Ray equation (parametric) $\mathbf{r}(t) = \mathbf{e} + t\mathbf{d}$
- Sphere (implicit representation) $\|\mathbf{x} - \mathbf{c}\|^2 - r^2 = 0$
- Plug in:
  - $\|\mathbf{r}(t) - \mathbf{c}\|^2 - r^2 = 0$
  - $\|\mathbf{e} + t\mathbf{d} - \mathbf{c}\|^2 - r^2 = 0 \quad (\|\mathbf{x}\|^2 = \mathbf{x} \cdot \mathbf{x})$
  - $(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - r^2 = 0$
  - $\underbrace{(\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - r^2}_{const.} + \underbrace{2(t\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}))}_{t(2\mathbf{d}\cdot(\mathbf{e}-\mathbf{c}))} + \underbrace{(t\mathbf{d}) \cdot (t\mathbf{d})}_{t^2(\mathbf{d}\cdot\mathbf{d})} = 0$

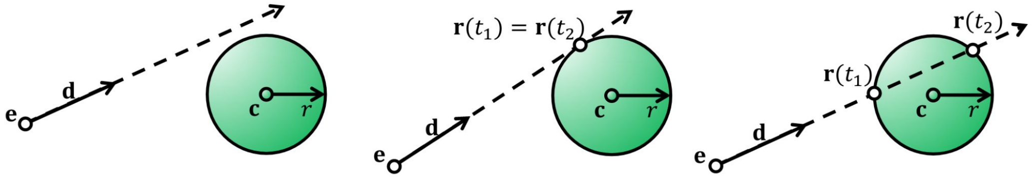


Exercise 3: Ray Tracing

Determination of Intersection

Ray-sphere intersection

- $\underbrace{(\mathbf{e}-\mathbf{c})\cdot(\mathbf{e}-\mathbf{c})-r^2}_{const.} + \underbrace{2(t\mathbf{d}\cdot(\mathbf{e}-\mathbf{c}))}_{t(2\mathbf{d}\cdot(\mathbf{e}-\mathbf{c}))} + \underbrace{(t\mathbf{d})\cdot(t\mathbf{d})}_{t^2(\mathbf{d}\cdot\mathbf{d})} = 0$
- Quadratic equation:
  - $t_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
    with $a = \mathbf{d}\cdot\mathbf{d}$, $b = 2\mathbf{d}\cdot(\mathbf{e}-\mathbf{c})$, $c = (\mathbf{e}-\mathbf{c})\cdot(\mathbf{e}-\mathbf{c})-r^2$
  - Discriminant $b^2 - 4ac$



discriminant < 0 → no solution

discriminant $= 0$ → one solution

discriminant > 0 → two solutions