Fundamental of Simulation Methods

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Problem Set 5: FFT-based convolution

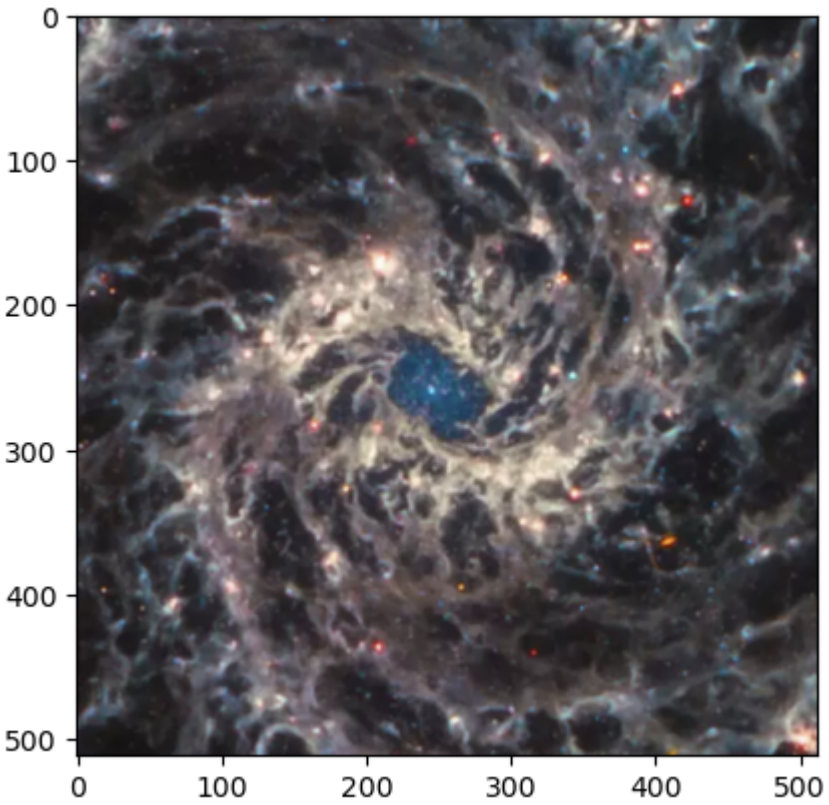
a) nicht gemacht b)

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats

#Reads a square image in 8-bit/color PPM format from the given file. Note
def readImage(filename):
    f = open(filename,"rb")

    f.readline()
    s = f.readline()
    f.readline()
    (pixel, pixel) = [t(s) for t,s in zip((int,int),s.split())]

    data = np.fromfile(f,dtype=np.uint8,count = pixel*pixel*3)
    img = data.reshape((pixel,pixel,3)).astype(np.double)

    f.close()

    return img, pixel


#Writes a square image in 8-bit/color PPM format.
def writeImage(filename, image):
    f = open(filename,"wb")

    pixel = image.shape[0]
    header= "P6\n%d %d\n%d\n"%(pixel, pixel, 255)
    f.write(bytearray(header, 'ascii'))

    image = image.astype(np.uint8)

    image.tofile(f)

    f.close()


# function to compute the power spectrum
def _compute_spectrum_2D(fourier_2D_data):

    npix = fourier_2D_data.shape[0]

    fourier_amplitudes = np.abs(fourier_2D_data)**2
```

```
    kfreq = np.fft.fftfreq(npix) * npix
    kfreq2D = np.meshgrid(kfreq, kfreq)
    knrm = np.sqrt(kfreq2D[0]**2 + kfreq2D[1]**2)

    knrm = knrm.flatten()
    fourier_amplitudes = fourier_amplitudes.flatten()

    kbins = np.arange(0.5, npix//2+1, 1.)
    kvals = 0.5 * (kbins[1:] + kbins[:-1])

    Abins, _, _ = stats.binned_statistic(knrm, fourier_amplitudes,
                                         statistic = "sum",
                                         bins = kbins)
    Abins = Abins * np.pi * (kbins[1:]**2 - kbins[:-1]**2)
    return kvals, Abins
```

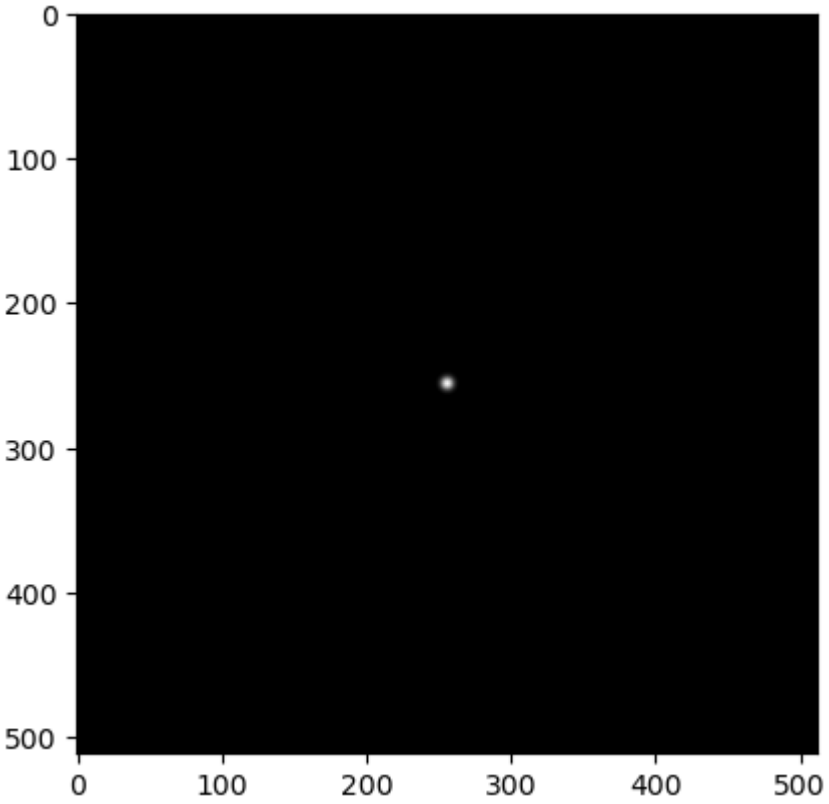
In [ ]:
```
img, pixel = readImage("ngc628-original.ppm")
print(img.sum())
img = img/img.max()

plt.imsave("normal.png",img/img.max())
plt.imshow(img/img.max())
plt.show()
```

```
61850618.0
```



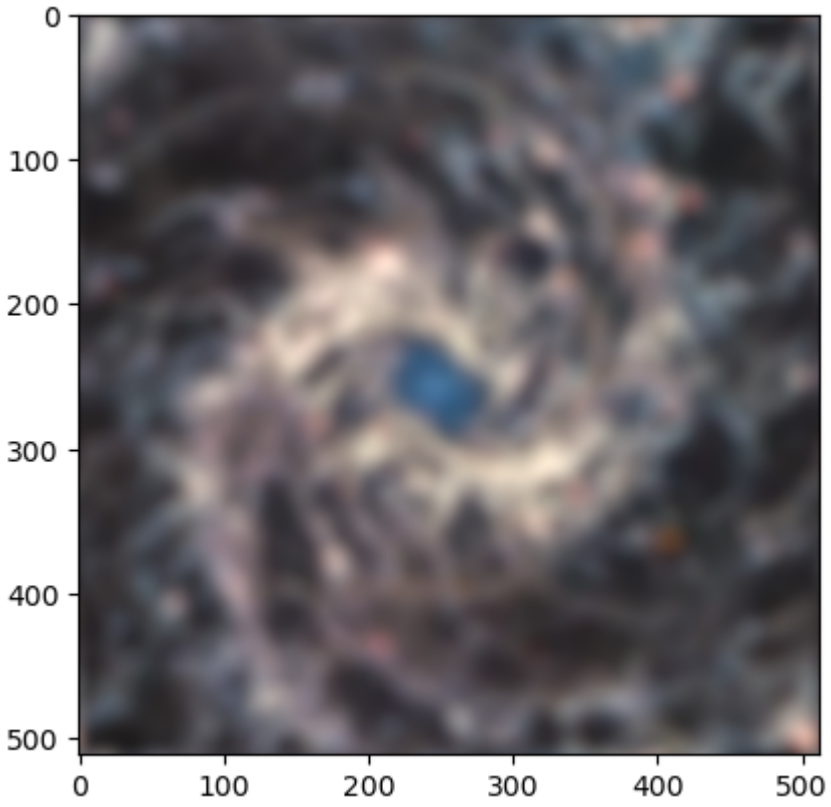
In [ ]:
```
def Kernel(x,y, h):
    r = np.sqrt(x**2+y**2)
    if 0 <= r/h  < 0.5:
        return 1 - 6*(r/h)**2 + 6*(r/h)**3
    elif 0.5 <=  r/h < 1:
        return 2*(1-r/h)**3
    else:
        return 0
```

```
#Now we set up our desired smoothing kernel. We'll use complex number for
kernel_real = np.zeros((pixel,pixel),dtype=np.complex128)

hsml = 10.

nx,ny,cs = img.shape

#now set the values of the kernel
for i in np.arange(pixel):
    for j in np.arange(pixel):
        ix, iy = i-nx//2, j-ny//2
        kernel_real[i,j] = Kernel(ix, iy,hsml)

# Normalization
kernel_real = kernel_real/np.sum(kernel_real)

plt.imshow(kernel_real.real, cmap="gray")
plt.show()
```



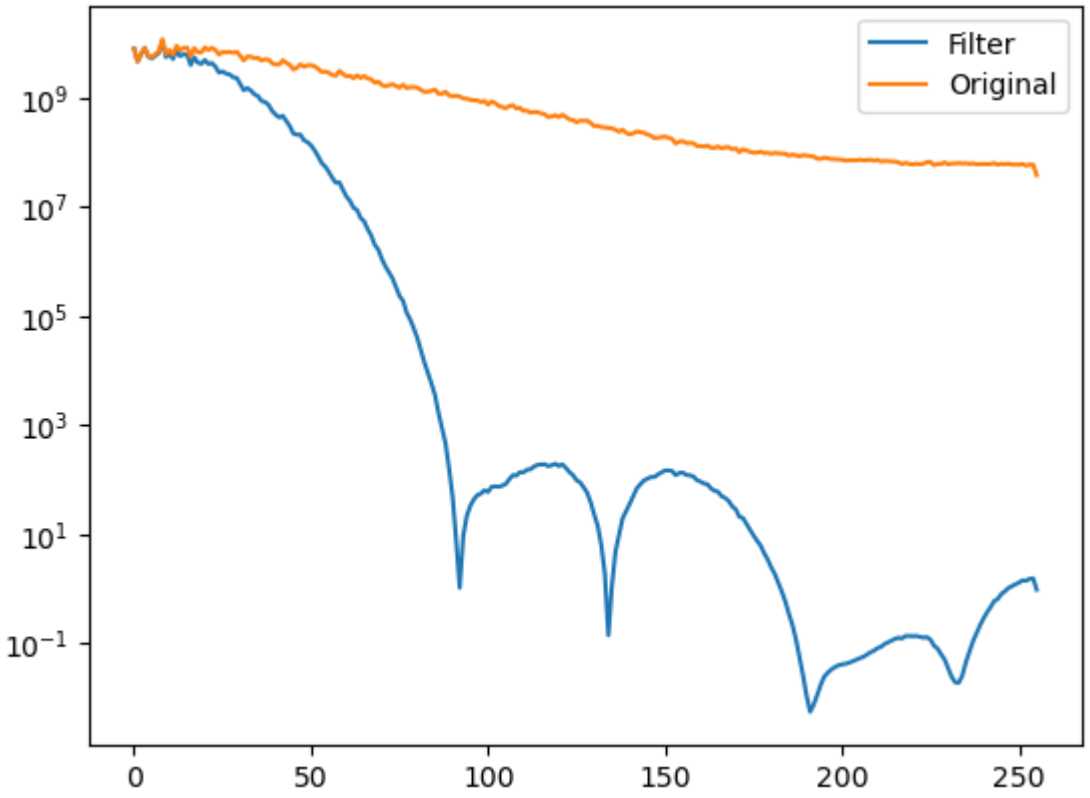
In [ ]:
```
kernel_real = np.fft.ifftshift(kernel_real)

#Let's calculate the Fourier transform of the kernel
kernel_kspace = np.fft.fft2(kernel_real)

#further space allocations for image transforms
color_real = np.zeros((pixel,pixel),dtype=np.complex128)

# variables for the spectra
# - store spectrum of the original image and the smoothed one
# - three spectra for each color
spectrum_original = np.zeros((3,pixel//2))#pixel, pixel))
spectrum_filter   = np.zeros((3,pixel//2))#pixel, pixel))

#we now convolve each color channel with the kernel using FFTs
```

```
for colindex in np.arange(3):
    #copy input color into complex array
    color_real[:,:].real = img[:,:,colindex]

    #forward transform
    color_kspace = np.fft.fft2(color_real)

    #store image spectrum for original image
    _, spectrum_original[colindex] = _compute_spectrum_2D(color_kspace) #

    #multiply with kernel in Fourier space
    color_kspace = color_kspace * np.conj(kernel_kspace)

    #store image spectrum for smoothed image
    _, spectrum_filter[colindex] =  _compute_spectrum_2D(color_kspace)#np

    #backward transform
    color_real = np.fft.ifft2(color_kspace)

    #copy real value of complex result back into color array
    img[:,:,colindex] = color_real.real

img = img/img.max()

plt.imshow(img)
plt.imsave("smooethed.png",img)
plt.show()
print(img.sum())
writeImage("ngc628-smoothed.ppm", img)
```




```
280681.99246776744
```

In [ ]:
```
plt.plot(spectrum_filter.sum(axis=0), label="Filter")
plt.plot(spectrum_original.sum(axis=0), label="Original")
plt.legend()
plt.yscale("log")
```



In [ ]:
```
def Kernel(x,y, h):
    r = np.sqrt(x**2+y**2)
    if r/h <= 1:
        return 1
    else:
        return 0

#Now we set up our desired smoothing kernel. We'll use complex number for
kernel_real = np.zeros((pixel,pixel),dtype=np.complex128)

hsml = 10.

nx,ny,cs = img.shape

#now set the values of the kernel
for i in np.arange(pixel):
    for j in np.arange(pixel):
        ix, iy = i-nx//2, j-ny//2
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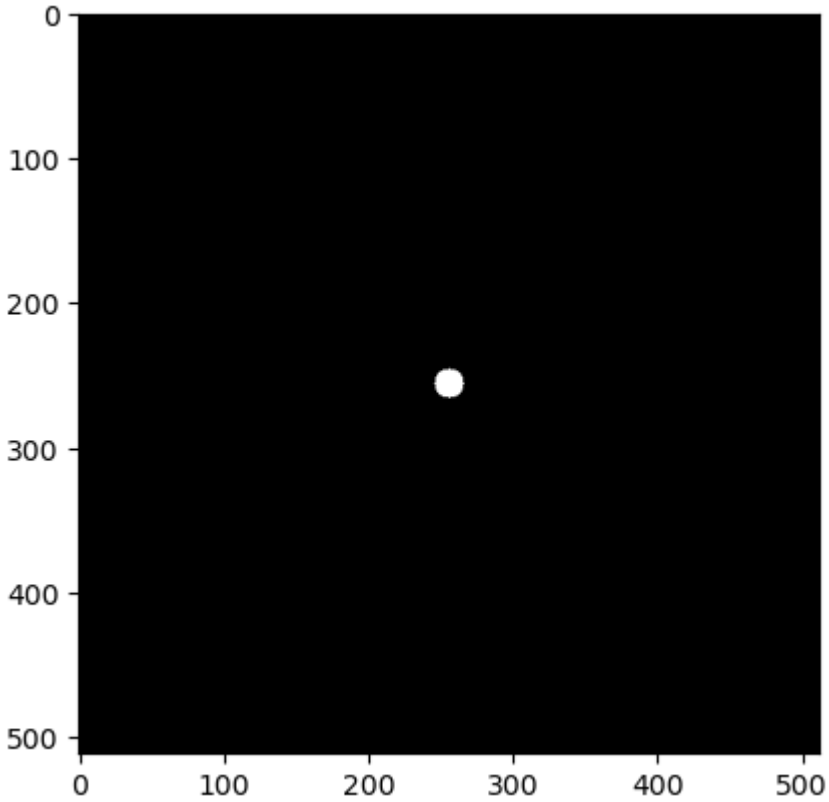

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```
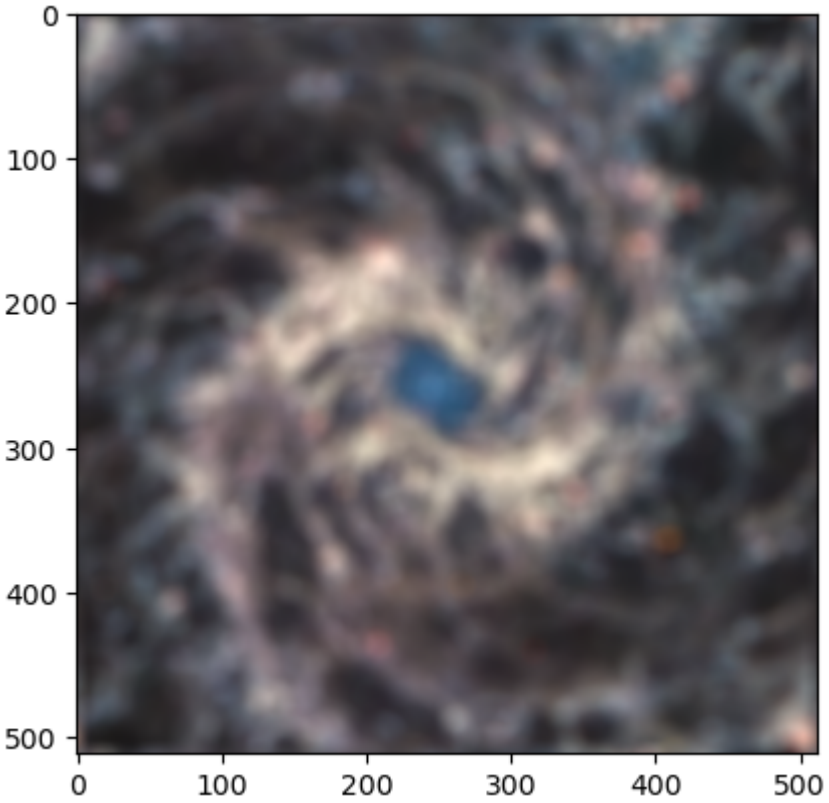

```
    #copy real value of complex result back into color array
    img[:,:,colindex] = color_real.real

img = img/img.max()

plt.imshow(img)
plt.imsave("smooethed.png",img)
plt.show()

writeImage("ngc628-tophat.ppm", img)
```



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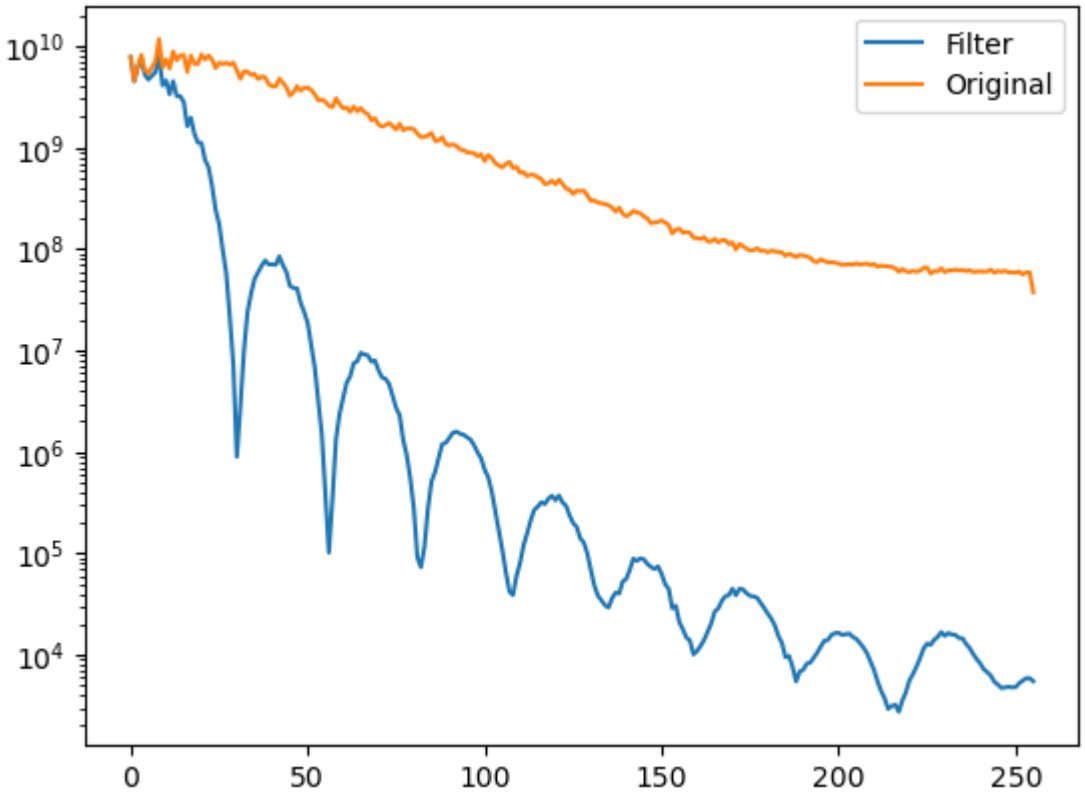
Filter
Original
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
0
50
100
150
200
250

In [ ]: