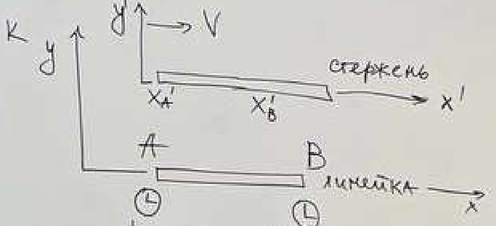
Задача 1

Дано:

ΔX_1

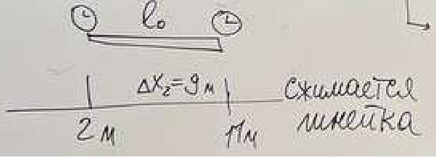
ΔX_2

ищем l_0



$t_A = t_B$

$\Delta X_1 = 4\text{м} = X_B - X_A$



(L) $K \to K'$

$$l_0 = \frac{\Delta X_1}{\sqrt{1-\frac{V^2}{c^2}}} \quad (1)$$

$$l = l_0\sqrt{1-\frac{V^2}{c^2}} < l_0$$

сокращение длины движущегося отрезка

$$\Delta X_2 = \frac{l_0}{\sqrt{1-\frac{V^2}{c^2}}} \quad (2)$$

$$X'_A = \frac{X_A - Vt_A}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$X'_B = \frac{X_B - Vt_B}{\sqrt{1-\frac{V^2}{c^2}}}$$

$t_A = t_B$ по условию!

$$X'_B - X'_A = l_0 = \frac{X_B - X_A}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{\Delta X_1}{\sqrt{1-\frac{V^2}{c^2}}} > \Delta X_2$$

длина стержня там, где он покоится

$$1 - \frac{V^2}{c^2} = \frac{\Delta X_1}{\Delta X_2}$$

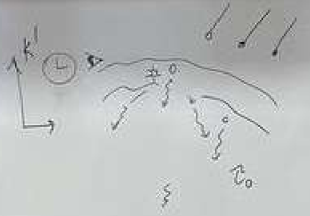
$$V = c\sqrt{1 - \Delta X_1/\Delta X_2} \quad (**)$$

$$\frac{l_0}{\Delta X_1} = \frac{\Delta X_2}{l_0}$$

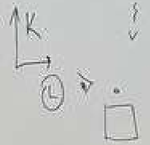
$$l_0 = \sqrt{\Delta X_1 \Delta X_2} \quad (*)$$

$$\sqrt{\Delta X_1 \Delta X_2} = \frac{\Delta X_1}{\sqrt{1-V^2/c^2}}$$

$$\sqrt{\frac{\Delta X_2}{\Delta X_1}} = \frac{1}{\sqrt{1-V^2/c^2}}$$



Задача 2



$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

в K-системе

$S - ?$

$\tau = 3\cdot10^{-6}$ с в K

$\tau_0 = 2{,}2\cdot10^{-6}$ с в K'

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \frac{\tau_0^2}{\tau^2}$$

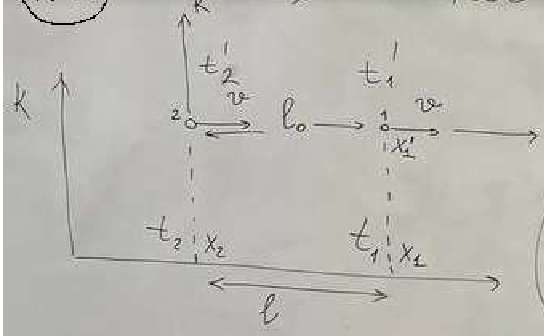
$$v = c\sqrt{1-\frac{\tau_0^2}{\tau^2}}$$

в K-системе

$$S = c\cdot\tau\sqrt{1-\frac{\tau_0^2}{\tau^2}}$$

1.3

$K' \longrightarrow V = v = 0{,}99c$



$t_1' = t_2'$

$K' \to K \cdot L^{-1}$

$$t_1 = \frac{t_1' + v x_1'/c^2}{\sqrt{1 - v^2/c^2}}$$

$$t_2 = \frac{t_2' + v x_2'/c^2}{\sqrt{1 - v^2/c^2}}$$

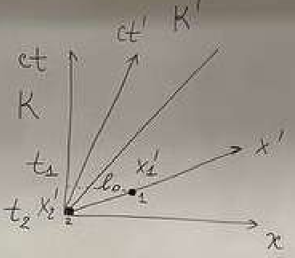


Диаграмма Минковского

$$t_1 - t_2 = \frac{(x_1' - x_2')\, v/c^2}{\sqrt{1 - v^2/c^2}} = \frac{l v/c^2}{(1 - v^2/c^2)}$$

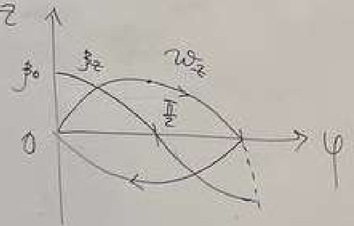
$$l_0 = x_1' - x_2' = \frac{l}{\sqrt{1 - v^2/c^2}}$$

2.1

$\vec{\beta} = \vec{\beta}_0 \cos\varphi$

$\vec{\beta}_0 = const$

$\omega_z(\varphi) - ?$



Oz вдоль $\vec{\beta}_0$

В проекциях:

$\beta_z = \beta_0 \cos\varphi$

$$\beta_z = \frac{d\omega_z}{dt} = \frac{d\omega_z}{d\varphi} \cdot \overset{\omega_z}{\frac{d\varphi}{dt}} = \frac{\omega_z d\omega_z}{d\varphi}$$

Тело совершает колебания около $\varphi = \frac{\pi}{2}$

$$\frac{\omega_z d\omega_z}{d\varphi} = \beta_0 \cos\varphi$$

$$\int_0^{\omega_z} \omega_z d\omega_z = \int_0^{\varphi} \beta_0 \cos\varphi \, d\varphi$$

$$\frac{\omega_z^2}{2} = \beta_0 \sin\varphi, \quad 0 \le \varphi \le \pi$$

$$\omega_z = \pm\sqrt{2\beta_0 \sin\varphi}$$

2

$$w = \sqrt{\left(\frac{v}{r}\right)^2 + \left(\frac{v}{h}\right)^2} = \frac{v}{r}\sqrt{1+\left(\frac{r}{h}\right)^2}$$

$$\vec{w} = \vec{w}_\perp + \vec{w}_\parallel$$

$\vec{w}_\parallel = const$

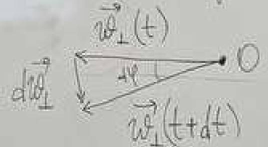
$|\vec{w}_\perp| = const$

$\vec{w}_\perp \neq const$

$\rightarrow \dfrac{d\vec{w}}{dt} = \dfrac{d\vec{w}_\perp}{dt}$

$w_\parallel = \frac{v}{h}$; $\vec{w}_\parallel = const$

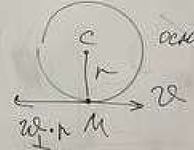
Вид сверху

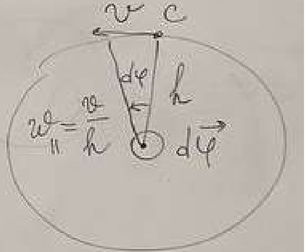


$$d\vec{w}_\perp = [d\vec{\varphi}; \vec{w}_\perp] = [\vec{w}_\parallel; \vec{w}_\perp]\, dt$$

$$d\vec{\varphi} = \vec{w}_\parallel\, dt$$

$$\vec{w} \uparrow\uparrow \vec{MO}$$

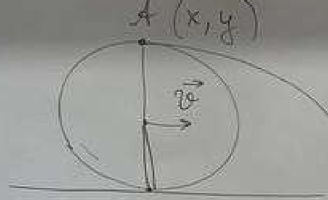




$\rightarrow \vec{\beta} = [\vec{w}_\parallel; \vec{w}_\perp]$, $\quad \vec{w}_\parallel \perp \vec{w}_\perp$

$$\beta = \frac{v}{h}\cdot\frac{v}{r} = \frac{v^2}{rh}$$

3



$b = const$

$\omega = const$

$v_A - ?$

$a_A - ?$

$x = b(\omega t - \sin \omega t)$

$y = b(1 - \cos \omega t)$

$S - ?$

$\boxed{\frac{1-\cos\alpha}{2} = \sin^2\frac{\alpha}{2}}$

$v = \sqrt{v_x^2 + v_y^2}$

$v_x = \frac{dx}{dt} = b\omega(1 - \cos\omega t)$

$v_y = \frac{dy}{dt} = b\omega \sin\omega t$

$$\boxed{\begin{aligned} a &= \sqrt{a_x^2 + a_y^2} = b\omega^2 \\ |a_x| &= \left|\frac{dv_x}{dt}\right| = b\omega^2 \sin\omega t \\ |a_y| &= \left|\frac{dv_y}{dt}\right| = b\omega^2 \cos\omega t \end{aligned}}$$

$$v = b\omega\sqrt{(1-\cos\omega t)^2 + \sin^2\omega t} =$$

$$= b\omega\sqrt{1 - 2\cos\omega t + 1} = b\omega\sqrt{2(1-\cos\omega t)} =$$

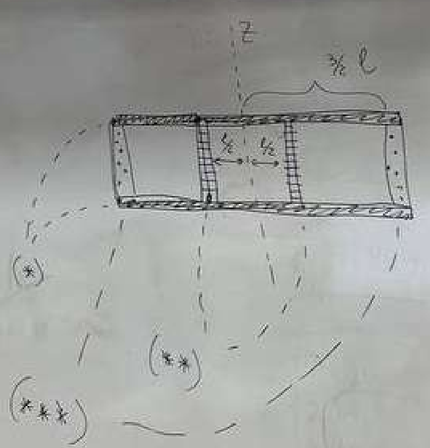
$$= b\omega\sqrt{2\cdot 2\sin^2\frac{\omega t}{2}} = 2b\omega\left|\sin\frac{\omega t}{2}\right|$$

$$S = \int v\,dt = \int \frac{v}{\omega}\,d\varphi = \frac{2b\omega}{\omega}\int_0^{2\pi}\sin\frac{\varphi}{2}\,d\varphi = 4b\underbrace{\int_0^{2\pi}\sin\left(\frac{\varphi}{2}\right)d\left(\frac{\varphi}{2}\right)}_{2} = 8b$$

$$\frac{d\varphi}{dt} = \omega \rightarrow dt = \frac{d\varphi}{\omega}$$

Задача 1

10 стержней $J_z - ?$



| — 1 стержень, m, l

$(**)\quad I = MR^2$
$M = m$
$R = \frac{l}{2}$
$\times 2$

$$\boxed{I_2 = \frac{ml^2}{2}}$$

$(***)\quad I = MR^2$
$M = m,\ R = \frac{3}{2}l$
$\times 2$

$$\boxed{I_3 = \frac{9}{2}ml^2}$$

$J = J_1 + J_2 + J_3$

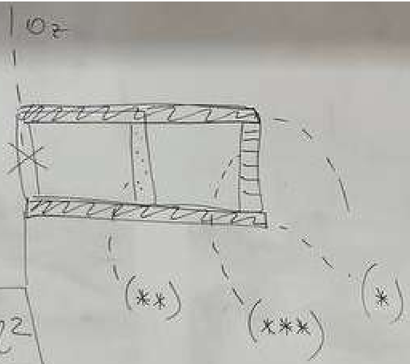
$(*)\quad I = \frac{ML^2}{12}$

$M = 3m$
$L = 3l$
$\times 2$

$$\boxed{I_1 = \frac{9}{2}ml^2}$$

$J = 9{,}5\ ml^2$

Задача (2)



(*) $I = \frac{ML^2}{3}$

$\left.\begin{matrix} M = 2m \\ L = 2\ell \end{matrix}\right) \times 2$ $\boxed{J_1 = \frac{16}{3} m\ell^2}$

(**) $\left.\begin{matrix} I = MR^2 \\ M = m, \ R = \ell \end{matrix}\right) \times 1$ $\boxed{J_2 = m\ell^2}$

$I = J_1 + J_2 + J_3 = \left(\frac{16}{3} + \frac{3}{3} + \frac{12}{3}\right) m\ell^2 = \frac{31}{3} m\ell^2$

(***) $\left.\begin{matrix} I = MR^2 \\ M = m, \ R = 2\ell \end{matrix}\right) \times 1$

$\boxed{I_3 = 4m\ell^2}$

Задача (3)

$\frac{53}{12}$ $\frac{60}{12}$

$$\frac{113}{12} ml^2 = J = J_1 + J_2 + J_3 + J_4 = \left(\frac{13}{3} + \frac{1}{12} + \frac{1}{2} + \frac{9}{2}\right) ml^2$$

(*) $J = \frac{13}{12} ML^2$, $M = m$, $L = l$ $\quad \times 4$

$$\boxed{J_1 = \frac{13}{3} ml^2}$$

(****) $J = MR^2$, $M = m$, $R = \frac{3}{2} l$ $\quad 2$

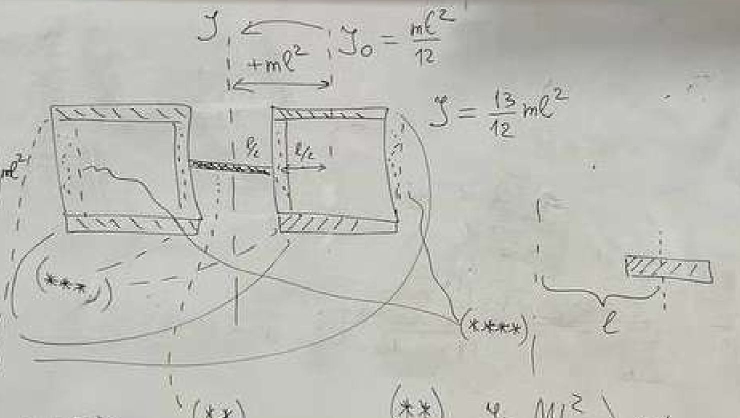
$$\boxed{J_4 = \frac{9}{2} ml^2}$$

(***) $J = MR^2$, $M = m$, $R = \frac{l}{2}$ $\quad \times 2$

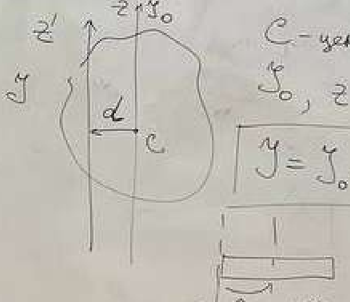
$$\boxed{J_3 = \frac{ml^2}{2}}$$

(**) $J = \frac{Ml^2}{12}$, $M = m$, $L = l$ $\quad \times 1$

$$\boxed{J_2 = \frac{ml^2}{12}}$$



Теорема Штейнера



C - центр масс!

J_0, z через C

$$\boxed{J = J_0 + md^2}$$

расстояние между осями

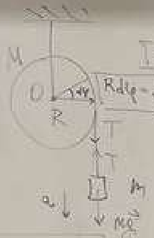
$\frac{ml^2}{3}$ $\quad \frac{l}{2}$ $\quad \frac{ml^2}{12}$

(1)

$T_M = \frac{I\omega^2}{2} = \frac{MR^2\omega^2}{4}$

$T_m = \frac{mv^2}{2}$ $v_0 = 0$

$T = T_M + T_m$



$\boxed{R\,d\varphi = dy}$

$I = \frac{MR^2}{2}$

$\omega(t) - ?$

$T(t) - ?$

Тело m : Oy : $mg - T = ma = m\varepsilon R$

цилиндр M : Oz : $\frac{dL_z}{dt} = M_z \rightarrow L_z = I\omega$, $I = \frac{MR^2}{2}$; $M_z = TR$

Относ-но точки O

$I\frac{d\omega}{dt} = TR$, $\varepsilon = \frac{d\omega}{dt}$

Кинематическая связь

$\omega = \frac{v}{R}$; $\boxed{\varepsilon = \frac{a}{R}}$

$v = R\omega$

$T = mg - m\varepsilon R$

$T = \frac{I\varepsilon}{R}$

$\rightarrow mg - m\varepsilon R = \frac{I\varepsilon}{R} \longrightarrow \varepsilon = \frac{mg}{mR + \frac{I}{R}} = \frac{mg}{mR + \frac{MR^2}{2R}}$

$\varepsilon = \frac{mg}{R(m + M/2)} = const = \frac{d\omega}{dt}$

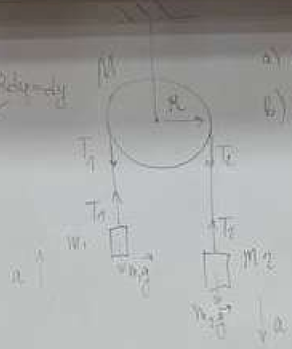
1) $\boxed{\omega = \varepsilon t = \frac{mgt}{R(m + \frac{M}{2})}}$

2) $v = \omega R = \frac{mgt}{(m + \frac{M}{2})}$

$T = \frac{MR^2}{4} \cdot \frac{m^2g^2t^2}{R^2(m + M/2)^2}$ (3)

(4) $\frac{m^2g^2t^2}{2(m + \frac{M}{2})^2} = \frac{\frac{1}{2}g^2t^2m^2(\frac{M}{2} + m)}{(m + M/2)^2} = \frac{mg^2t^2}{2} \cdot \frac{m}{m + M/2}$

(2)



$\boxed{\varepsilon = \frac{a}{R}}$

a) ε - ?

b) $\frac{T_1}{T_2}$ - ?

a) ЗДД m_1: $T_1 - m_1 g = m_1 a = m_1 \varepsilon R$, $\quad T_1 = m_1 \varepsilon R + m_1 g$ (1)

ЗДД m_2: $m_2 g - T_2 = m_2 a = m_2 \varepsilon R$, $\quad T_2 = m_2 g - m_2 \varepsilon R$ (2)

$\longrightarrow T_2 - T_1$

нддр. M: $\boxed{I\varepsilon} = M_z = T_2 R - T_1 R = M_{2z} - M_{1z} = (T_2 - T_1)R = (m_2 g - m_2 \varepsilon R - m_1 \varepsilon R - m_1 g)R$

$I\varepsilon = m_2 g R - m_2 \varepsilon R^2 - m_1 \varepsilon R^2 - m_1 g R$ (3)

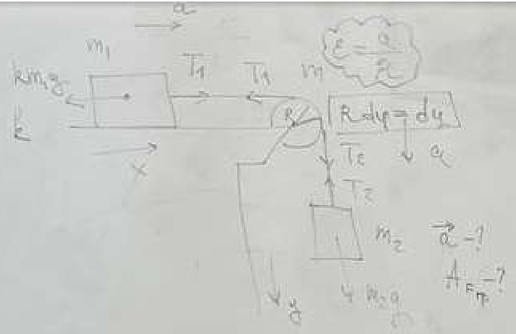
$\varepsilon = \frac{a}{R}$ кин. связь

$$\varepsilon (I + m_1 R^2 + m_2 R^2) = (m_2 g - m_1 g) R$$

$$\varepsilon = \frac{m_2 - m_1}{\frac{MR^2}{2} + m_1 R^2 + m_2 R^2} \cdot gR = \frac{m_2 - m_1}{m_1 + m_2 + M/2} \cdot \frac{g}{R}$$

b) $$\frac{T_1}{T_2} = \frac{m_1 \varepsilon R + m_1 g}{m_2 g - m_2 \varepsilon R} = \frac{m_1 \frac{(m_2 - m_1)}{m_1 + m_2 + M/2} g + m_1 g}{m_2 g - m_2 \frac{(m_2 - m_1)}{m_1 + m_2 + M/2} g} = \frac{m_1 m_2 - \cancel{m_1^2} + \cancel{m_1^2} + m_1 m_2 + m_1 \frac{M}{2}}{m_1 m_2 + \cancel{m_2^2} + m_2 \frac{M}{2} - \cancel{m_2^2} + m_1 m_2} = \frac{m_1 \left(2 m_2 + \frac{M}{2}\right)}{m_2 \left(2 m_1 + \frac{M}{2}\right)}$$

3



груз m_1: $T_1 - km_1g = m_1a \longrightarrow T_1 = m_1a + km_1g$ (1)

груз m_2: $m_2g - T_2 = m_2a \longrightarrow T_2 = m_2g - m_2a$ (2)

блок m: $I\varepsilon = M_z = M_2 - M_1 = T_2R - T_1R = (T_2 - T_1)R$ (3)

$$I\frac{a}{R} = (T_2 - T_1)R = (m_2g - m_2a - m_1a - km_1g)R$$

$$Ia = m_2gR^2 - m_2aR^2 - m_1aR^2 - km_1gR^2$$

$$(I + m_1R^2 + m_2R^2)a = \left(\frac{mR^2}{2} + m_1R^2 + m_2R^2\right)a = m_2gR^2 - km_1gR^2$$

$$\vec{a} = \frac{m_2 - km_1}{\frac{m}{2} + m_1 + m_2}\vec{g}; \quad a_y = \frac{m_2 - km_1}{\frac{m}{2} + m_1 + m_2}g; \quad S = \frac{at^2}{2}$$

б) $A_{F_{тр}} = -F_{тр} \cdot S = -km_1g \cdot \frac{at^2}{2} =$

$$= -km_1g^2\frac{t^2}{2} \cdot \frac{m_2 - km_1}{\frac{m}{2} + m_1 + m_2}$$

(1)

$a_m = 49{,}3$ см/с^2

$T = 2$ с

$\underline{x_0 = 25 \text{ мм}} = x(t=0) = 2{,}5$ см

$x(t) = \underline{x_m} \sin(\omega_0 t + \varphi_0)$

$\omega_0 = \sqrt{\frac{k}{m}}$

$\dot{x} = v = x_m \omega_0 \cdot \cos(\omega_0 t + \varphi_0)$, $\boxed{v_m = x_m \omega_0}$

Ну

(1) $2\pi\nu = \frac{2\pi}{T} = \omega_0$

$\omega_0 = \frac{2\pi}{T} = \pi\,(с^{-1})$

$\ddot{x} = a = -x_m \omega_0^2 \cdot \sin(\omega_0 t + \varphi_0)$

(2) $\boxed{a_m = x_m \omega_0^2}$

$x_m = \frac{a_m}{\omega_0^2} = \frac{49{,}3 \text{ см/с}^2}{\pi^2 (с^{-1})^2} = 5$ (см)

(3) При $t = 0$ $\underset{x_0}{x} = x_m \cdot \sin(\omega_0 \overset{0}{t} + \varphi_0) \to$

$\sin \varphi_0 = \frac{x_0}{x_m} = \frac{2{,}5}{5} = \frac{1}{2}$; $\varphi_0 = \frac{\pi}{6}$

Ответ: $x(t) = 5 \sin\left(\pi t + \frac{\pi}{6}\right)$

$x(t) = 5 \cos\left(\pi t - \frac{\pi}{3}\right)$

2

$v = \frac{dx}{dt}$

$\varphi_0 = 0$

$x_1 = 2{,}4$ см $\quad$ $x_2 = 2{,}8$ см

$v_1 = 3$ см/с $\quad$ $v_2 = 2$ см/с

$t_1 \quad t_2$

$x_m - ?$

$T - ?$

$$x(t) = x_m \sin(\omega_0 t + \varphi_0) = x_m \sin(\omega_0 t) \to \sin \omega_0 t = \frac{x}{x_m}$$

$$\frac{dx}{dt} = \dot{x}(t) = v(t) = x_m \omega_0 \cdot \cos(\omega_0 t) \to \cos \omega_0 t = \frac{v}{x_m \omega_0}$$

для любого t!

$$\sin^2\alpha + \cos^2\alpha = 1 \to \left(\frac{x}{x_m}\right)^2 + \left(\frac{v}{x_m\omega_0}\right)^2 = 1, \quad x^2 + \frac{v^2}{\omega_0^2} = x_m^2$$

$$\begin{cases} x_1^2 + \frac{v_1^2}{\omega_0^2} = x_m^2 & \text{при } t_1 \\ x_2^2 + \frac{v_2^2}{\omega_0^2} = x_m^2 & \text{при } t_2 \end{cases}$$

$$x_1^2 + \frac{v_1^2}{\omega_0^2} = x_2^2 + \frac{v_2^2}{\omega_0^2} \to x_2^2 - x_1^2 = \frac{v_1^2 - v_2^2}{\omega_0^2}$$

$$\boxed{\omega_0^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

$$x_m^2 = x_1^2 + \frac{v_1^2(x_2^2 - x_1^2)}{v_1^2 - v_2^2} = \frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}$$

$$\boxed{x_m = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}}$$

$v_m = x_m \omega_0$

$a_m = x_m \omega_0^2$

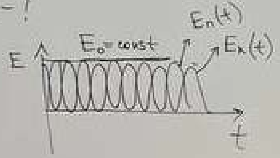
③

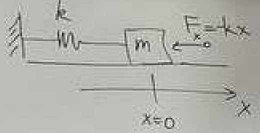
$x_m = 2$ см

$E_0 = E_к + E_п = 0{,}3$ мкДж

$F = 22{,}5$ мкН

$x - ?$





$$x(t) = x_m \sin(\omega_0 t + \varphi_0)$$

$$\dot{x}(t) = v(t) = x_m \omega_0 \cos(\omega_0 t + \varphi_0)$$

$$\frac{m\omega_0^2}{2} \qquad \omega_0^2 = \frac{k}{m}$$

$$E_к = \frac{mv^2}{2} = \frac{m x_m^2 \omega_0^2}{2} \cos^2(\omega_0 t + \varphi_0) = \frac{k x_m^2}{2} \cdot \cos^2(\omega_0 t + \varphi_0)$$

$$E_п = \frac{kx^2}{2} = \frac{k x_m^2}{2} \sin^2(\omega_0 t + \varphi_0) = \frac{k x_m^2}{2} \cdot \sin^2(\omega_0 t + \varphi_0)$$

$$E_0 = E_к + E_п = \frac{k x_m^2}{2} \left(\cos^2(\omega_0 t + \varphi_0) + \sin^2(\omega_0 t + \varphi_0)\right) = \frac{k x_m^2}{2}$$

средние значения:

$$\langle E_к \rangle = \frac{k x_m^2}{4}$$

$$\langle E_п \rangle = \frac{k x_m^2}{4}$$

Значит, $k = \frac{2E_0}{x_m^2}$ и $|\vec{F}| = kx \rightarrow \boxed{x = \frac{F}{k} = \frac{F x_m^2}{2E_0}}$