① $a = a_0 - bS$, $S_0 - ?$, $v_{max} - ?$

Мы знаем:

$$a = \frac{dv}{dt} = \frac{dv}{dS} \cdot \frac{dS}{dt} = \frac{v\,dv}{dS}$$

$$\frac{v\,dv}{dS} = a_0 - bS, \rightarrow v\,dv = (a_0 - bS)\,dS$$

$$\int_A^B d\frac{v^2}{2} = 0$$

$$\int_0^0 v\,dv = \int_{A\,0}^{B\,S_0} (a_0 - bS)\,dS = 0$$

$$a_0 S_0 - \frac{bS_0^2}{2} = 0 \Rightarrow \boxed{S_0 = \frac{2a_0}{b}}$$

$v(S) - ?$

$$\frac{v^2}{2} = \int_0^v d\frac{v^2}{2} = \int_0^S (a_0 - bS)\,dS = a_0 S - \frac{bS^2}{2} \Rightarrow v = \sqrt{2a_0 S - bS^2} = v(S)$$

v_{max} при $\frac{dv}{dS} = 0 \Rightarrow \frac{1}{2}\frac{2a_0 - 2bS}{\sqrt{2a_0 S - bS^2}} = 0 \Rightarrow S\Big|_{v_{max}} = \frac{a_0}{b} = \frac{S_0}{2}$

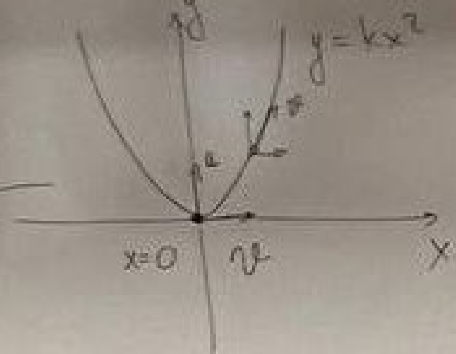
$$v_{max} = \sqrt{2a_0 \frac{a_0}{b} - b\frac{a_0^2}{b^2}} \Leftarrow \frac{v^2}{2}$$

$$\boxed{v_{max} = \frac{a_0}{\sqrt{b}}}$$

1.1

② $\boxed{y = kx^2}$, $v = const$

$a(x=0) - ?$



Мы знаем

$\begin{cases} v_y = 0 \\ v_x = v \end{cases}$ $\quad a_\tau = 0$, $\quad a_n = \frac{v^2}{R}$

$\boxed{k = const}$

$\boxed{a = a_y = a_n}$

$y = kx^2$

$\frac{d}{dt} \to \dot{y} = \underbrace{2kx}_{1} \cdot \underbrace{\dot{x}}_{2}$

$\frac{d}{dt} \to \ddot{y} = 2k\,\dot{x}^2 + \underbrace{2kx \cdot \ddot{x}}_{=0} \quad \Big| \; x = 0$

$\ddot{y} = a_y = a_n = 2k \cdot (\dot{x})^2 = \boxed{2kv^2 = a}$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$2kx$ $\quad \dot{x}$

$\dot{x} = \frac{dx}{dt}$

$\ddot{x} = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

(3)

$$|\vec{a_\tau}| = |\vec{a_n}| \quad \text{Дано} \qquad r = const$$

Мы знаем:

$a_\tau = \frac{dv}{dt} < 0$ замедление!

$a_n = \frac{v^2}{r} > 0 \quad a_\tau = -a_n$

$a(s)$ — ?

$v(s)$ — ?

$a = \sqrt{a_\tau^2 + a_n^2} = a_n\sqrt{2}$

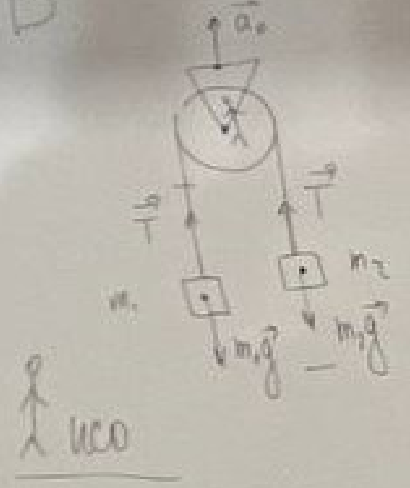
(2) $\boxed{a = \frac{v^2(s)}{r}\sqrt{2} = \frac{v_0^2 e^{-\frac{2s}{r}}}{r}\sqrt{2}}$

$$\frac{dv}{dt} = -\frac{v^2}{r} = \frac{dv}{dS} \cdot \frac{dS}{dt} = \frac{v\,dv}{dS}$$

$$\frac{v\,dv}{dS} = -\frac{v^2}{r} \rightarrow \int_{v_0}^{v} \frac{dv}{v} = -\int_0^S \frac{dS}{r} \longrightarrow \ln\frac{v}{v_0} = -\frac{S}{r} \Rightarrow \boxed{v = v_0 e^{-\frac{S}{r}}} \; (1)$$

1.3

① D



2 з-н Н работает в ИСО

$$\begin{cases} m_1\vec{a}_1 = m_1\vec{g} + \vec{T} & (1) \\ m_2\vec{a}_2 = m_2\vec{g} + \vec{T} & (2) \end{cases}$$

$\vec{a}_1 \neq -\vec{a}_2$ переход

m_1: $\vec{a}_1 = \vec{a}_0 + \vec{a}_{отн1}$

m_2: $\vec{a}_2 = \vec{a}_0 + \vec{a}_{отн2}$

абс = отн + пер

Есть кинематическая связь:

$$\vec{a}_{отн2} = -\vec{a}_{отн1}$$

$$\boxed{\begin{aligned} \vec{a}_1 &= \vec{a}_0 + \vec{a}_{отн1} \\ \vec{a}_2 &= \vec{a}_0 - \vec{a}_{отн1} \end{aligned}} \quad (3)$$

$$\vec{a}_1 = \vec{a}_0 + \vec{a}_{отн1} = = \frac{m_1 - m_2}{m_1 + m_2}(\vec{g} - \vec{a}_0) + \vec{a}_0 = = \frac{2m_2\vec{a}_0 + (m_1 - m_2)\vec{g}}{m_1 + m_2}$$

$$\vec{T} = m_1(\vec{a}_0 + \vec{a}_{отн1}) - m_1\vec{g}$$

$$\vec{T} = m_2(\vec{a}_0 - \vec{a}_{отн1}) - m_2\vec{g}$$

$$m_1\vec{a}_0 + \underbrace{m_1\vec{a}_{отн1}} - m_1\vec{g} = m_2\vec{a}_0 - \underbrace{m_2\vec{a}_{отн1}} - m_2\vec{g}$$

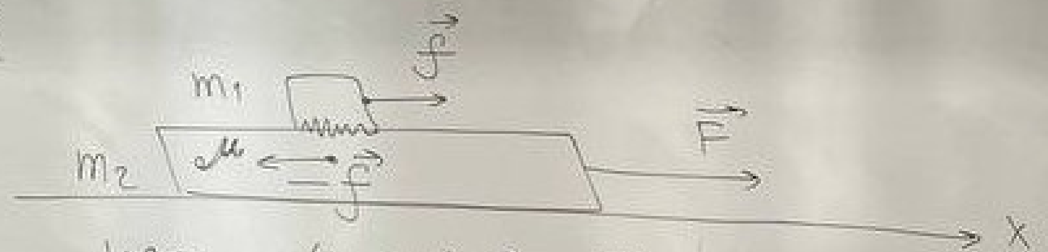
$$\vec{a}_{отн1} = \frac{(m_1 - m_2)\vec{g} + (m_2 - m_1)\vec{a}_0}{m_1 + m_2}$$

$$\vec{a}_{отн1} = \frac{(m_1 - m_2)}{m_1 + m_2}(\vec{g} - \vec{a}_0)$$

$$(\vec{a}_1)\Big|^{\uparrow y} = \frac{2m_2 a_0 - (m_1 - m_2)g}{m_1 + m_2}$$

2.1

② D



ИСО: $(m_1+m_2)\vec{a}_0 = \vec{F}$

Движение системы

Ox: $\boxed{(m_1+m_2)a_0 = \alpha t} \Longrightarrow a_0 = \dfrac{\alpha t}{m_1+m_2}$ ускорение центра масс

$\alpha = const$

В ИСО

$$\begin{cases} m_1\vec{a}_1 = \vec{f} & \text{брусок} \\ m_2\vec{a}_2 = \vec{F} - \vec{f} & \text{доска} \end{cases}$$

Если проскальзывания нет $\rightarrow \vec{a}_1 = \vec{a}_2 = \vec{a}_0$

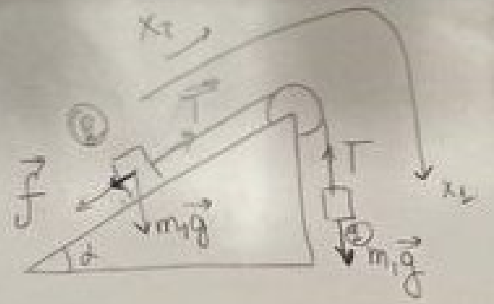
$|\vec{f}| \leq \mu m_1 g$ Пусть $f = \mu m_1 g$

Ox: $a_1 = \mu g = a_0$ брусок m_1 вот-вот сдвинется

Ox: $\mu g = \dfrac{\alpha t_0}{m_1+m_2} \rightarrow t_0 = \dfrac{\mu(m_1+m_2)}{\alpha} g$

альфа

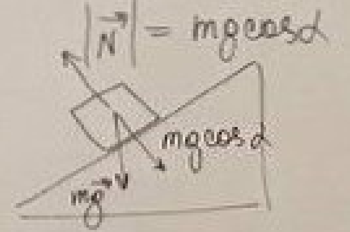
(5) D



① $m_1 g$

② $m_2 g \sin\alpha = \frac{3}{2} m_1 g \cdot \frac{1}{2} = \frac{3}{4} m_1 g < m_1 g$

m_1 движется вниз!



$\cos 30^\circ = \frac{\sqrt{3}}{2} \approx 0,866$

$\sin 30^\circ = \frac{1}{2} = 0.5$

$\boxed{m_1}$: $m_1\vec{g} + \vec{T} = m_1\vec{a}_1$

$\boxed{m_2}$: $\vec{T} + m_2\vec{g} + \vec{f} = m_2\vec{a}_2$

$\vec{a}_1\Big|_{x_1} = \vec{a}_2\Big|_{x_2} \equiv a$

В проекциях на направление дв-я:

$$\begin{cases} m_1 a = m_1 g - T \\ m_2 a = T - m_2 g \sin\alpha - \mu m_2 g \cos\alpha \end{cases}$$

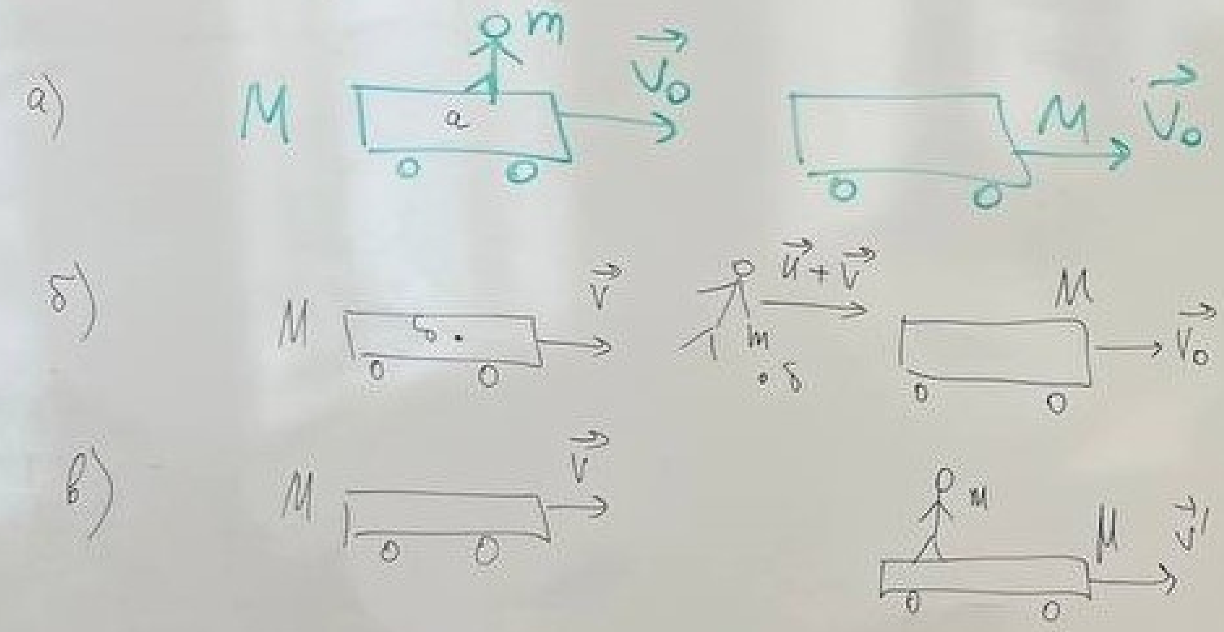
$$(m_1 + m_2) a = m_1 g - m_2 g (\sin\alpha + \mu\cos\alpha)$$

$$a = \frac{m_1 g - m_2 g(\sin\alpha + \mu\cos\alpha)}{m_1 + m_2} = \frac{\frac{m_1}{m_2} - (\sin\alpha + \mu\cos\alpha)}{\frac{m_1}{m_2} + 1} \cdot g$$

$$a = \frac{2/3 - \left(\frac{1}{2} + 0.1 \cdot 0,866\right)}{2/3 + 1} \cdot g \approx 0.05 g > 0$$

2.3

① ЗСИ, ЗСЭ



(а→б) $(M+m)\vec{v}_0 = M\vec{v} + m(\vec{u}+\vec{v}) = (M+m)\vec{v} + m\vec{u}$

$$\vec{v} = \vec{v}_0 - \frac{m}{m+M}\vec{u}$$

Скор человека в ИСО $(\vec{u}+\vec{v}) = \vec{v}_0 - \frac{m}{m+M}\vec{u} + \vec{u} = \vec{v}_0 + \frac{M}{m+M}\vec{u}$

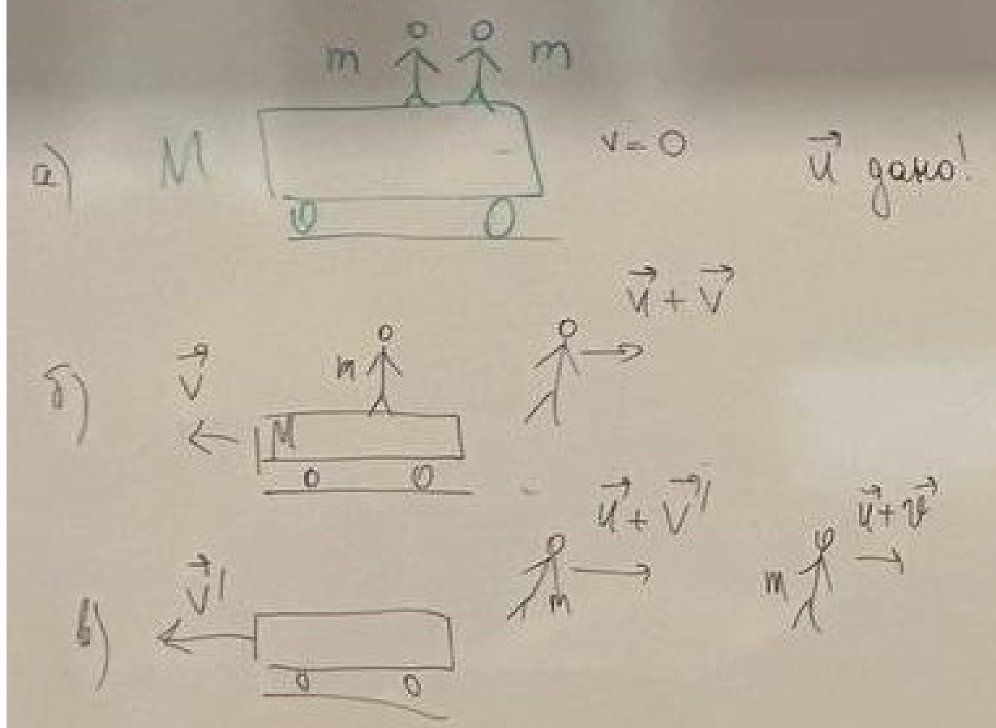
(б→в) $m\left(\vec{v}_0 + \frac{M}{m+M}\vec{u}\right) + M\vec{v}_0 = (m+M)\vec{v}\,'$

$$(m+M)\vec{v}_0 + \frac{mM}{m+M}\vec{u} = (m+M)\vec{v}\,'$$

$$\vec{v}\,' = \vec{v}_0 + \frac{mM}{(m+M)^2}\vec{u}$$

3.1

② ЗСИ, ЗСЭ



(а → б) $$0 = (m+M)\vec{v} + m(\vec{u}+\vec{v})$$

$$\vec{v} = -\frac{m\vec{u}}{M+2m}$$

(б → в) $$\underbrace{(m+M)\cdot\left(-\frac{m\vec{u}}{M+2m}\right)}_{\text{это } \vec{v}} = M\vec{v}' + m(\vec{u}+\vec{v}') = (m+M)\vec{v}' + m\vec{u}$$

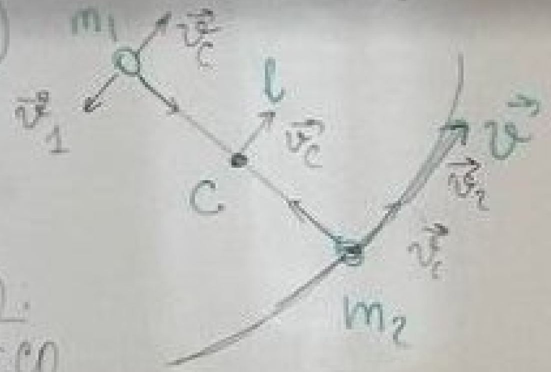
$$(m+M)\vec{v}' = -m\vec{u}\left[\frac{m+M}{M+2m}+1\right] = -m\vec{u}\,\frac{2M+3m}{(M+2m)}$$

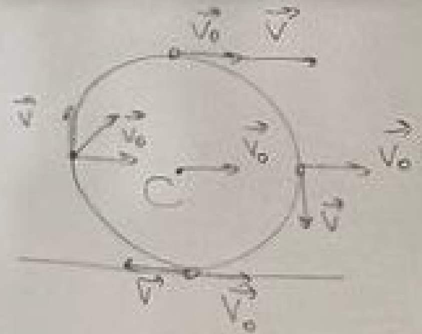
$$\vec{v}' = -m\vec{u}\,\frac{(2M+3m)}{(M+2m)(M+m)}$$

3.2

3

$v_1 = v_c$ (покоится)





ИСО:
лаб. СО
С-система

Решим задачу в С-сист.

$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \vec{0}$$

$m_1 r_1 = m_2 r_2$

$$T_1 = T_2 = m_1\frac{v_1^2}{r_1} = m_2\frac{v_2^2}{r_2} = m_1 a_{n_1} = m_2 a_{n_2}$$

$r_1 + r_2 = l$

$r_1 m_1 = r_2 m_2$

$$r_1 = \frac{m_2}{m_1 + m_2} l$$

$$r_2 = \frac{m_1}{m_1 + m_2} l$$

в С-системе

$\vec{P}_0 = 0$

$m_1 v_1 = m_2 v_2$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

$v_1 = v_c$

$v_c + v_2 = v$ известно

$v_1 + v_2 = v$

$$v_1 = \frac{m_2}{m_1 + m_2} v$$

$$\rightarrow v_2 = \frac{m_1}{m_1 + m_2} v$$

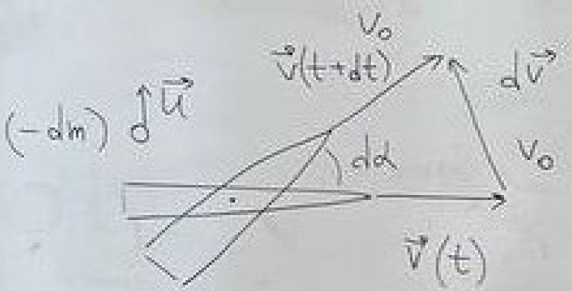
$$T = m_1 \frac{\left(\frac{m_2}{m_1 + m_2} v\right)^2}{\left(\frac{m_2}{m_1 + m_2} l\right)} = \frac{m_1 m_2}{m_1 + m_2} \frac{v^2}{l}$$

$R - ?$

$$R = \frac{v^2 m_2}{T} = \frac{\cancel{v^2} m_2}{\frac{m_1 m_2}{m_1 + m_2} \cancel{v^2}} l = \left(\frac{m_1 + m_2}{m_1}\right) l$$

3.3

①



dm<0 масса ракеты уменьшается

ракета газ ΔО

$$(m+dm)(\vec v + d\vec v) + (-dm)(\vec u + \vec v + d\vec v) = m\vec V$$

отн-но корабля

после

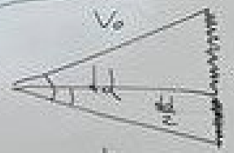
$$\cancel{m\vec v} + m\,d\vec v + \cancel{dm\,\vec v} + \cancel{dm\,d\vec v} - dm\,\vec u - \cancel{dm\,\vec v} - \cancel{dm\,d\vec v} = \cancel{m\vec V}$$

$$\boxed{m\,d\vec v - dm\,\vec u = 0}$$

$$d\vec v = \frac{dm}{m}\vec u \longrightarrow$$ переход к модулям $$dv = -\frac{dm}{m}\cdot u = V_0\,d\alpha$$

знак минус, т.к. dm<0

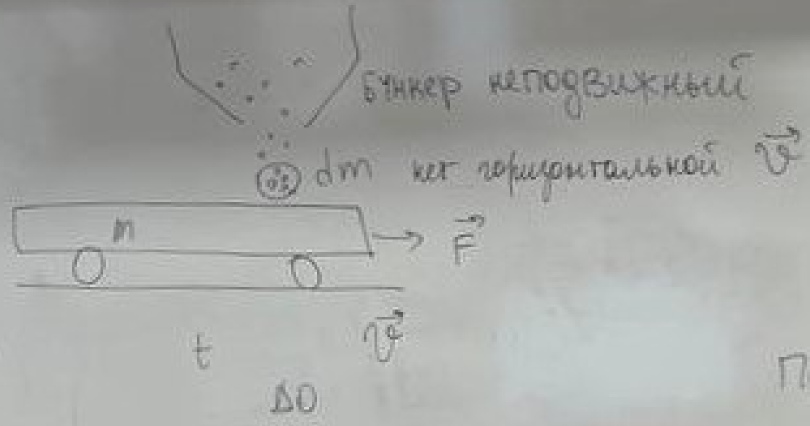
$V = const = V_0$

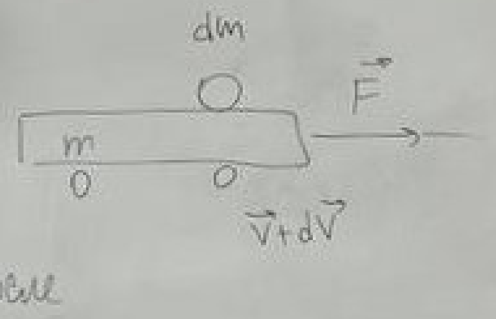


$$dv = 2V_0\sin\frac{d\alpha}{2} \approx 2V_0\,\frac{d\alpha}{2} = V_0\,d\alpha$$

$$\int_0^{\Delta\alpha} d\alpha = \frac{u}{v_0}\int_{m_0}^{m}\frac{dm}{m} \longrightarrow \Delta\alpha = \frac{u}{v_0}\ln\frac{m_0}{m}$$

2





$m(t) = m_0 + \mu t$

зависимость массы платформы от времени.

$d\vec p = \vec F dt$ внешняя сила есть!

$$(m+dm)(\vec v + d\vec v) - m\vec v = \vec F dt$$

$$\cancel{m\vec v} + \underbrace{m\, d\vec v + dm\, \vec v} + \cancel{dm\, d\vec v} - \cancel{m\vec v} = \vec F dt$$

мало

$$d(m\vec v) = \vec F dt \quad \frac{d\vec p}{dt} = \vec F$$

$\Delta m\vec v = \vec F t$ при $t = 0$ $\vec v(0) = 0$

$$m\vec v = \vec F t \to \vec v = \frac{\vec F t}{m(t)} = \frac{\vec F t}{m_0 + \mu t}$$

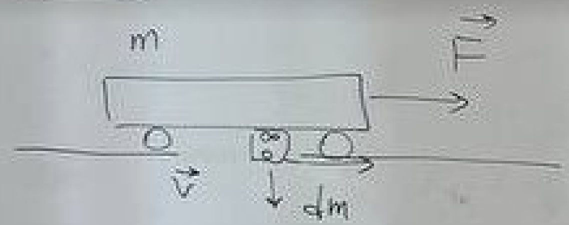
При $t \to \infty$

$$v \to \frac{\vec F}{\mu}$$

4.2

$\boxed{dm<0}$

3



До: $m, \vec{v}$

После: $(m+dm)(\vec{v}+d\vec{v})$ тележка

$(-dm)\vec{v}$ песок в ЛСО

Дано: $m(t)=m_0-\mu t$

$$d\vec{p}=\vec{F}dt$$

$$\underbrace{(m+dm)(\vec{v}+d\vec{v})+(-dm)\vec{v}}_{\text{после}} - \underbrace{m\vec{v}}_{\text{до}} = \vec{F}dt$$

$$\cancel{m\vec{v}}+md\vec{v}+\cancel{dm\vec{v}}+\underbrace{\cancel{dm\,d\vec{v}}}_{\text{мало}}-\cancel{dm\vec{v}}-\cancel{m\vec{v}}=\vec{F}dt$$

$$md\vec{v}=\vec{F}dt$$

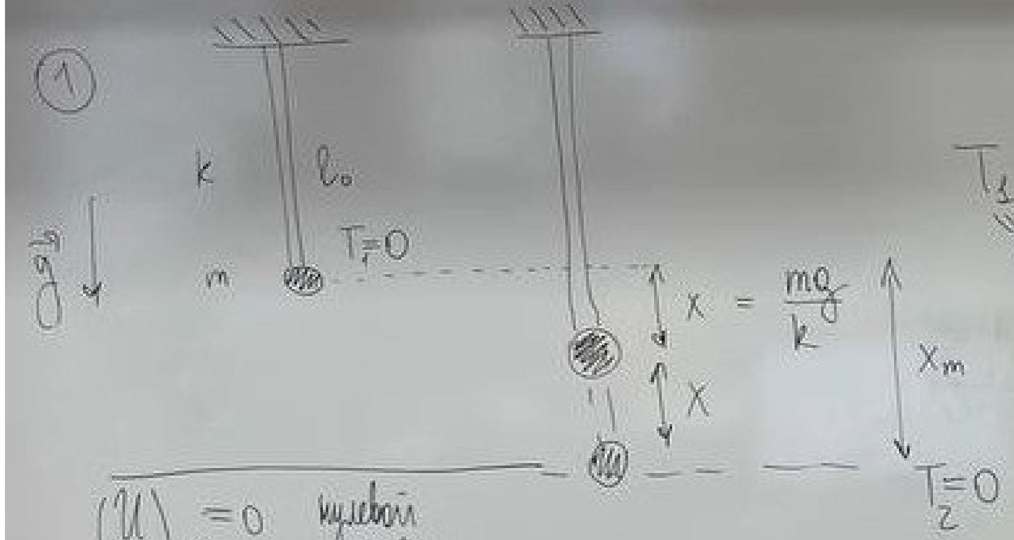
$$m(t)dv=F\,dt$$

$$\int_0^v dv=\frac{F\,dt}{m(t)}=\frac{F\,dt}{m_0-\mu t}=\frac{F}{(-\mu)}\int_{t=0}^{t} d\left(\ln(m_0-\mu t)\right)$$

$$v(t)=-\frac{F}{\mu}\ln\left(m_0-\mu t\right)\Big|_{t=0}^{t=t} = \frac{F}{\mu}\ln\frac{m_0}{m_0-\mu t}$$

4.3

1



$(U)_{mg} = 0$ нулевой уровень

$E = const$

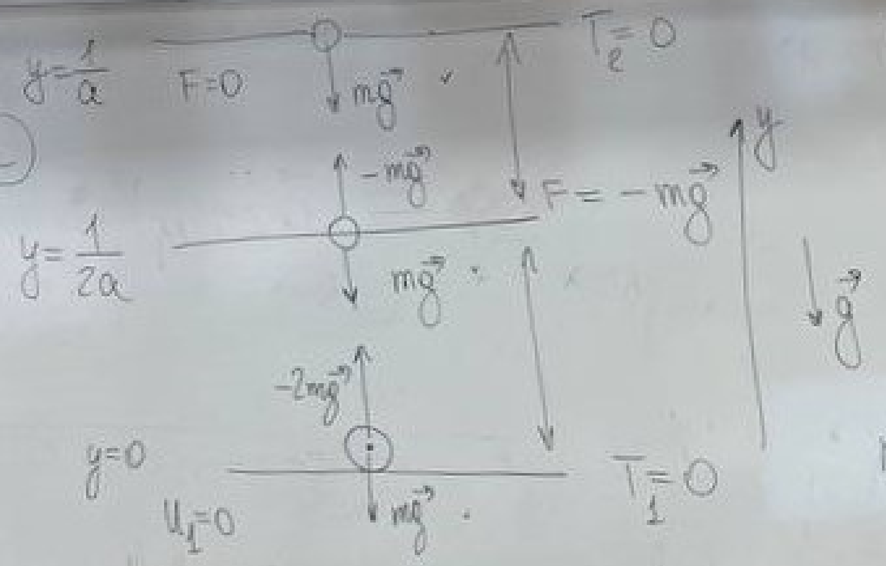
$$\underbrace{T_1}_{=0} + (U_1)_{mg} + \underbrace{(U_1)_{kx}}_{=0} = \overbrace{(U_2)_{mg}}^{=0} + (U_2)_{kx} + \underbrace{T_2}_{=0}$$

T — кин. эн.

U — потенц. эн.

$$mg x_m = \frac{k x_m^2}{2}$$

$$x_m = \frac{2mg}{k}$$



$\boxed{\Delta E = A_F}$ изменение мех. энергии

$\vec{F} = -2m\vec{g}(1-ay)$

потенц. эн. в поле тяжести

$\Delta E = E_2 - E_1 = U_2 - U_1 = mgy = A_F$

$mgy = 2mgy - mgay^2$

$\boxed{y = \frac{1}{a}}$ весь путь подъема от остановки до остановки $S = y - 0 = \frac{1}{a}$

$A_F(y) = \int_1^2 \delta A_F = (\vec{F}, d\vec{r}) = F_y\, dy =$

$F_y = 2mg(1-ay)$

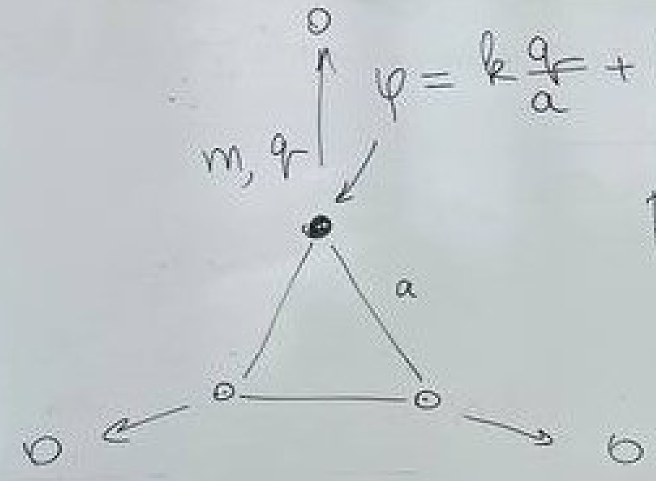
$= \int_0^y 2mg(1-ay)\, dy = 2mgy - mgay^2$

$A_F\left(y = \frac{1}{2a}\right) = \cancel{2}mg\frac{1}{\cancel{2}a} - mg\cancel{a}\frac{1}{4a^{\cancel{2}}} = \frac{3}{4}\cdot\frac{mg}{a}$

$\Delta U = mg\cdot\frac{1}{2a} = \frac{mg}{2a}$ в поле тяжести.

5.2

3



$$\varphi = k\frac{q}{a} + k\frac{q}{a} = 2k\frac{q}{a}$$

В поле конс. кулоновской силы

$$U_1 = 3k\frac{q^2}{a}$$ потенц. энергия

$$\boxed{* \quad U_1 = \frac{1}{2}\sum_{i\neq j}\varphi_{ij}\cdot q_i = \frac{1}{2}\cdot 2k\frac{q}{a}\cdot 3 = 3\frac{kq^2}{a}}$$

$$A_k = U_1/3 = \frac{kq^2}{a}$$

$E = const$ сохр. полной мех. энергии

$$E_1 = E_2 = E_\infty$$

$$U_1 = U_2 + T_2$$

$T_1 = 0$

$$T_2 = \frac{mv^2}{2} + \frac{mv^2}{2} + \frac{mv^2}{2}$$

$$\longrightarrow \cancel{3}k\frac{q^2}{a} = \cancel{3}k\frac{q^2}{r} + \frac{mv^2}{2}\cdot\cancel{3}$$

$$mv^2 = 2kq^2\left(\frac{1}{a} - \frac{1}{r}\right)$$

$$v = \sqrt{\frac{2kq^2}{m}\frac{(r-a)}{ra}}$$

5.3