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518. Coin Change II (/problems/coin-change-2/)

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You are given coins of different denominations and a total amount of money. Write a function to compute the number of combinations that make up that amount. You may assume that you have infinite number of each kind of coin.

Example 1:

```
Input: amount = 5, coins = [1, 2, 5]
Output: 4
Explanation: there are four ways to make up the amount:
5=5
5=2+2+1
5=2+1+1+1
5=1+1+1+1+1
```

Example 2:

```
Input: amount = 3, coins = [2]
Output: 0
Explanation: the amount of 3 cannot be made up just with coins of 2.
```

Example 3:

```
Input: amount = 10, coins = [10]
Output: 1
```

Note:

You can assume that

- 0 <= amount <= 5000
- 1 <= coin <= 5000
- the number of coins is less than 500
- the answer is guaranteed to fit into signed 32-bit integer

Solution

Approach 1: Dynamic Programming

Template

This is a classical dynamic programming problem.

Here is a template one could use:

- Define the base cases for which the answer is obvious.
- Develop the strategy to compute more complex case from more simple one.
- Link the answer to base cases with this strategy.

Example

Let's pic up an example: amount = 11, available coins - 2 cent, 5 cent and 10 cent. Note, that coins are unlimited.

Base Cases: No Coins or Amount = 0

If the total amount of money is zero, there is only one combination: to take zero coins.

Another base case is no coins: zero combinations for `amount > 0` and one combination for `amount == 0`.

Base cases

amount =	0	1	2	3	4	5	6	7	8	9	10	11
combinations using no coins =	1	0	0	0	0	0	0	0	0	0	0	0

2 Cent Coins

Let's do one step further and consider the situation with one kind of available coins: 2 cent.

amount =	0	1	2	3	4	5	6	7	8	9	10	11
combinations using no coins =	1	0	0	0	0	0	0	0	0	0	0	0
combinations with 2 cents =	1	0	1	0	1	0	1	0	1	0	1	0

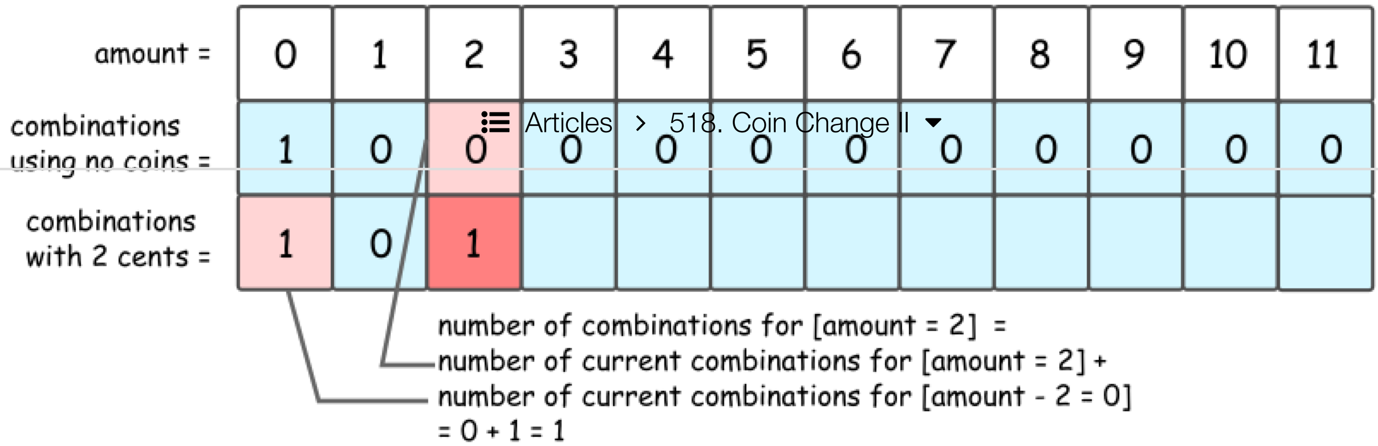
It's quite evident that there could be 1 or 0 combinations here, 1 combination for even amount and 0 combinations for the odd one.

> The same answer could be received in a recursive way, by computing the number of combinations for all amounts of money, from 0 to 11.

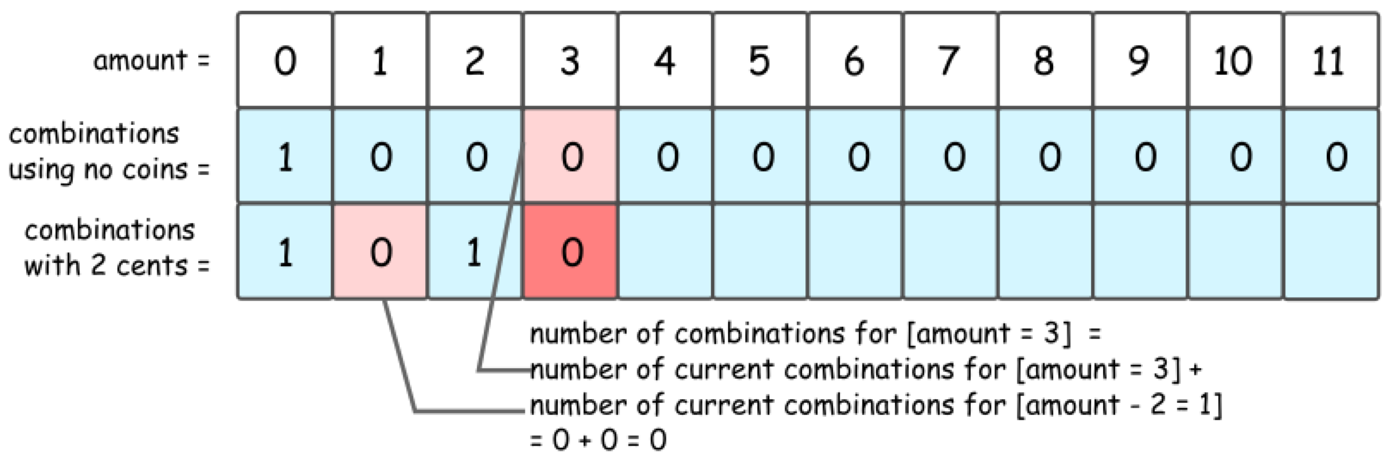
First, that's quite obvious that all amounts less than 2 are *not* impacted by the presence of 2 cent coins. Hence for `amount = 0` and for `amount = 1` one could reuse the results from the figure 2.

Starting from `amount = 2`, one could use 2 cent coins in the combinations. Since the amounts are considered gradually from 2 to 11, at each given moment one could be sure to add not more than one coin to the previously known combinations.

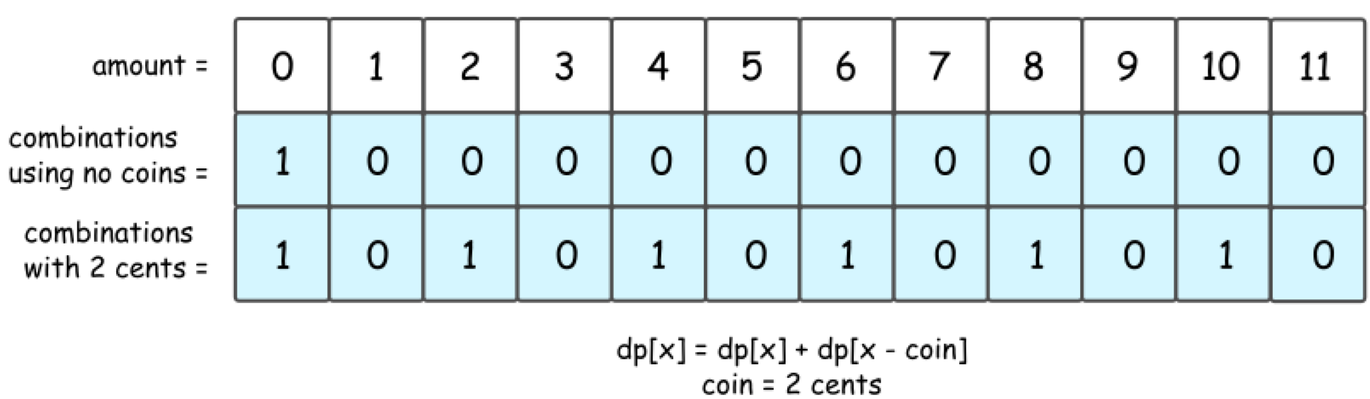
So let's pick up 2 cent coin, and use it to make up `amount = 2`. The number of combinations with this 2 cent coin is a number combinations for `amount = 0`, i.e. 1.



Now let's pick up 2 cent coin, and use it to make up `amount = 3`. The number of combinations with this 2 cent coin is a number combinations for `amount = 1`, i.e. 0.

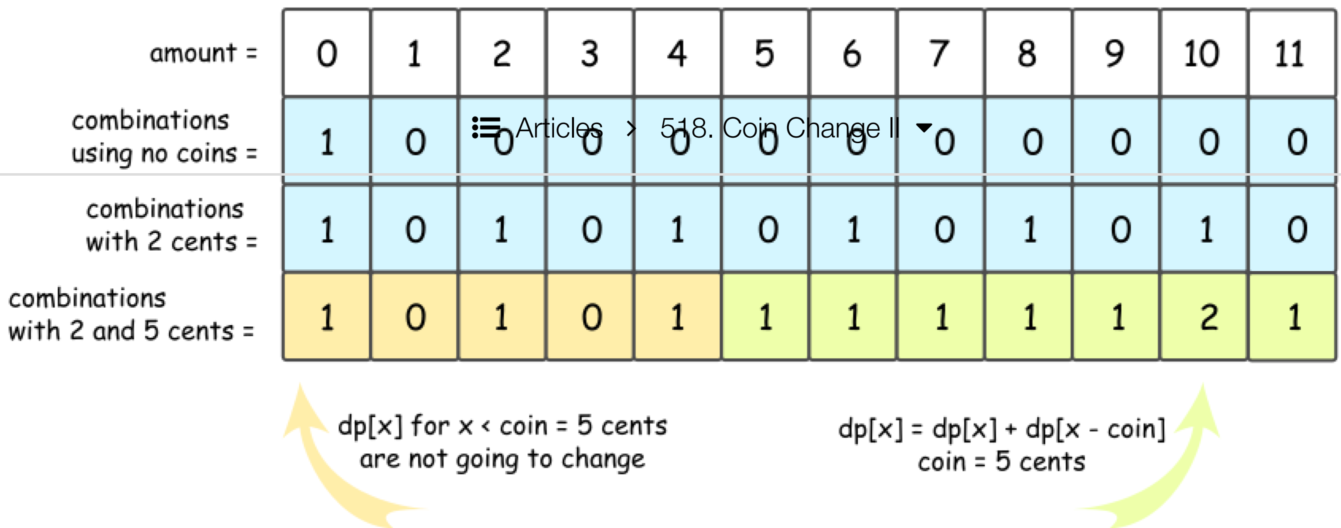


That leads to DP formula for number of combinations to make up the `amount = x`: `dp[x] = dp[x] + dp[x - coin]`, where `coin = 2 cents` is a value of coins we're currently adding.



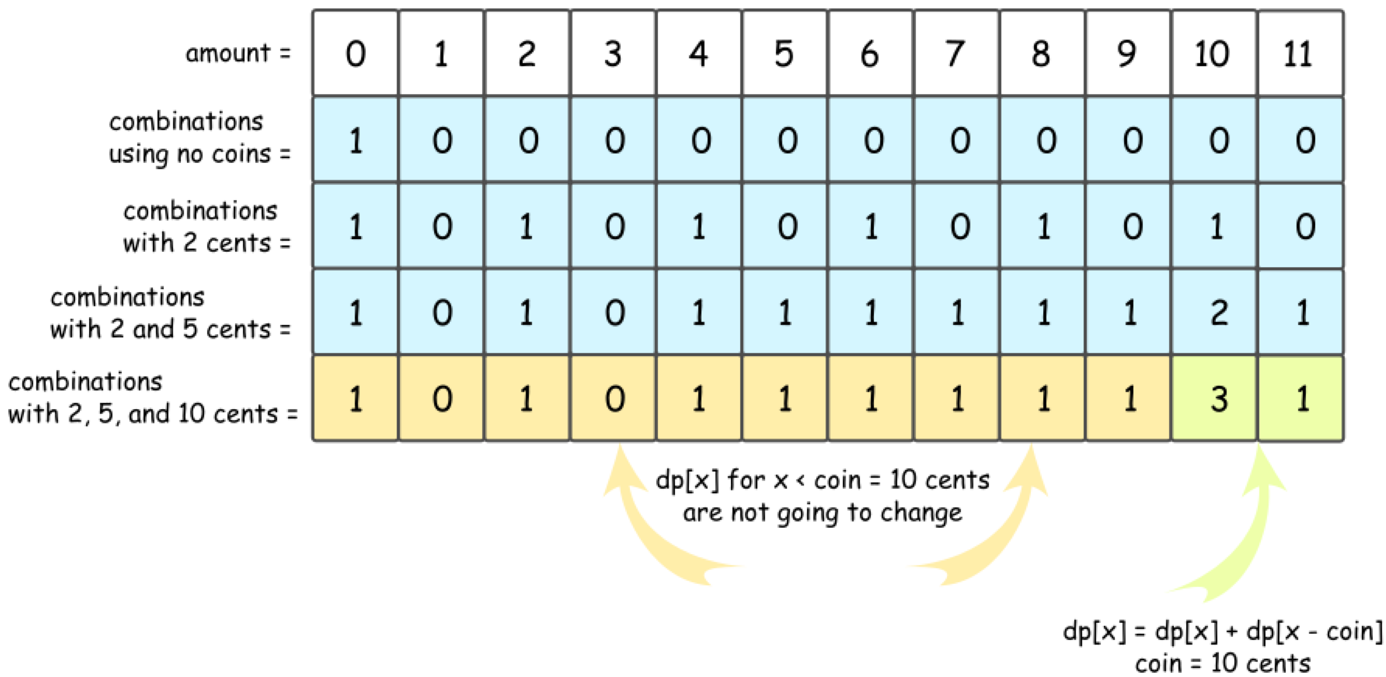
2 Cent Coins + 5 Cent Coins + 10 Cent Coins

Now let's add 5 cent coins. The formula is the same, but do not forget to add `dp[x]`, number of combinations with 2 cent coins.



amount =	0	1	2	3	4	5	6	7	8	9	10	11
combinations using no coins =	1	0	0	0	0	0	0	0	0	0	0	0
combinations with 2 cents =	1	0	1	0	1	0	1	0	1	0	1	0
combinations with 2 and 5 cents =	1	0	1	0	1	1	1	1	1	1	2	1

The story is the same for 10 cent coins.



amount =	0	1	2	3	4	5	6	7	8	9	10	11
combinations using no coins =	1	0	0	0	0	0	0	0	0	0	0	0
combinations with 2 cents =	1	0	1	0	1	0	1	0	1	0	1	0
combinations with 2 and 5 cents =	1	0	1	0	1	1	1	1	1	1	2	1
combinations with 2, 5, and 10 cents =	1	0	1	0	1	1	1	1	1	1	3	1

Now the strategy is here:

- Add coins one-by-one, starting from base case "no coins".
- For each added coin, compute recursively the number of combinations for each amount of money from 0 to `amount`.

Algorithm

- Initiate number of combinations array with the base case "no coins": `dp[0] = 1`, and all the rest = 0.
- Loop over all coins:

- For each coin, loop over all amounts from 0 to `amount`:
  - For each amount x, compute the number of combinations: `dp[x] += dp[x - coin]`.

- Return `dp[amount]`.

Implementation

Java | Python

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```
class Solution:
    def change(self, amount: int, coins: List[int]) -> int:
        dp = [0] * (amount + 1)
        dp[0] = 1

        for coin in coins:
            for x in range(coin, amount + 1):
                dp[x] += dp[x - coin]
        return dp[amount]
```

Complexity Analysis

- Time complexity: $\mathcal{O}(N \times \text{amount})$, where N is a length of coins array.
- Space complexity: $\mathcal{O}(\text{amount})$ to keep dp array.

Analysis written by @liaison (https://leetcode.com/liaison/) and @andvary (https://leetcode.com/andvary/)

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