

# study\_transition\_matrix\_MH.r

max

2022-05-29

```
get_csize <- function(clade){
  length(strsplit(clade, "'')[[1]])
}
get_stuff <- function(clade){

  compute_all_errors <- function(clade){

    csize <- get_csize(clade)
    clade.ind <- clade_index(clade, all.splits)

    S1.c.pos <- which(complete.clade.matrix[, clade.ind] == 1)
    S0.c.pos <- setdiff(1:K, S1.c.pos)

    grid.pos <- subset(expand.grid(S1.c.pos, S1.c.pos),
                      Var1 < Var2)

    all.errors <- do.call(rbind,
                        apply(grid.pos, 1,
                            function(row){
                              x.pos <- row[1]
                              y.pos <- row[2]
                              sum_x <- sum(trans.mat.MH[x.pos, S0.c.pos])
                              sum_y <- sum(trans.mat.MH[y.pos, S0.c.pos])
                              return(data.frame(
                                index.x = x.pos,
                                index.y = y.pos,
                                Nx = neighbourhood.sizes[x.pos],
                                Ny = neighbourhood.sizes[y.pos],
                                A0cx = A0s.sizes[x.pos],
                                A0cy = A0s.sizes[y.pos],
                                sum.x = sum_x,
                                sum.y = sum_y,
                                error = sum_x - sum_y
                              ))
                            })
    )
    all.errors$ntaxa <- n
    all.errors$clade_size <- csize
    all.errors$clade <- clade
    all.errors <- tibble::tibble(all.errors)
    return(all.errors)
  }
}
```

```

compute_all_dists <- function(clade){
  clade.ind <- clade_index(clade, all.splits)
  S1.c.pos <- which(complete.clade.matrix[, clade.ind] == 1)
  grid.pos <- subset(expand.grid(S1.c.pos, S1.c.pos),
                     Var1 < Var2)
  all.dists <- do.call(rbind,
                      apply(grid.pos, 1,
                           function(row){
                             x.pos <- row[1]
                             y.pos <- row[2]
                             return(data.frame(
                               index.x = x.pos,
                               index.y = y.pos,
                               rspr_dist = rspr(
                                 all.trees[[x.pos]],
                                 all.trees[[y.pos]]
                               )
                             ))
                           })
  )
  all.dists$ntaxa <- n
  all.dists$clade <- clade
  all.dists <- tibble::tibble(all.dists)
  return(all.dists)
}
###

clade.nice <- paste0("t", strsplit(clade, "")[[1]])
clade.ind <- clade_index(x = clade, all.splits = all.splits)
hasclade <- which(complete.clade.matrix[, clade.ind] == 1)
parts <- lapply(all.trees[hasclade],
               safe_f_c, clade = clade.nice)

Nxprimes <- unlist(lapply(parts,
                          function(x) neighbourhood_size(x$x_prime)$n_size))
Nphixcs <- unlist(lapply(parts,
                          function(x) neighbourhood_size(x$phi_x_c)$n_size))

A0s.sizes <- rep(NA, K)
A0s.sizes[hasclade] <- neighbourhood.sizes[hasclade] - Nxprimes + Nphixcs

all.errors <- compute_all_errors(clade)
dists <- compute_all_dists(clade)

all.errors.dt <- tibble::tibble(
  merge(all.errors, dists,
        by = c("index.x", "index.y",
              "ntaxa", "clade") )
)
all.errors.dt <- all.errors.dt %>%
  mutate(bound = A0cx/Nx - (5*A0cy)/(6*Ny))
return(all.errors.dt)
}

```

```
#####
library(PhyloMarkovChains)
library(phangorn)

## Loading required package: ape

library(tidyverse)

## -- Attaching packages ----- tidyverse 1.3.1.9000 --

## v ggplot2 3.3.5.9000      v purrr  0.3.4
## v tibble  3.1.6          v dplyr  1.0.8
## v tidyr   1.2.0          v stringr 1.4.0
## v readr   2.1.2          v forcats 0.5.1

## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

n <- 6
all.splits <- sapply(allSplits(n + 1), paste, collapse = "")
J <- length(all.splits)

all.trees <- allTrees(n, rooted = TRUE)
K <- length(all.trees)

compute <- FALSE

if(compute){
  comp.time <- system.time(
    SPR.mat <- as.matrix(rspr_matrix(all.trees) > 0) + 0
  )
  save(SPR.mat,
       file = paste0("saved_data/SPR_matrix_n=", n, ".RData"))
}else{
  load(paste0("saved_data/SPR_matrix_n=", n, ".RData"))
}

####
# Neighbourhood stuff
incidence.mat <- SPR.mat
neighbourhood.sizes <- colSums(incidence.mat)

neigh.ratios <- matrix(NA, nrow = K, ncol = K)
diag(neigh.ratios) <- 0
for(i in 1:K){
  neigh.ratios[i, ] <- sapply(neighbourhood.sizes[i]/neighbourhood.sizes,
                             function(x) min(1, x))/
    neighbourhood.sizes[i]
}
```

```
trans.mat.MH <- neigh.ratios * incidence.mat
diag(trans.mat.MH) <- 1-rowSums(trans.mat.MH)
diag(trans.mat.MH)
```

```
## [1] 0.04428220 0.01307692 0.01865385 0.04428220 0.03245262 0.04428220
## [7] 0.04428220 0.04428220 0.04428220 0.01307692 0.00000000 0.00000000
## [13] 0.01307692 0.01307692 0.01307692 0.00000000 0.00000000 0.01865385
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```

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## [943] 0.01307692 0.04428220 0.04428220
```

```
#####
```

```
complete.Splits <- parallel::mclapply(all.trees,
                                      get_clades, mc.cores = 12)

complete.indexes <- parallel::mclapply(complete.Splits,
                                      function(x) charmatch(x, all.splits),
                                      mc.cores = 12)

complete.clade.matrix <- do.call(rbind, parallel::mclapply(complete.indexes,
                                                           clade_indicators,
                                                           L = J,
                                                           mc.cores = 12))

colnames(complete.clade.matrix) <- all.splits
colMeans(complete.clade.matrix)
```

```
##          1          2          12          3          13          23          123
## 1.00000000 1.00000000 0.11111111 1.00000000 0.11111111 0.11111111 0.04761905
##          4          14          24          124          34          134          234
## 1.00000000 0.11111111 0.11111111 0.04761905 0.11111111 0.04761905 0.04761905
##        1234          5          15          25          125          35          135
## 0.04761905 1.00000000 0.11111111 0.11111111 0.04761905 0.11111111 0.04761905
##        235        1235          45          145          245        1245          345
## 0.04761905 0.04761905 0.11111111 0.04761905 0.04761905 0.04761905 0.04761905
##        1345        2345        12345          6          16          26          126
## 0.04761905 0.04761905 0.11111111 1.00000000 0.11111111 0.11111111 0.04761905
##          36          136          236          1236          46          146          246
## 0.11111111 0.04761905 0.04761905 0.04761905 0.11111111 0.04761905 0.04761905
##        1246          346          1346          2346          12346          56          156
## 0.04761905 0.04761905 0.04761905 0.04761905 0.11111111 0.11111111 0.04761905
##        256          1256          356          1356          2356          12356          456
## 0.04761905 0.04761905 0.04761905 0.04761905 0.04761905 0.11111111 0.04761905
##        1456          2456          12456          3456          13456          23456          123456
## 0.04761905 0.04761905 0.11111111 0.04761905 0.11111111 0.11111111 1.00000000
```

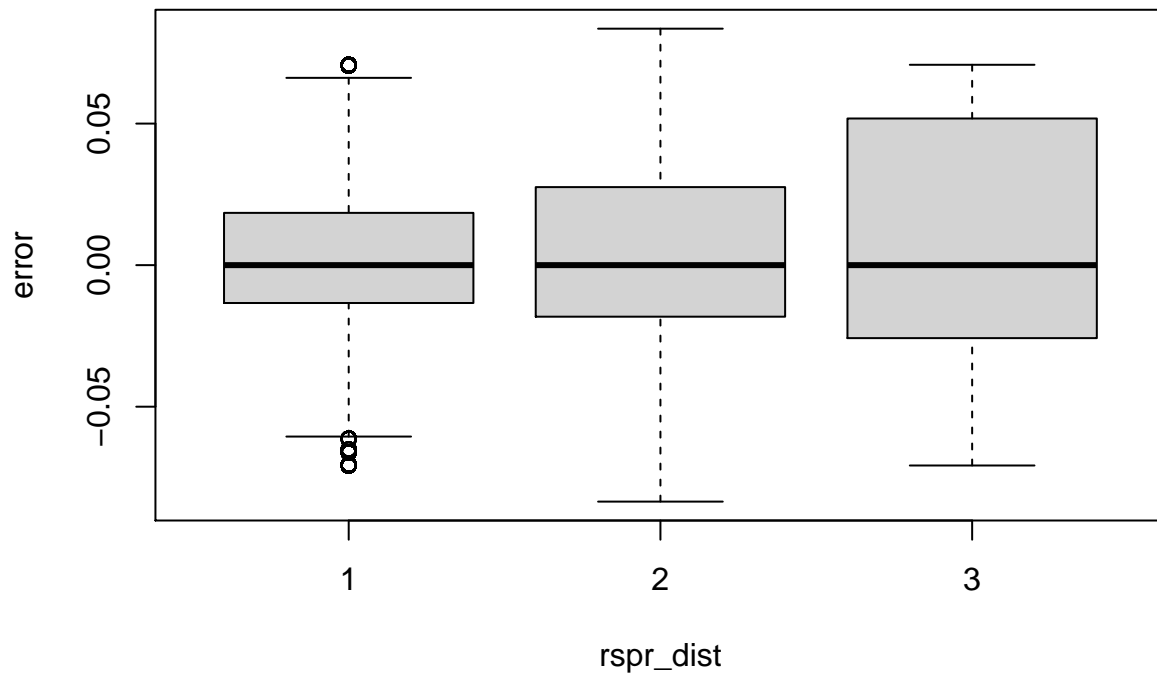
```
## Computing |A0c|
```

```
clade.sizes <- unlist(lapply(all.splits, get_csize))
nontrivial <- intersect(which(clade.sizes > 1), which(clade.sizes < n))
all.ntclades <- all.splits[nontrivial]
all.errors <- parallel::mclapply(all.ntclades,
                                function(clade) get_stuff(clade), mc.cores = 8)
```

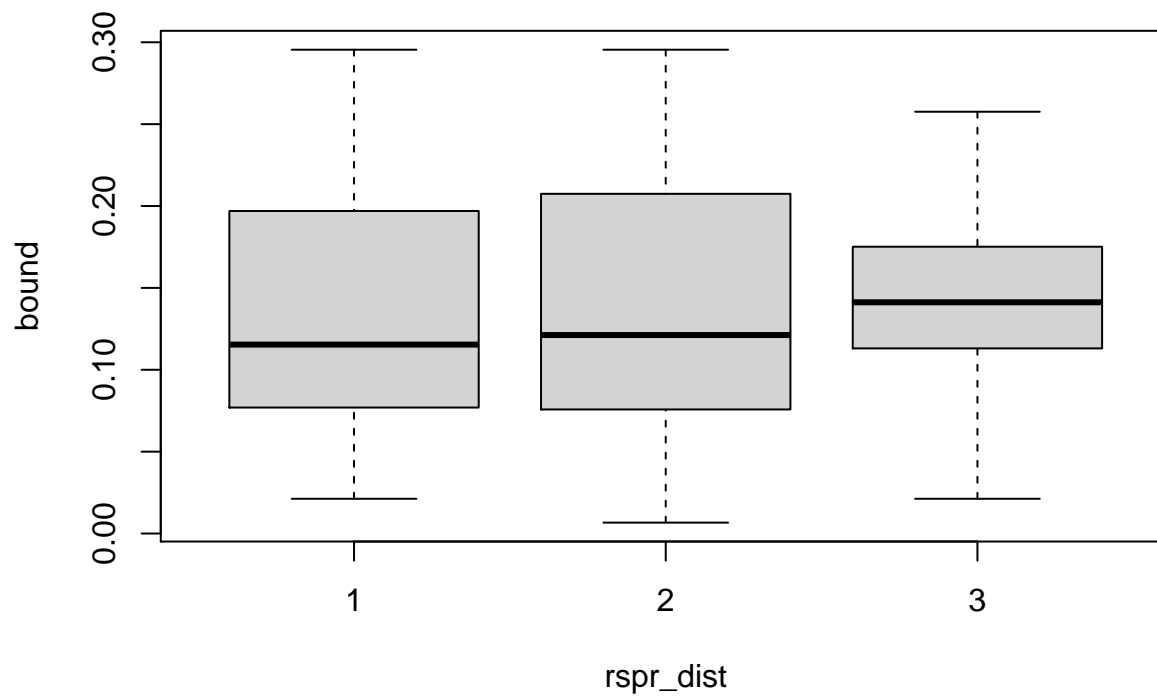
```
all.errors.dt <- do.call(rbind, all.errors)
```

```
## Plotting
```

```
boxplot(error ~ rspr_dist, all.errors.dt)
```

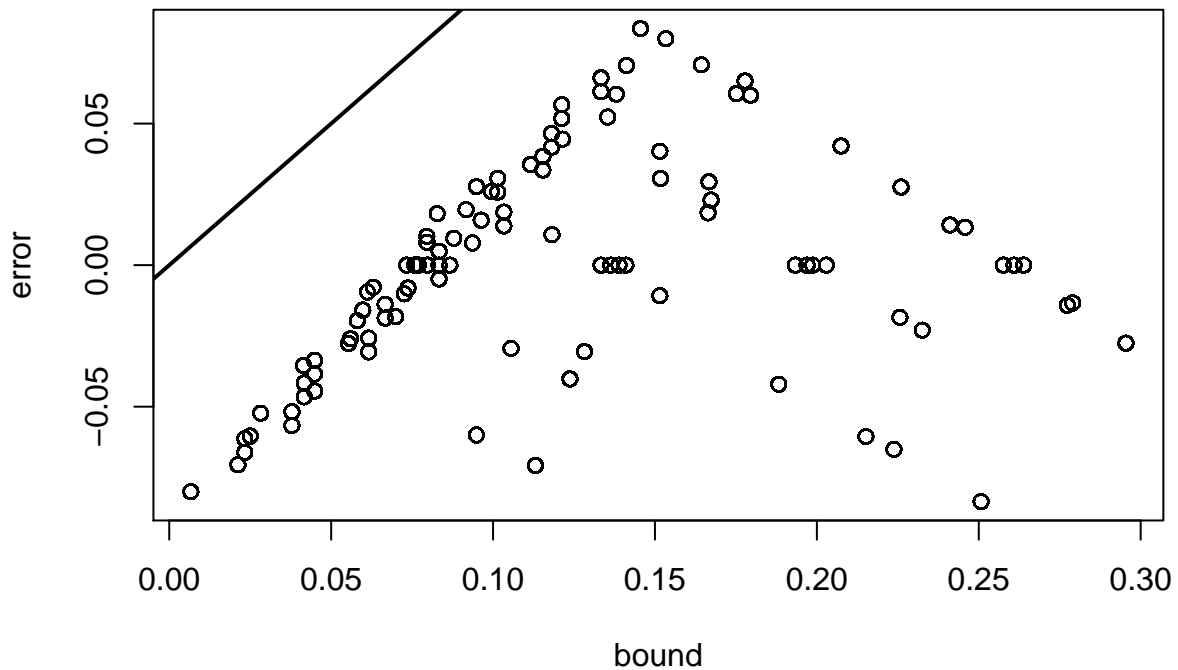


```
boxplot(bound ~ rspr_dist, all.errors.dt)
```



```
plot(error~bound, all.errors.dt)
abline(a = 0, b = 1, lwd = 2)
```





```
range(all.errors.dt$error)
```

```
## [1] -0.08352285 0.08352285
```

```
range(all.errors.dt$bound)
```

```
## [1] 0.006666667 0.295454545
```

```
all.errors.dt[which.min(all.errors.dt$error), ]
```

```
## # A tibble: 1 x 14
##   index.x index.y ntaxa clade    Nx    Ny A0cx A0cy sum.x sum.y  error
##   <int>   <int> <dbl> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1    125    187     6 1234    46   50   56   58 0.666 0.749 -0.0835
## # ... with 3 more variables: clade_size <int>, rspr_dist <dbl>, bound <dbl>
```

```
all.errors.dt[which.max(all.errors.dt$error), ]
```

```
## # A tibble: 1 x 14
##   index.x index.y ntaxa clade    Nx    Ny A0cx A0cy sum.x sum.y  error
##   <int>   <int> <dbl> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1    187    377     6 1234    50   46   58   56 0.749 0.666 0.0835
## # ... with 3 more variables: clade_size <int>, rspr_dist <dbl>, bound <dbl>
```

```
all.errors.dt[which.min(all.errors.dt$bound), ]
```

```
## # A tibble: 1 x 14
##   index.x index.y ntaxa clade    Nx    Ny A0cx A0cy sum.x sum.y  error
##   <int>   <int> <dbl> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1    787    797     6 12    50   50   22   26 0.434 0.514 -0.0800
## # ... with 3 more variables: clade_size <int>, rspr_dist <dbl>, bound <dbl>
```

```
all.errors.dt[which.max(all.errors.dt$bound), ]
```

```
## # A tibble: 1 x 14
##   index.x index.y ntaxa clade    Nx    Ny A0cx  A0cy sum.x sum.y  error
##   <int>   <int> <dbl> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1     108     153     6 12345   48   44   76    68 0.398 0.426 -0.0276
## # ... with 3 more variables: clade_size <int>, rspr_dist <dbl>, bound <dbl>
```