

MSM_metaAnalysis_Gaussian_example.r

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```
# Leo Bastos & Luiz Max Carvalho (2019)
# This example was taken from Malta et al. (2010)

source("pooling_aux.r")

## Loading required package: ggplot2
## Registered S3 methods overwritten by 'ggplot2':
##   method      from
##   [.quosures   rlang
##   c.quosures   rlang
##   print.quosures rlang
meta <- read.csv("../data/meta_analysis_Malta_2010.csv")

meta$SampleSize

## [1] 658 461 621 1165 642 849
K <- nrow(meta)
av <- meta$HIV + 1
bv <- meta$SampleSize - meta$HIV + 1

mv <- meta$HIV/meta$SampleSize
vv <- mv*(1-mv)/meta$SampleSize
sv <- sqrt(vv)

cbind(
  t(apply(cbind(av, bv), 1, stat_beta) ),
  t(apply(cbind(mv, sv), 1, stat_gauss) )
)

##           av           mv
## [1,] 0.06818182 0.05024034 0.08859908 0.06686930 0.04778309 0.08595551
## [2,] 0.24190065 0.20402034 0.28189269 0.24078091 0.20175148 0.27981034
## [3,] 0.09951846 0.07727883 0.12419170 0.09822866 0.07482038 0.12163695
## [4,] 0.24164524 0.21752169 0.26660751 0.24120172 0.21663549 0.26576795
## [5,] 0.09006211 0.06920620 0.11332765 0.08878505 0.06678311 0.11078698
## [6,] 0.11750881 0.09675464 0.13996512 0.11660777 0.09501866 0.13819689

# Individual entropies
entropies <- rep(NA, K)
for(k in 1:K) entropies[k] <- entropy_gauss(mv[k], vv[k])

entropies

## [1] -3.212777 -2.497427 -3.008653 -2.960370 -3.070612 -3.089554
#####
PaperMSMGauss.tbl <- data.frame(mean.prior = rep(NA, 6), lower.prior = NA,
                                upper.prior = NA)
```

```

rownames(PaperMSMGauss.tbl) <- c("equal_weights", "maximum_entropy", "minimum_KL",
                                "hierarchical_Dirichlet", "hierarchical_LogisticNormal", "Sample_size")

AlphasMSMGauss.tbl <- data.frame(matrix(NA, nrow = 3, ncol = length(av)))
rownames(AlphasMSMGauss.tbl) <- c("maximum_entropy", "minimum_KL", "Sample_size")
colnames(AlphasMSMGauss.tbl) <- paste("alpha_", 0:(K-1), sep = "")

library(ggplot2)

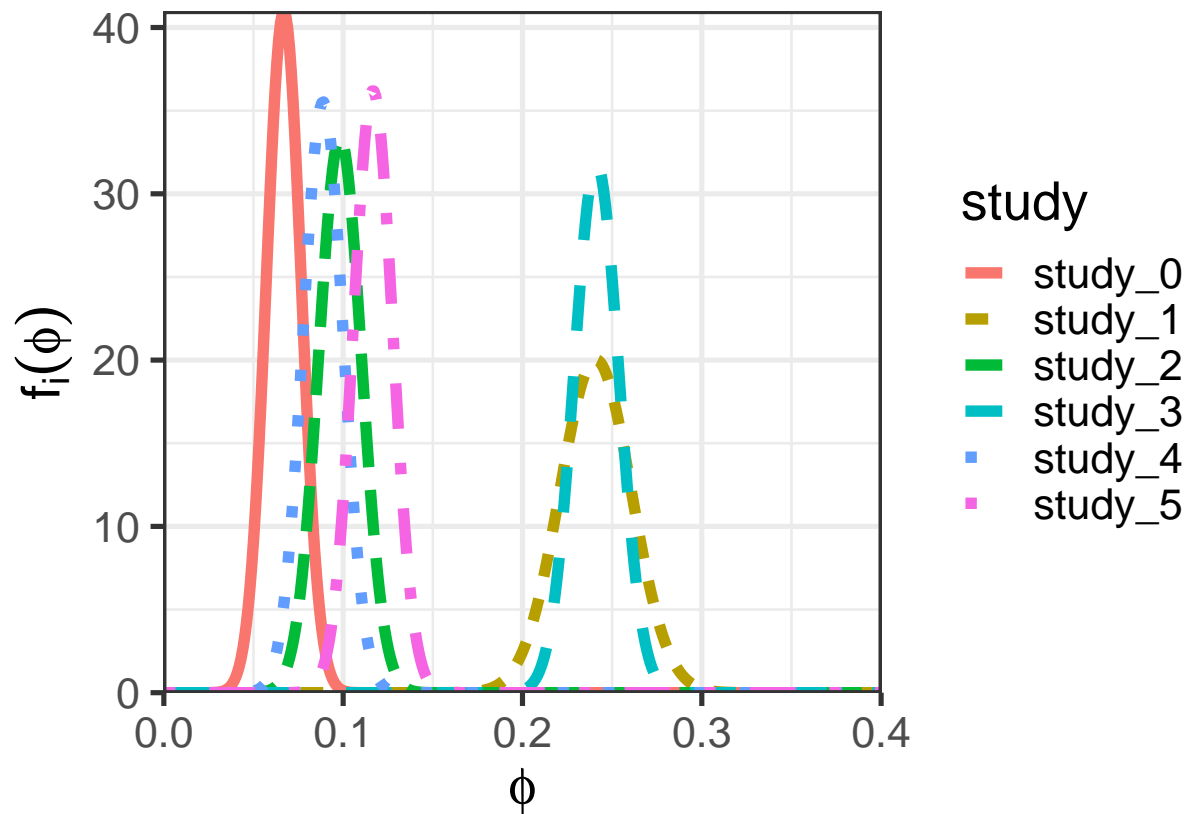
phi.grid <- seq(0, 1, length.out = 1000)
study.densities <- vector(K, mode = "list")
for(k in 1:K){
  study.densities[[k]] <- data.frame(phi = phi.grid,
                                    dens = dnorm(phi.grid, mean = mv[k], sd = sv[k] ),
                                    study = paste("study_", k-1, sep = ""))
}
study.densities.df <- do.call(rbind, study.densities)
study.densities.df$distribution <- "Gaussian"
write.csv(study.densities.df, file = "../data/output/MSM_Gaussian_expert_densities.csv", row.names = F)

study_priors <- ggplot(study.densities.df, aes(x = phi, y = dens,
                                              linetype = study, colour = study)) +
  geom_line(size = 2) +
  scale_x_continuous(expression(phi), expand = c(0, 0), limits = c(0, .4)) +
  scale_y_continuous(expression(f[i](phi)), expand = c(0, 0)) +
  theme_bw(base_size = 20)

study_priors

## Warning: Removed 3600 rows containing missing values (geom_path).

```



```
ggsave(study_priors, filename = "../plots/study_densities_MSMGaussian.pdf")
```

```
## Saving 6.5 x 4.5 in image
```

```
## Warning: Removed 3600 rows containing missing values (geom_path).
```

```
##### Equal weights
```

```
alphaEqual <- rep(1/K, K)
```

```
ab.Equal.star <- pool_par_gauss(alphaEqual, mv, vv)
```

```
# Prior
```

```
(PaperMSMGauss.tbl[1, 1:3] <- stat_gauss(ab.Equal.star))
```

```
## [1] 0.12205302 0.09879808 0.14530795
```

```
##### Maximum entropy
```

```
## WARNING: For the Gaussian case, we do not need to optimise, if you don't believe the maths, just run
```

```
# N <- 1000 ## could increase to, say, 10000 in order to make sure, but it's fine
```

```
# ent.many.startingPoints <- matrix(rnorm(n = (K-1)*N, mean = 0, sd = 100), ncol = K-1, nrow = N)
```

```
# many.ents <- lapply(1:N, function(i) {
```

```
#   optim(ent.many.startingPoints[i, ], optentgauss_inv, mp = mv, vp = vv)
```

```
# })
```

```
# optimised.ents <- unlist(lapply(many.ents, function(x) x$value))
```

```
#
```

```
# hist(optimised.ents)
```

```
# abline(v = optimised.ents[which.min(optimised.ents)], lty = 2, lwd = 2)
```

```
#
```

```
# alphaMaxEnt.opt <- alpha_01(many.ents[[which.min(optimised.ents)]]$par)
```

```

## Maximum entropy "analytical" solution,
alphaMaxEnt.opt <- rep(0, K)
alphaMaxEnt.opt[which.max(vv)] <- 1
round(alphaMaxEnt.opt, 2)

## [1] 0 1 0 0 0 0

( AlphasMSMGauss.tbl[1, ] <- alphaMaxEnt.opt )

## [1] 0 1 0 0 0 0
ab.MaxEnt.star <- pool_par_gauss(alphaMaxEnt.opt, mv, vv)

# Prior
(PaperMSMGauss.tbl[2, 1:3] <- stat_gauss(ab.MaxEnt.star))

## [1] 0.2407809 0.2017515 0.2798103

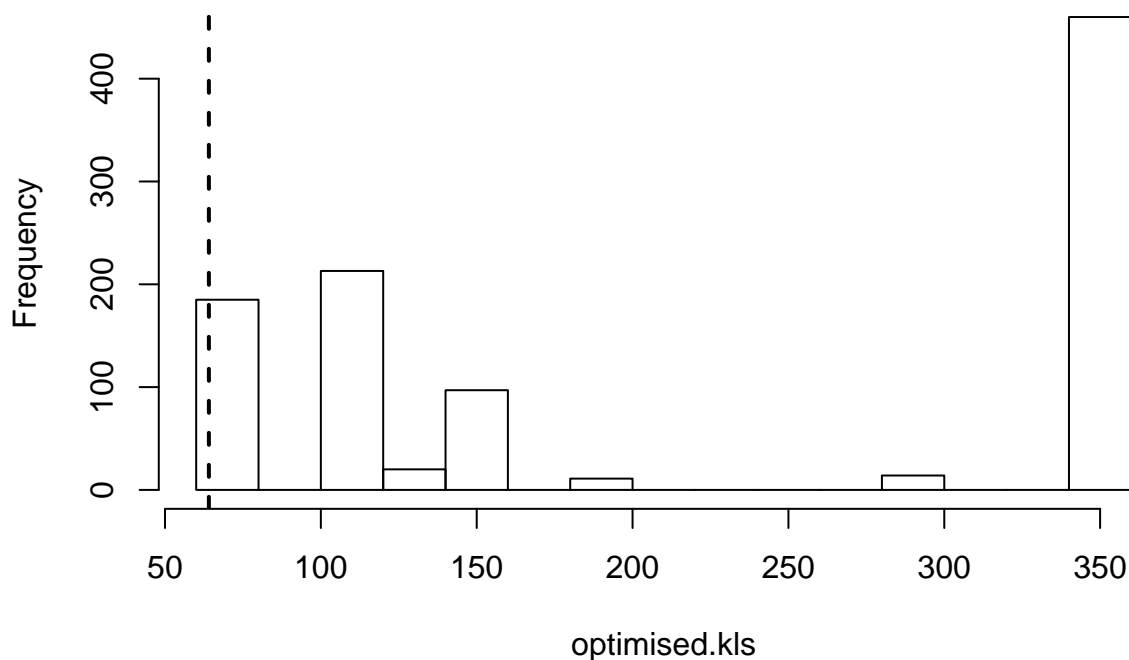
##### Minimum KL

N <- 1000 ## could increase to, say, 10000 in order to make sure, but it's fine
kl.many.startingPoints <- matrix(rnorm(n = (K-1)*N, mean = 0, sd = 100), ncol = K-1, nrow = N)
many.kls <- lapply(1:N, function(i) {
  optim(kl.many.startingPoints[i, ], optklgauss_inv, mp = mv, vp = vv, type = "fp")
})
optimised.kls <- unlist(lapply(many.kls, function(x) x$value))

hist(optimised.kls)
abline(v = optimised.kls[which.min(optimised.kls)], lty = 2, lwd = 2)

```

Histogram of optimised.kls



```

alphaKL.opt <- alpha_01(many.kls[[which.min(optimised.kls)]]$par)

round(AlphasMSMGauss.tbl[2, ] <- alphaKL.opt, 2)

## [1] 0.17 0.83 0.00 0.00 0.00 0.00
ab.KL.star <- pool_par_gauss(alphaKL.opt, mv, vv)

# Prior
(PaperMSMGauss.tbl[3, 1:3] <- stat_gauss(ab.KL.star))

## [1] 0.1601554 0.1287551 0.1915557

##### Hierarchical priors
require("LearnBayes")

## Loading required package: LearnBayes
M <- 100000
X <- c(1, 1, 1, 1, 1, 1)/10
alpha.MC.dirichlet <- rdirichlet(M, X)
alpha.MC.logisticNormal <- rlogisticnorm(N = M,
                                         m = digamma(X)-digamma(X[K]),
                                         Sigma = constructSigma(X))

apply(alpha.MC.dirichlet, 2, mean)

## [1] 0.1673374 0.1665987 0.1669955 0.1671761 0.1656557 0.1662365
apply(alpha.MC.logisticNormal, 2, mean)

## [1] 0.1671627 0.1676562 0.1669083 0.1665868 0.1661364 0.1655496
apply(alpha.MC.dirichlet, 2, sd)

## [1] 0.2955351 0.2949382 0.2955590 0.2954523 0.2937906 0.2946246
apply(alpha.MC.logisticNormal, 2, sd)

## [1] 0.3442450 0.3445844 0.3438166 0.3436327 0.3431704 0.3427884
gauss.par.dirichlet <- apply(alpha.MC.dirichlet, 1, function(w) pool_par_gauss(w, mv, vv))
gauss.par.logisticNormal <- apply(alpha.MC.logisticNormal, 1, function(w) pool_par_gauss(w, mv, vv))

phi.par.dirichlet <- apply(gauss.par.dirichlet, 2, function(x) rnorm(1, x[1], x[2]))
phi.par.logisticNormal <- apply(gauss.par.logisticNormal, 2, function(x) rnorm(1, x[1], x[2]))
# Prior
PaperMSMGauss.tbl[4, 1] <- mean(phi.par.dirichlet)
PaperMSMGauss.tbl[4, 2:3] <- quantile(phi.par.dirichlet, c(.025, .975))

PaperMSMGauss.tbl[5, 1] <- mean(phi.par.logisticNormal)
PaperMSMGauss.tbl[5, 2:3] <- quantile(phi.par.logisticNormal, c(.025, .975))

##### Using sample sizes
alphas.sampleSize <- meta$SampleSize/sum(meta$SampleSize)

( AlphasMSMGauss.tbl[3, ] <- alphas.sampleSize )

```

```
## [1] 0.1496815 0.1048681 0.1412648 0.2650136 0.1460419 0.1931301
```

```
ab.sampleSize <- pool_par_gauss(alphas.sampleSize, mv, vv)
```

```
# Prior
```

```
( PaperMSMGauss.tbl[6, 1:3] <- stat_gauss(ab.sampleSize) )
```

```
## [1] 0.1322952 0.1093096 0.1552809
```

```
#### Finally, tables!
```

```
round(PaperMSMGauss.tbl, 3)
```

```
##
##               mean.prior lower.prior upper.prior
## equal_weights      0.122      0.099      0.145
## maximum_entropy    0.241      0.202      0.280
## minimum_KL         0.160      0.129      0.192
## hierarchical_Dirichlet 0.133      0.063      0.250
## hierarchical_LogisticNormal 0.138      0.059      0.259
## Sample_size        0.132      0.109      0.155
```

```
round(AlphasMSMGauss.tbl, 3)
```

```
##
##      alpha_0 alpha_1 alpha_2 alpha_3 alpha_4 alpha_5
## maximum_entropy 0.000 1.000 0.000 0.000 0.000 0.000
## minimum_KL      0.171 0.829 0.000 0.000 0.000 0.000
## Sample_size     0.150 0.105 0.141 0.265 0.146 0.193
```

```
round(PaperMSMGauss.tbl, 2)
```

```
##
##               mean.prior lower.prior upper.prior
## equal_weights      0.12      0.10      0.15
## maximum_entropy    0.24      0.20      0.28
## minimum_KL         0.16      0.13      0.19
## hierarchical_Dirichlet 0.13      0.06      0.25
## hierarchical_LogisticNormal 0.14      0.06      0.26
## Sample_size        0.13      0.11      0.16
```

```
round(AlphasMSMGauss.tbl, 2)
```

```
##
##      alpha_0 alpha_1 alpha_2 alpha_3 alpha_4 alpha_5
## maximum_entropy 0.00 1.00 0.00 0.00 0.00 0.00
## minimum_KL      0.17 0.83 0.00 0.00 0.00 0.00
## Sample_size     0.15 0.10 0.14 0.27 0.15 0.19
```

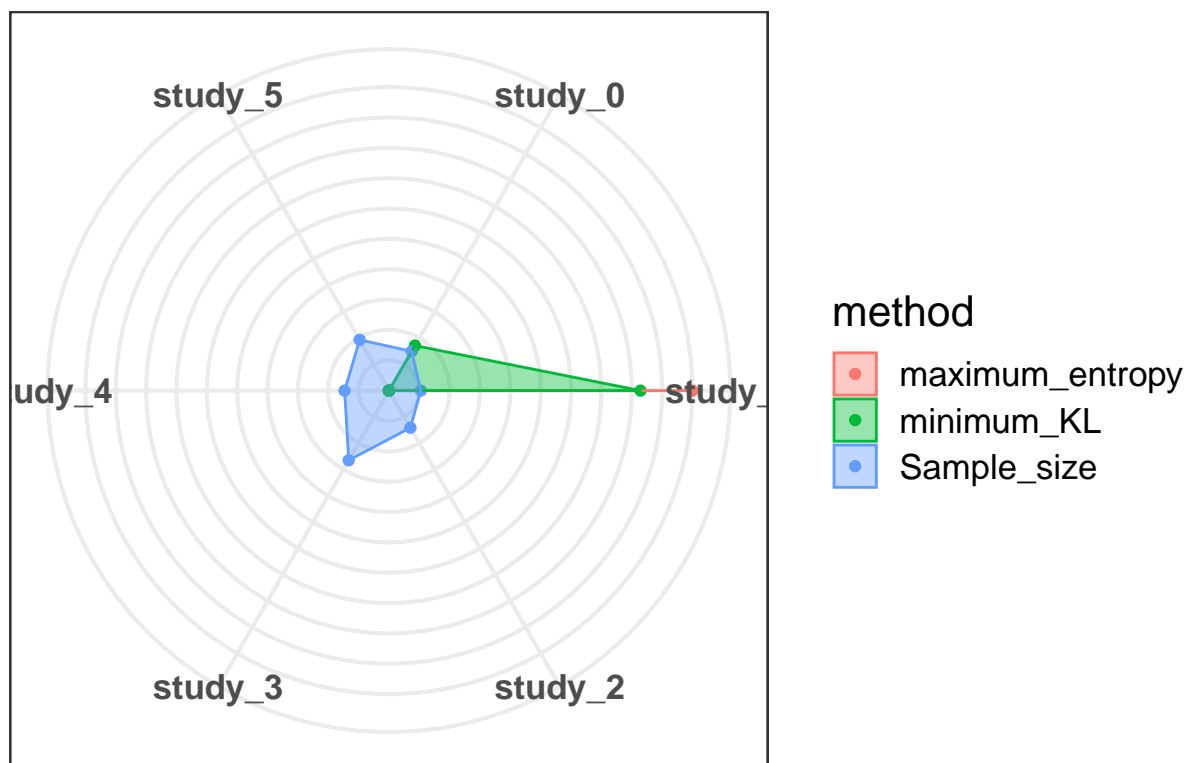
```
write.csv(round(PaperMSMGauss.tbl, 3), file = "../data/output/MSM_Gaussian_stat.csv", row.names = TRUE)
```

```
write.csv(round(AlphasMSMGauss.tbl, 3), file = "../data/output/MSM_Gaussian_weights.csv", row.names = TRUE)
```

```
##### Plotting
```

```
posterior_studies <- data.frame(
  alpha = as.numeric(c(AlphasMSMGauss.tbl[1, ], AlphasMSMGauss.tbl[2, ], AlphasMSMGauss.tbl[3, ])),
  lwr = rep(NA, 18),
  upr = rep(NA, 18),
  study = rep(paste("study_", 0:(K-1), sep = ""), 3),
  method = rep(c("maximum_entropy", "minimum_KL", "Sample_size"), each = K)
)
```

```
####
radar_alphas <- ggplot(data = posterior_studies,
  aes(x = study, y = alpha, group = method, colour = method, fill = method)) +
  geom_point() +
  geom_polygon(alpha = 0.4) +
  theme_bw(base_size = 16) +
  scale_y_continuous(expand = c(0, 0), limits = c(0, 1),
    breaks = number_ticks(10)) +
  coord_radar() +
  theme(axis.title.x = element_blank(),
    axis.ticks.x = element_blank(),
    axis.text.x = element_text(face = "bold"),
    axis.title.y = element_blank(),
    axis.text.y = element_blank(),
    axis.ticks.y = element_blank())
)
radar_alphas
```



```
ggsave(plot = radar_alphas, filename = "../plots/alphas_radar_MSMGaussian.pdf")
```

```
## Saving 6.5 x 4.5 in image
```

```
#####
# Now let's look at marginal likelihoods for the pooled priors

pars <- list(equal_weights = ab.Equal.star,
  maximum_entropy = ab.MaxEnt.star,
  minimum_KL = ab.KL.star,
  sample_Size = ab.sampleSize)
```

```

pars

## $equal_weights
## [1] 0.12205302 0.01186498
##
## $maximum_entropy
## [1] 0.24078091 0.01991334
##
## $minimum_KL
## [1] 0.16015539 0.01602084
##
## $sample_Size
## [1] 0.13229523 0.01172759

apply(AlphasMSMGauss.tbl, 1, get_ratio)

## maximum_entropy      minimum_KL      Sample_size
##                Inf          4.838260          1.372203

J <- length(pars)
posterior.densities.list <- vector(J, mode = "list")
for (j in 1:J){
  posterior.densities.list[[j]] <- data.frame(
    phi = phi.grid,
    dens = dnorm(phi.grid, mean = pars[[j]][1], sd = pars[[j]][2]),
    method = names(pars)[j]
  )
}

posterior.densities.df <- do.call(rbind, posterior.densities.list)
posterior.densities.df$distribution <- "Gaussian"

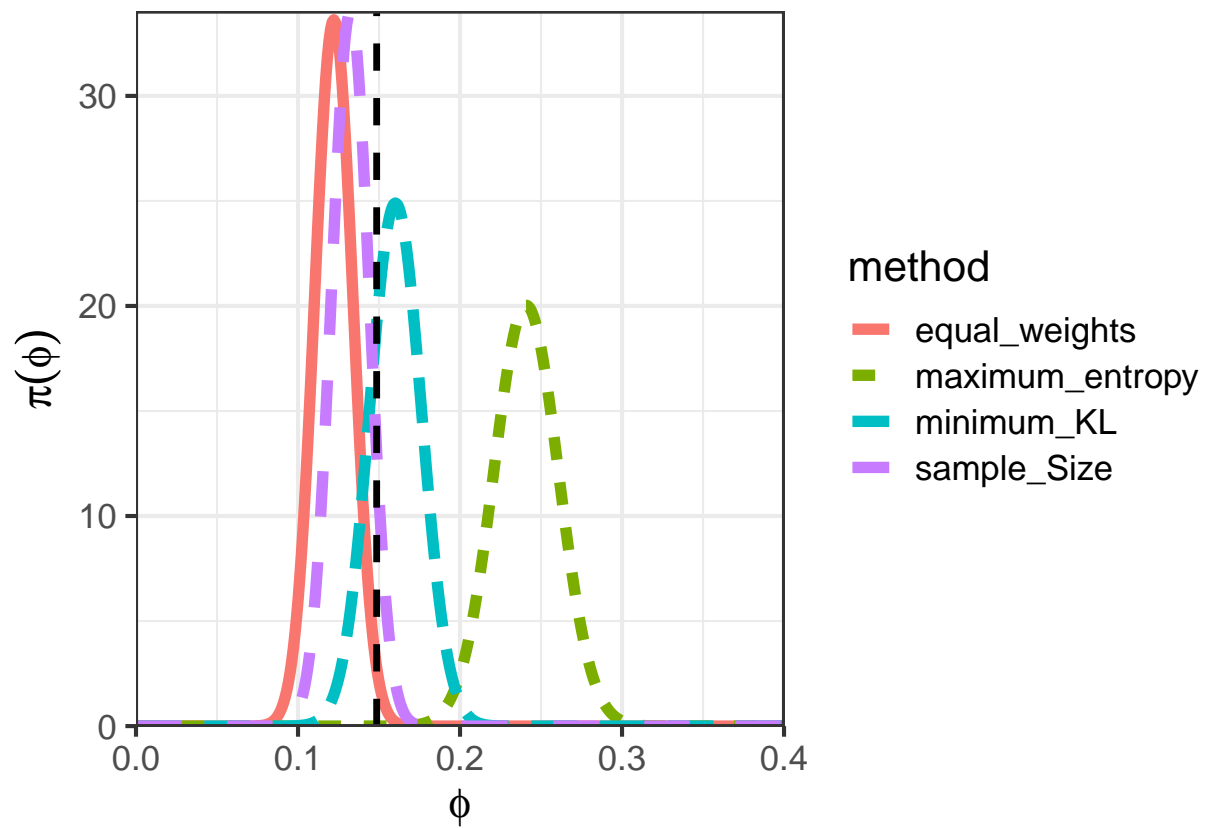
write.csv(posterior.densities.df, "../data/output/MSM_Gaussian_densities.csv", row.names = FALSE)

method_posteriors <- ggplot(posterior.densities.df, aes(x = phi, y = dens,
                                                         linetype = method, colour = method)) +
  geom_line(size = 2) +
  scale_x_continuous(expression(phi), expand = c(0, 0), limits = c(0, .4)) +
  scale_y_continuous(expression(pi(phi)), expand = c(0, 0)) +
  geom_vline(xintercept = sum(meta$HIV)/sum(meta$SampleSize), linetype = "dashed", size = 1.2) +
  theme_bw(base_size = 16)

method_posteriors

## Warning: Removed 2400 rows containing missing values (geom_path).

```

```
ggsave(method_posteriors, filename = "../plots/method_posterior_densities_MSMGaussian.pdf")
```

```
## Saving 6.5 x 4.5 in image
```

```
## Warning: Removed 2400 rows containing missing values (geom_path).
```