Homework 17

Max Bolger

October 2021

1 Problem 1

Problem - Find a way of calculating $1+x+x^2+x^3+...+x^n$ where n is a non negative integer

Solution - This solution assumes $1+x+x^2+x^3+...+x^n$ is equivalent to $x^0+x^1+x^2+x^3+...+x^n$ as $x^0=1$ and $x^1=x$

```
def func(x,n):
  sum = 0
  if n < 0:
    raise Exception('Sorry, n must be a non negative integer!')
  else:
    for i in range(n+1):
      sum+=(pow(x,i))

  return int(sum)
```

Listing 1: This function uses a loop to solve the problem

This can be written as $\frac{1-x^{n+1}}{1-x}$. However, there is only one exception. If $x=1$, then this equation is ignored and the answer is $n+1$. This is a *geometric series* since it has $n+1$ terms.

Definition 1 (Geometric series) *A series whose terms form a geometric progression.*

```
def formula(x,n):
  sum = 0
  if n < 0:
    raise Exception('Sorry, n must be a non negative integer!')
  elif x == 1:
    sum = n+1
  else:
    sum = (1-(pow(x,n+1)))/(1-x)

  return int(sum)
```

Listing 2: This function uses the mathematical formula to solve the problem

We can write a third function that combines these two functions and checks if they are indeed equivalent

```python
def test(x,n):
  sum_func = 0
  if n < 0:
    raise Exception('Sorry, n must be a non negative integer!')
  else:
    for i in range(n+1):
      sum_func+=(pow(x,i))

  if x == 1:
    sum_formula = n+1
  else:
    sum_formula = (1-(pow(x,n+1)))/(1-x)

  if sum_formula == sum_func:
    print('Equivalent')
  else:
    print('Not equivalent')

  return int(sum_formula), int(sum_func)
```

Listing 3: A function that tests if my function and the mathematical formula to solve this problem are equal

Lastly, we can test our function.

```python
>>>import random
>>> x = random.randint(0,10)
>>> n = random.randint(0,10)
>>> print(f'{x},{n}')
>>> test(x,n)
9,8
Equivalent
(48427561, 48427561)
```

Listing 4: Generating random numbers to test my function with

2 Problem 2

Problem - *True or False*: The *power set* of the natural numbers is <u>infinite but countable</u>.

Solution - *False*. The power set of the natural numbers is <u>uncountably infinite</u>. *Cantor's Theorem* states that for any set A there is no onto function $A \to \mathcal{P}(A)$. In this case, our set A is all real numbers and the power set of all real numbers can't be countable as there is no onto function. Additionally, $|\mathcal{P}(s)|= 2^{|s|}$

Definition 2 (Power set) *The set of all subsets of a set.*

Definition 3 (Cantor's Theorem) *For any set A, the set of all subsets of A (the power set of A, denoted by $\mathcal{P}(A)$) has a strictly greater cardinality than A itself.*