1 (a) The system is Linear. We let (u_1, y_1) and (u_2, y_2) be trajectories, so

$$y_1[n] = x_1[n+1] - n\, x_1[n]$$
$$y_2[x] = x_2[n+1] - n\, x_2[n]$$

Lett $a, b \in R$ and define

$$x_3[n] = a x_1[n] + b x_2(n)$$
$$y_3[n] = a y_1[n] + b y_2[n]$$

Then
$$y_3[n] = a(x_1[n+1] - n x_1[n]) + b(x_2[n+1] + \cancel{A} x_2[n])$$
$$= a x_1[n+1] + b x_2[n+1] - n(\cancel{a}\, x_1[n] + b x_2[n])$$
$$= x_3[n+1] - n x_3[n].$$

So (x_3, y_3) is also a trajectory, and the system is linear. $\boxed{5}$

(b) The system is NOT time-invariant. To show this we introduce $x[n] = \delta[n]$. Then the output is $y[n] = \delta[n+1] - n\,\delta[n]$.

Now let $x_1[n] = \delta[n+1]$, with the output $y_1[n] = \delta[n+2] - n\,\delta[n+1]$.
But $y[n+1] = \delta[n+2] - (n+1)\,\delta[n+1]$
and $y_1[-1] = 1$. Thus $y_1[n] \neq y[n+1]$
while $y[0] = 0$. So $(x(n+1), y(n+1))$ is not a trajectory. $\boxed{5}$

(c) The system is not causal, as the output at time n requires knowledge of the input ~~at time~~ ~~n+1~~ future time n+1 $\boxed{2}$

(2) The unit pulse response $h[n]$ satisfies

(a) $$h[n] + 2h[n-1] = 5\delta[n] \quad \text{— (1)}$$

If we apply $h[n] = 5(-2)^n u[n]$

we obtain

$$5(-2)^n u[n] + 10(-2)^{n-1} u[n-1]$$
$$= 5(-2)^n \left[u[n] - u[n-1] \right]$$
$$= 5(-2)^n \delta[n]$$
$$= 5\delta[n] \quad , \text{ as } \delta[n] = 1 \text{ when } n = 0$$

So $h[n]$ does satisfy (1) and is the unit pulse response.

$\boxed{5}$

(b) Assuming $x[n] = 0$ and $y[-1] = 10$

The natural response is obtained by recursion

$$y[0] + 2y[-1] = 0$$
$$\Rightarrow y[0] = -20$$
$$y[1] + 2y[0] = 0$$
$$\Rightarrow y[1] = (-20)(-2)$$
$$= 40 .$$

In general

$$y[n] = -2y[n-1]$$
$$= (-2)^n (-20) , \quad \text{for } n \geq 0 .$$

$\boxed{5}$

(c) The input is $x[n] = u[n] - u[n-2]$
$$= \delta[n] + \delta[n-1]$$

As the system is LTI, and $y[-1] = 0$,

we obtain output

$$y[n] = h[n] + h[n-1]$$
$$= 5(-2)^n u[n] + 5(-2)^{n-1} u[n-1]$$
$$= 5\delta[n] + \tfrac{5}{2}(-2)^n u[n-1] .$$

$\boxed{5}$

(d) As $(-2)^n \to \infty$ as $n \to \infty$, the output is unbounded.

$\boxed{1}$

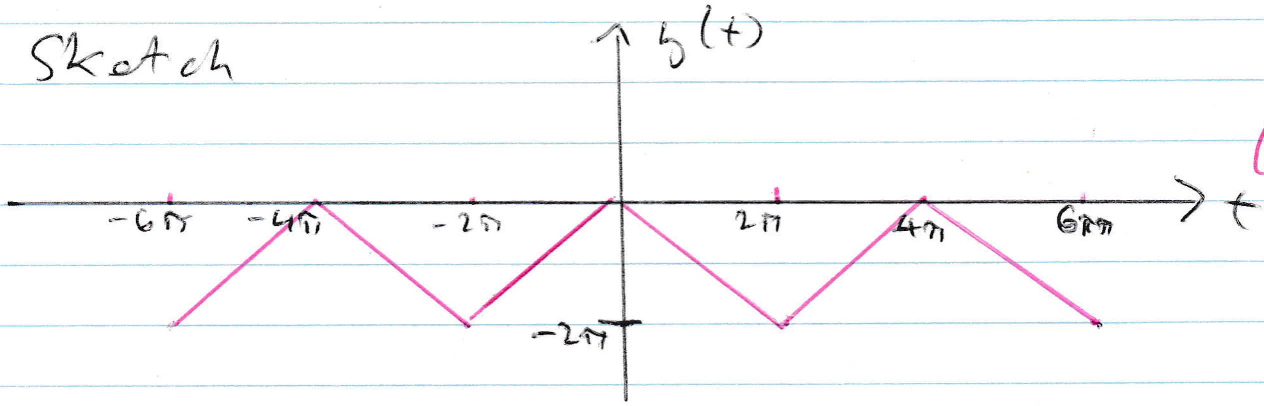
3

(a) ~~Sketch~~ The signal is periodic because it is obtained from $f(t)$ using the shift-and-summation formula, with

$$y(t) = \sum_{k=-\infty}^{+\infty} f(t-dk).$$

Since $d = 4\pi$, which is larger than the support of f (which is $[-2\pi, 2\pi]$) the signal has period $T = 4\pi$. (3)

(b) Sketch



(3)

(c) The signal satisfies the three Dirichlet conditions

(1) ~~System~~ signal is bounded on $[-2\pi, 2\pi]$ and hence it is absolutely integrable (1)

(2) Signal has maxima at $t=0$ and minima at $t = \pm 2\pi$. ~~Only~~ Hence it has finitely many maxima/minima on one ~~period~~ period (1)

(3) Function has no points of discontinuity (1)

$\boxed{3}$

(d) First we find a_1 and b_1, the coefficients of the trigonometric Fourier series. Signal y has period $T = 4\pi$, so $\omega_0 = \frac{2\pi}{T} = \frac{1}{2}$.

As y is an even function

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} y(t)\cos\left(\frac{t}{2}\right)dt$$

③ (d) So $a_1 = \frac{1}{\pi}\int_0^{2\pi} -t\cos(\frac{t}{2})\,dt$

Using Integration by Parts

$u = -t, \quad u' = -1, \quad v' = \cos(\frac{t}{2}), \quad v = 2\sin(\frac{t}{2})$

So

$$a_1 = \frac{1}{\pi}\Big[u(t)v(t)\Big]_0^{2\pi} - \frac{1}{\pi}\int_0^{2\pi} u'(t)v(t)\,dt$$

$$= \frac{1}{\pi}\Big[-2t\sin(\frac{t}{2})\Big]_0^{2\pi} + \frac{1}{\pi}\int_0^{2\pi} 2\sin(\frac{t}{2})\,dt$$

$$= \frac{1}{\pi}\Big[-2t\sin(\frac{t}{2}) - 4\cos(\frac{t}{2})\Big]_0^{2\pi}$$

$$= \frac{1}{\pi}(-2\pi\sin(\pi) - 4\cos(\pi)) - \frac{1}{\pi}(0 - 4\cos(0))$$

$$= \frac{8}{\pi}.$$

(3)

~~Hence~~ Also $b_1 = 0$ as y is an even function

(1)

Hence $A_1 = \sqrt{a_1^2 + b_1^2}$

$= \frac{8}{\pi}$

(1)

$\theta_1 = \tan^{-1}(\frac{b_1}{a_1})$

$= \tan^{-1}(0)$

$= 0.$

(1)

$\boxed{6}$

(4) We want to prove that for all integer $N >$

$$\sum_{i=0}^{N-1} \delta[n-i] = u[n] - u[n-N].$$

Firstly we let $N = 1$; we need to prove

$$\delta[n] = u[n] - u[n-1]. \quad (1)$$

To do this let $n = 0$: Then

$$\delta[0] = 1 = u[0] - u[-1] \checkmark \quad (1)$$

Let $n \not< 0$: Then

$$\delta[n] = 0 = u[n] - u[n-1] \quad (1)$$

If $n < 0$: $\delta[n] = 0 = 0 - 0 = u[n] - u[n-1]$

Next Assume this is true for some $N = k$

$$\sum_{i=0}^{k-1} \delta[n-i] = u[n] - u[n-k] \quad (1$$

Consider $N = k+1$:

$$\sum_{i=0}^{k} \delta[n-i] = \sum_{i=0}^{k-1} \delta[n-i] + \delta[n-k] \quad (1$$

$$= u[n] - u[n-k] + \delta[n-k] \quad ($$

$$= u[n] - u[n-(k+1)] - \delta[n-k] + \delta[n-k$$

(1)

$$= u[n] - u[n-(k+1)]$$

(1)

Hence its true for all $N > 0$, by Induction

$\boxed{8}$