THE UNIVERSITY OF MELBOURNE

Semester 2 Assessment
7 September 2020

Department of Electrical and Electronic Engineering
ELEN30012 SIGNALS AND SYSTEMS

Time allowed: 70 minutes

Instruction to students:

Attempt **ALL** questions. Write your test answers using pens and A4 paper. Tablets and other electronic software may not be used for writing your test answers. Start each question on a new sheet of paper. Only write on one side of each sheet of paper.

The questions carry weight in proportion to the marks in brackets after the question numbers. These marks total 50 marks. **You must show your work in order to receive credit!**

The format of the test is open book, meaning that students may have unlimited access to printed or online materials.

Students may use any computational software for numerical calculations, however the marking scheme will require that students show the details of their working and calculations as if a CASIO FX-82 or CASIO FX-100 calculator had been used.

Your work must be scanned and uploaded to CANVAS as a single pdf file. This must be done by 12.10 p.m. Late submissions will not be marked.

Question 1 (12 marks)

Consider the discrete-time system with trajectories (x, y) defined by

$$y[n] = x[n+1] - nx[n]$$

(a) Is this a linear system? Justify your answer with either a proof or a counterexample.

(b) Is this a time-invariant system? Justify your answer with either a proof or a counterexample.

(c) Is this a causal system? Give a reason for your answer.

Question 2 (16 marks)

A linear time-invariant discrete-time system has first-order difference equation

$$y[n] + 2y[n-1] = 5x[n]$$

(a) Show that the unit pulse response is

$$h[n] = 5(-2)^n u[n]$$

(b) If $y[-1] = 10$, find the natural response of the system for all $n \geq 0$.

(c) Assuming $y[-1] = 0$, find the output of the system when the input is

$$x[n] = u[n] - u[n-2]$$

Simplify your answer as much as possible.

(d) Is the output in part (c) bounded? Briefly explain your answer.

Question 3 (14 marks)

Consider the continuous-time signal f given by

$$f(t) = -|t|(u(t + 2\pi) - u(t - 2\pi))$$

and introduce the continuous-time signal y as

$$y(t) = \sum_{k=-\infty}^{\infty} f(t - 4\pi k)$$

(a) Explain why the signal y is periodic, and state its period. Briefly justify your answers.

(b) Sketch y on the interval $-6\pi \leq t \leq 6\pi$.

(c) Explain why the signal y has a Fourier series representation, giving brief reasons.

(d) Find A_1 and θ_1, the first pair of coefficients in the cosine-with-phase Fourier series for y.

Question 4 (8 marks)

Use Mathematical Induction to prove that for any integer n and positive integer N,

$$\sum_{i=0}^{N-1} \delta[n - i] = u[n] - u[n - N]$$