Question 1

(a) Linear

(b) Time-invariant

(c) $H(z) = \dfrac{-z}{z^2 - 3z + 1}$.

(d) Causal, has memory.

Question 2

(a) $f = 5$

(b) $y_1[n] = \cos\left(\dfrac{5\pi n}{4}\right) + \cos\left(\dfrac{5\pi n}{2}\right)$

(c) $L = 8$.

Question 3

(a) $T_0 = \pi$.

(b) $c_k^v = \begin{cases} \frac{-1}{2k^2}, & k \neq 0 \\ 2, & k = 0 \end{cases}$

(c) $|H(\omega)| = \begin{cases} 5, & |\omega| \geq \pi \\ 0, & |\omega| < \pi \end{cases}$ $\qquad \angle H(\omega) = \begin{cases} \pi - \omega, & |\omega| \geq \pi \\ 0, & |\omega| < \pi \end{cases}$

(d) $a_0^y = A_1^y = \theta_1^y = 0$ and for $k \geq 1$, $A_k^y = \frac{5}{k^2}$, $\theta_k^y = -2k$.

(e)(i) f is periodic. (ii) $c_k^f = \begin{cases} \frac{-e^{-6jk}}{2k^2}, & k \neq 0 \\ 2, & k = 0 \end{cases}$

Question 4

(a) $X(\Omega) = e^{j4\Omega}$.

(b) $y[n] = \dfrac{1}{2}[\delta[n+4] + \delta[n-4]]$

(c) $g[n] = \dfrac{1}{4j}[\delta[n+8] - \delta[n-8]]$

Question 5

(a) $\sin(3t)$.

(b) $\dfrac{9}{s^2 + 81}$

(c) $\dfrac{6s}{(s^2 + 9)^2}$

(d) $Y(s) = \dfrac{3}{2}\left[\dfrac{1}{(s - j2)^2 + 9} + \dfrac{1}{(s + j2)^2 + 9}\right]$.

(f) $Z(s) = \frac{9e^{-4s/3}}{s^2 + 81}$.

Question 6

$x_1 \longleftrightarrow X_7, \ x_2 \longleftrightarrow X_3, \ x_3 \longleftrightarrow X_5, \ x_4 \longleftrightarrow X_8,$
$x_5 \longleftrightarrow X_1, \ x_6 \longleftrightarrow X_6, \ x_7 \longleftrightarrow X_2, \ x_8 \longleftrightarrow X_4.$

Question 7

(a) $H(s) = \dfrac{s-2}{s^2+4}; \ A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C = \begin{bmatrix} -2 & 1 \end{bmatrix}.$

(d) $\bar{A} = \begin{bmatrix} j2 & 0 \\ 0 & -j2 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} \frac{-j}{4} \\ \frac{j}{4} \end{bmatrix}, \ \bar{C} = \begin{bmatrix} -2+j2 & -2-j2 \end{bmatrix}.$