

Question 1

$$\begin{aligned} x[-1] &= 1 \\ \text{(a) (i)} \quad x[0] &= 1 \\ x[1] &= 0 \\ x[2] &= 0 \end{aligned} \quad \text{(ii) } M = -1, n_0 = 0$$

(b) $x(t) = t^2 + 4t + 5$ is one possible answer.

(c) $x[n] = \cos\left(\frac{2\pi n}{3}\right)$ is one possible answer.

$$\text{(d) } y[n] = \begin{cases} \frac{1}{3} & n = 0 \\ \frac{2}{3} & n = 1 \\ 1, & \geq 2 \end{cases}$$

Question 2

$$\text{(a) } H(z) = \frac{2z+1}{z^2-3z-4}$$

(b) Unstable system because the system poles are at $z = -1, 4$, and the pole at $z = 4$ is outside of the unit circle in the complex plane.

$$\text{(c) } A = \begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 2]$$

Question 3

(a) Causal, as $h(t) < 0$ for $t < 0$. (b) $\int_{-\infty}^{\infty} |h(t)| dt = 2 < \infty$

$$\text{(c) } y(t) = x(t-2) + x(t-1) \quad \text{(d) } y(t) = \begin{cases} 2t-1, & t \geq 2 \\ t, & 1 \leq t < 2 \\ 0, & t < 1 \end{cases}$$

$$\text{(e) } H(\omega) = e^{-j2\omega} + e^{-j\omega}, \quad y(t) = \sqrt{2} \cos\left(\frac{\pi t}{2} - \frac{3\pi}{4}\right)$$

Question 5

(a) Neither even nor odd function. (b) $T_0 = 2\pi, \omega_0 = 1$

(c) $c_0^y = 2, c_2^y = 3/2, c_{-2}^y = 3/2, c_3^y = -j2, c_{-3}^y = 2j, c_k = 0$ for all other $k \in \mathbf{Z}$

(d) $c_0^y = 0, c_2^y = 15e^{-j10}, c_{-2}^y = 15e^{j10}, c_3^y = 20e^{-j(15+\frac{\pi}{2})}, c_{-3}^y = 20e^{j(15+\frac{\pi}{2})}, c_k = 0$ for all other $k \in \mathbf{Z}$

Question 6

See Workshop 6.

Question 7

(a) $|X(\Omega)| = \sqrt{2-2\cos(\Omega)}$ (b) $X_k = 1 - e^{-j2\pi k/N}$ for $k = 0, 1, \dots, N-1$.

(c) $|X_0| = 0, |X_1| = \sqrt{2}, |X_2| = 2, |X_3| = \sqrt{2}$