Student Number: ....

THE UNIVERSITY OF MELBOURNE

Department of Electrical and Electronic Engineering

ELEN 20005 FOUNDATIONS OF ELECTRICAL NETWORKS

Semester 2 Assessment

November 2017

Time allowed: 180 minutes

Reading time: 15 minutes: no writing or annotating allowed anywhere

This paper has 11 pages including the 2-page Formulae Sheet

Authorised materials:
This is a closed-book examination. Only the following calculators may be used:
Casio FX82 and Casio FX100

Instructions to invigilators:
All examination material (script book and test paper) is to be collected at the end of the exam.

Instruction to students:
Attempt **ALL** questions.

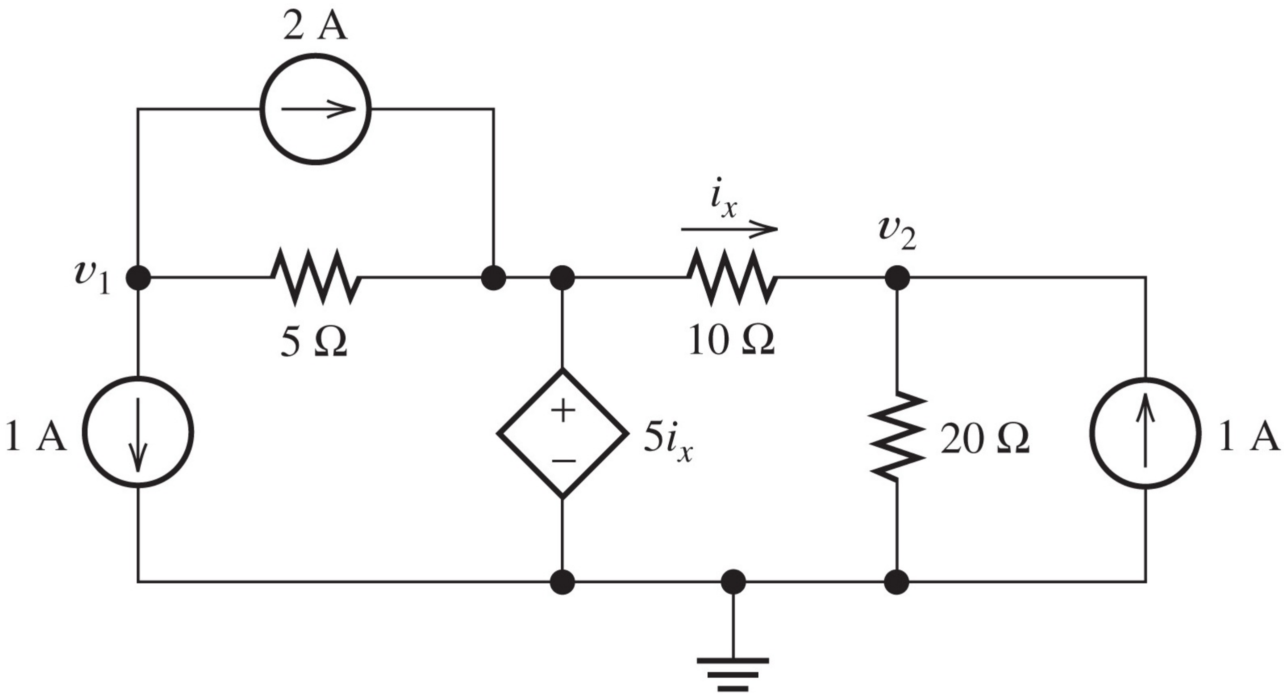
The marks given for each question are shown in brackets after the question numbers. The examination paper has a total of 100 marks.

Write your student number in the space provided on the all script books used and at the top of this page. This must be done during writing time. No annotating is allowed in reading time or after the end of writing time.

Answer all questions and show all working right-hand lined pages of the script book, except where you are instructed to write on this examination paper.

This paper is to be lodged with the Baillieu library.

Question 1 (12 marks)



(a) [**4 marks**] For the circuit above, use mesh current analysis to obtain the mesh currents. You will need to transcribe the circuit diagram into your script book and clearly label your mesh currents.

(b) [**4 marks**] Use your mesh currents to solve for the node voltages v_1 and v_2.

(c) [**2 marks**] Is the current controlled voltage source absorbing or delivering power? Justify your answer.

(d) [**1 marks**] A 4-band inductor has colour bands Orange/Orange/Red/Red. What is its inductance and tolerance?

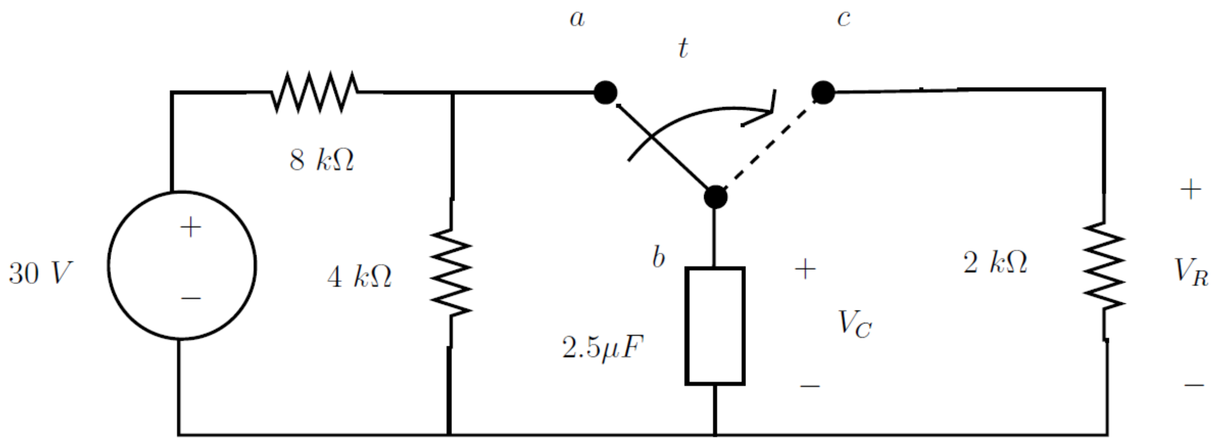
(e) [**1 marks**] A polyester capacitor has the alpha-numeric code 472J. What is its capacitance?

You may use the following colour codes and tolerances.

Silver = 10^{-2}	Gold = 10^{-1}	Black = 0	Brown = 1	Red = 2	Orange = 3
Yellow = 4	Green = 5	Blue = 6	Purple = 7	Grey = 8	White = 9
Tolerances	Silver = 10%	Gold = 5 %	Red = 2%	Brown = 1%	

Question 2 (10 marks)

The single pole, double throw switch in the following circuit has been in position a for a long time, giving a short circuit between node a and node b, and an open circuit between nodes b and c. At time $t = 0$ s it is instantaneously moved to position c, causing an open circuit between terminals a and b of the circuit, and a short circuit between nodes b and c.



(a) [**6 marks**] The rectangular box represents a capacitance of $C = 2.5\ \mu F$. What is $V_C(0^+)$, the voltage across the capacitor immediately after the switch is moved to node c? Find $V_C(t)$ for $t \geq 0$ and sketch its graph.

(b) [**2 marks**] After what time does $V_R(t) = 2\ V$?

(c) [**2 marks**] What is $w_C(0)$, the energy initially stored on the capacitor? Find $w_C(t)$ for $t \geq 0$ and sketch its graph.

Question 3 (12 marks)

A balanced three-phase Wye-connected source with positive phase sequence and line-to-neutral voltage $\mathbf{V}_{an} = 440\underline{/0^\circ}\ V$ is applied to a balanced Delta-connected load of $Z_\Delta = 12 + j9\ \Omega$.

(a) [**3 marks**] Draw the circuit diagram for this power circuit.

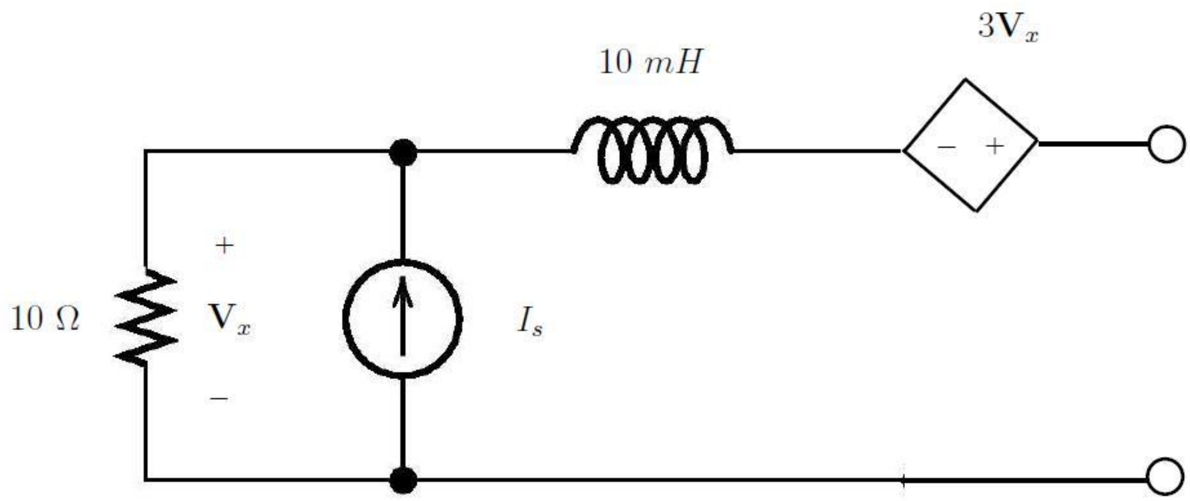
(b) [**3 marks**] Find the line voltage and line current in phase a of the source.

(c) [**3 marks**] Find the load voltage and load current in phase AB of the load.

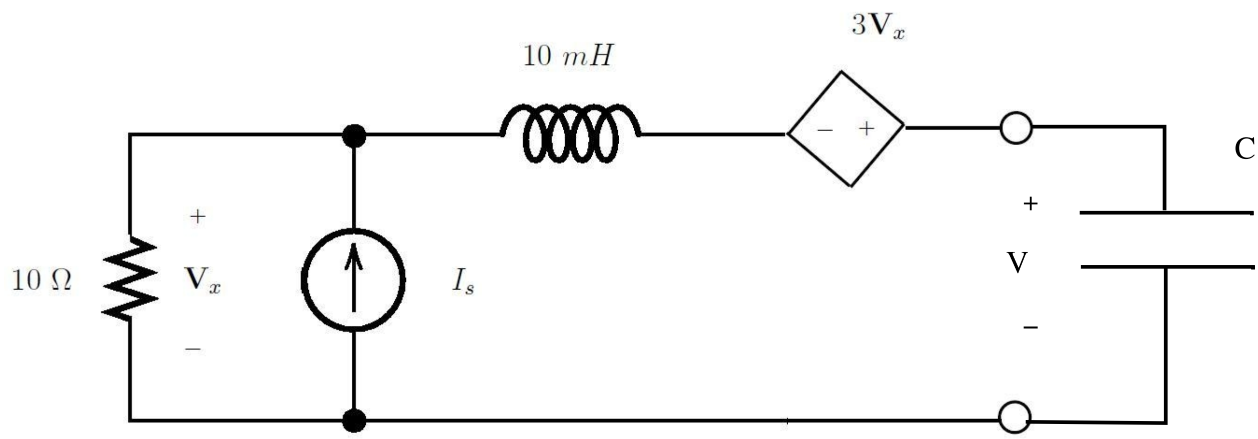
(d) [**3 marks**] Find the power angle and the average real and reactive powers supplied to phase AB of the load.

Question 4 (14 marks)

(a) [**8 marks**] Find and draw the Thévenin equivalent circuit at the output terminals for the circuit shown below, if $I_s(t) = 2\cos(1000t)\ A$.

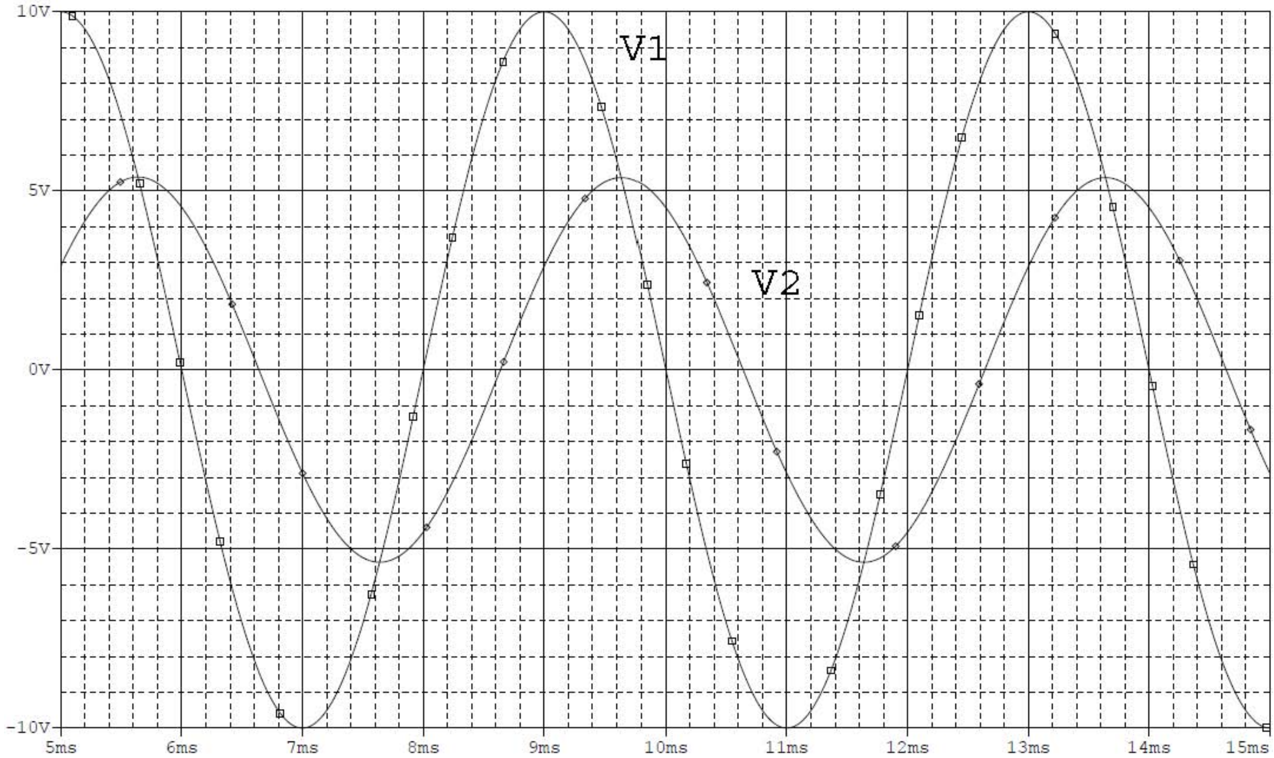


(b) [**6 marks**] Next assume a 100 μF capacitor C is added to the output terminals. Find $\mathbf{V}$, the capacitor voltage phasor, and hence write down $v(t)$, the voltage across the capacitor.



Question 5 (6 marks)

The following graph shows two sinusoidal voltage functions of the same frequency.



(a) [**4 marks**] Assume V_1 is a sinusoidal voltage signal in the form

$$V_1(t) = A_m \cos(\omega t - \theta)\ V$$

Estimate the values of A_m, ω and θ (in degrees).

(b) [**2 marks**] Assume that signal V_2 is a sinusoidal voltage signal of the same frequency as V_1. What is the phase difference between V_1 and V_2, in degrees? Is V_1 leading or lagging V_2?

Question 6 (15 marks)

(a) [**3 marks**] A simple RC series circuit consists of a sinusoidal voltage supply of $v_s(t) = V_m \cos(\omega t)$ delivering power to a load consisting of a resistor R and a capacitor C, connected in series. Draw and label the circuit diagram for this circuit, showing v_s, the reference current i and the capacitor voltage v_C. The voltage polarities and reference current direction should be such that the reference current direction enters the negative $(-)$ polarity of the voltage supply v_s, and the positive polarity $(+)$ of the capacitor voltage.

(b) [**5 marks**] Assume the circuit has the parameter values

$$V_m = 2\ V, \qquad \omega = 1000\pi\ rad/s, \qquad R = 560\ \Omega, \qquad C = 2.2\ \mu F$$

Find the sinusoidal steady state current $i(t)$ and capacitor voltage $v_C(t)$. What is the phase difference between $v_s(t)$ and $v_C(t)$?

(c) [**7 marks**] Your laboratory kit contains the following equipment:

- TG2000 digital function generator;
- TDS 1012B digital oscilloscope;
- A breadboard and connecting wires;
- One 560 Ω resistor, and a 2.2 μF capacitor;
- A pair of red and black bnc-banana cables, and two oscilloscope probes.

Explain using **words, graphs and equations** how you would do the following:

(i) build the RC circuit on your breadboard, and

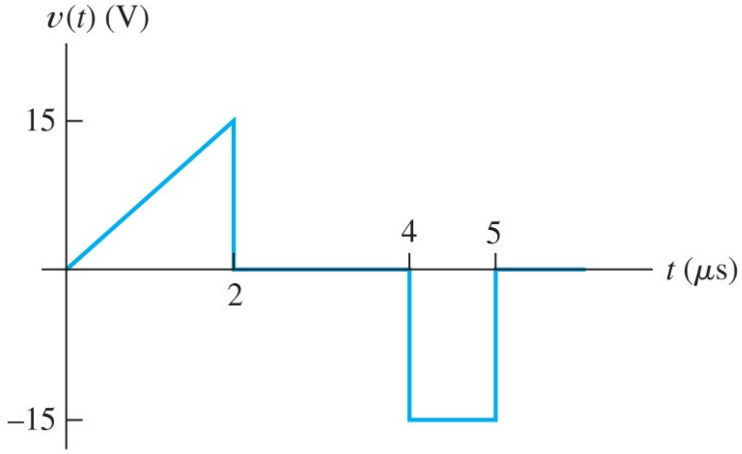
(ii) take suitable measurements to experimentally confirm your answer in part (b) for the phase difference between the supply voltage and the capacitor voltage.

Support your explanation with a diagram of your circuit drawn on the breadboard diagram on the next page. Draw and label the power supply, input voltage terminals, all components (these may be drawn as rectangular boxes), and the connecting wires. Also show where you would put the oscilloscope probes. The use of different colours is recommended. **Use a maximum of 150 words**.

Question 6 (continued)

Question 7 (10 marks)

The voltage through a 15 mH inductor is shown below. At $t = 0$, the current is 1 mA. Sketch the current, power and stored energy to scale versus time.



Question 8 (6 marks)

(a) [**2 marks**] Use Boolean algebra to express the logic function

$$Y = \overline{((A+\overline{C})+B\,C)} + \overline{(A+C)}$$

in sum-of-products (SOP) form.

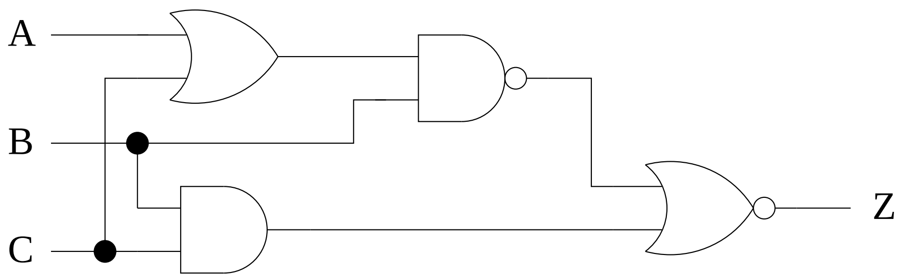
(b) [**2 marks**] Show how you would implement your logic function with suitable logic gates. Assume that AND, OR and INVERTOR gates are available.

(c) [**2 marks**] Use a truth table to prove or disprove the following Boolean equation:

$$X \;+\; \overline{X}\,Y \;\;=\;\; X \;+\; \overline{Y}$$

Question 9 (5 marks)

Consider the following circuit:



(a) [**1 marks**] Determine an expression for the output variable Z in terms of the input variables A, B and C.

(b) [**2 marks**] Using the above expression, determine the complete truth-table for Z.

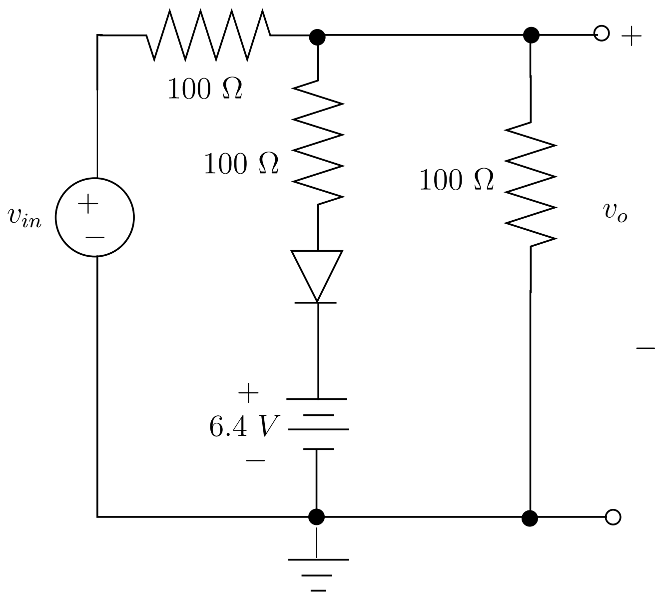
(c) [**2 marks**] Assume the following transition propagation delays (tpd) for logic gates:

- AND gate: 21 ns
- OR gate: 24 ns
- NAND and NOR gates: 17 ns
- Inverter (NOT) gate: 15 ns

Determine the worst-case propagation delay from input to output for the digital logic circuit above. Draw the circuit diagram in your script book and indicate the pathway for this delay.

Question 10 (10 marks)

The diode in the circuit below has a simple piecewise linear voltage current relationship with forward threshold $v_f = 0.6\ V$.



Find the output voltage v_o as a function of the input voltage v_{in} for all possible values of v_{in}.

END OF EXAMINATION

ELEN 20005 Foundations of Electrical Networks
Formulae for the Final Exam

1. **Current, Voltage, Power, Energy**

$$i(t) = \frac{dq}{dt}, \qquad v(t) = \frac{dw}{dq}, \qquad p(t) = \frac{dw}{dt} = v(t)i(t), \qquad w(t) = \int_{t_0}^{t} p(t)\ dt$$

2. **Ohm's Law and Kirchoff's Laws**

$$v = iR, \qquad \sum_{n=1}^{N} i_n = 0, \qquad \sum_{m=1}^{M} v_n = 0$$

3. **Resistive DC circuits**

$$p = vi = \frac{v^2}{R} = i^2 R \qquad R_{eq} = R_1 + R_2 + R_3, \qquad R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}, \qquad G = \frac{1}{R}$$

4. **Thévenin and Norton Equivalent DC Circuits**

$$v_T = v_{oc}, \qquad R_T = \frac{v_{oc}}{i_{sc}}, \qquad i_N = i_{sc}, \qquad R_L = R_t \text{ (max power transfer)}$$

5. **Capacitors**

$$i(t) = C\frac{dv}{dt}, \qquad q(t) = Cv(t), \qquad w(t) = \frac{1}{2}Cv^2(t),$$

$$C_{eq} = C_1 + C_2 + C_3 \text{ (parallel)} \qquad C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \text{ (series)}$$

6. **Inductors**

$$v(t) = L\frac{di}{dt}, \qquad w(t) = \frac{1}{2}Li^2(t),$$

$$L_{eq} = L_1 + L_2 + L_3 \text{ (series)} \qquad L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} \text{ (parallel)}$$

7. **First Order differential equations**

$$\frac{dx}{dt} + a_0 x = a_1 \qquad \text{has general solution} \qquad x(t) = K_1 e^{st} + K_2$$

8. **Phasors**

$$v(t) = V_m \cos(\omega t + \theta) \qquad \Longleftrightarrow \qquad \mathbf{V} = V_m \underline{/\theta} = V_m e^{j\theta}$$

$$i(t) = I_m \cos(\omega t + \theta) \qquad \Longleftrightarrow \qquad \mathbf{I} = I_m \underline{/\theta} = I_m e^{j\theta}$$

9. **Impedances**

$$\mathbf{V} = \mathbf{I}\,Z, \qquad Z_R = R, \qquad Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} = \frac{1}{\omega C}\underline{/-90^\circ}, \qquad Z_L = j\omega L = \omega L\underline{/90^\circ}$$

10. **Thévenin and Norton Equivalent AC Circuits**

$$\mathbf{V}_T = \mathbf{V}_{oc}, \qquad Z_t = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}}, \qquad \mathbf{I}_N = \mathbf{I}_{sc}$$

11. **Some Useful Sinusoid Identities**

$$\sin(z) = \cos(z - 90^\circ), \qquad -\cos(z) = \cos(z \pm 180^\circ),$$

12. **2 by 2 matrix inverse**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

13. **AC (Sinusoidal) Power**

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}} \qquad \theta = \theta_v - \theta_i$$

$$P = V_{rms}I_{rms}\cos(\theta) \qquad Q = V_{rms}I_{rms}\sin(\theta) \qquad P^2 + Q^2 = (V_{rms}I_{rms})^2$$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta)(1 + \cos(2\omega t)) + \frac{V_m I_m}{2}\sin(\theta)\sin(2\omega t) \qquad \text{(assuming } \theta_v = 0\text{)}$$

$$P = I_{rms}^2 R, \qquad Q = I_{rms}^2 X, \qquad P = \frac{V_{R\,rms}^2}{R}, \qquad Q = \frac{V_{X\,rms}^2}{X}$$

14. **Transformers**:

$$|v_2(t)| = \frac{N_2}{N_1}|v_1(t)|, \qquad |i_2(t)| = \frac{N_1}{N_2}|i_1(t)|, \qquad Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

15. **Three-Phase Power**:

$$\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{an}\underline{/30^\circ} \qquad \mathbf{I}aA = \sqrt{3}\mathbf{I}_{AB}\underline{/-30^\circ}$$

$$Z_Y = Z_y\underline{/\theta}, \qquad I_L = \frac{V_Y}{Z_y}, \qquad Z_\Delta = Z_d\underline{/\theta}, \qquad I_\Delta = \frac{V_L}{Z_d}$$

16. **Boolean Algebra**

DeMorgan's Theorem: $\overline{x + y} = \overline{x} \cdot \overline{y} \qquad \overline{x \cdot y} = \overline{x} + \overline{y}$