(112) $H(s) = \dfrac{32}{s^2+10s+16} = \dfrac{Y(s)}{V(s)}$

(a) $\Rightarrow$ $32\,V(s) = (s^2+10s+16)\,Y(s)$

So $32v(t) = \dfrac{d^2y}{dt^2} + 10\dfrac{dy}{dt} + 16y(t)$

describes the system

(b) Since $A(s) = s^2 + 10s + 16$

we see that $\omega_n^2 = 16 \Rightarrow \omega_n = 4$

Hence $\zeta = \dfrac{10}{2(4)} = 1.25 > 1$

So system is overdamped.

OR $A(s) = (s+8)(s+2)$

$\Rightarrow s = -8, -2$

Two real poles $\Rightarrow$ overdamped.

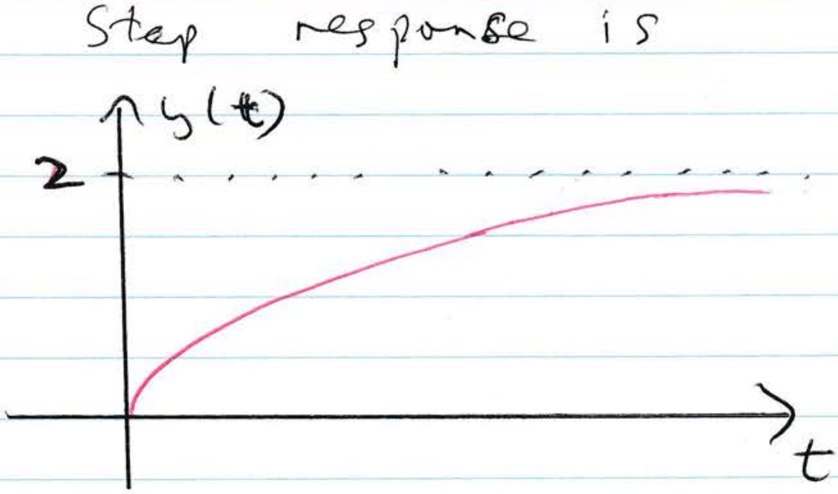
(c) $y_{ss} = H(0)$

$= \dfrac{32}{16}$

$= 2.$

$s = -2$ is the dominant pole.

Hence step response is



112 (d) $v(t) = \cos(4t + \pi)$

Then $H(j4) = \dfrac{32}{-16 + 40j + 16}$

$= -0.8j$

$\Rightarrow y_{ss}(t) = |H(j4)| \cos(4t + \pi + \angle H(j4$

$= 0.8 \cos(4t + \pi - \frac{\pi}{2})$

$= 0.8 \cos(4t + \frac{\pi}{2})$

(114) (a) $v[n] = \begin{cases} 1, & n=0 \\ 1, & n=1 \\ -1, & n=2 \\ -1, & n=3 \\ 0, & \text{otherwise} \end{cases}$

So $v[n] = \delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-$

and

$$y[n] = n\,u[n] - n\,u[n-4]$$

$$= \begin{cases} 1, & n=1 \\ 2, & n=2 \\ 3, & n=3 \end{cases}$$

So $y[n] = \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$

(b) Let $x[n] = \delta[n-q]$, $q > 0$.

Then $X(z) = \sum_{n=0}^{+\infty} \delta[n-q]\, z^{-q}$

$= z^{-q}$, by the sifting theore

(c) From (a) and (b) we have that

$$V(z) = 1 + z^{-1} - z^{-2} - z^{-3}$$

$$Y(z) = z^{-1} + 2z^{-2} + 3z^{-3}$$

Hence

$$H(z) = \frac{Y(z)}{V(z)}$$

$$= \frac{z^{-1} + 2z^{-1} + 3z^{-3}}{1 + z^{-1} - z^{-2} - z^{-3}}$$

$$= \frac{z^2 + 2z + 3}{z^3 + z^2 - z - 1}.$$

(115) Using the Left-Time Shift theorem gives
(a)

$$z^2 Y(z) + Y(z) = 2zV(z) - V(z)$$

$$\Rightarrow Y(z)(z^2+1) = V(z)(2z-1)$$

So $$H(z) = \frac{Y(z)}{V(z)} = \frac{2z-1}{z^2+1}$$

Then $z^2+1 = (z+j)(z-j)$
So poles are $p = \pm j$

These poles are on the unit disk. Hence the system is <u>marginally stable</u>.

(b) For the system step response.

$$y_{ss}[n] = H(1) = \frac{1}{2}$$ is the steady-state value.

(c) We know that the step ~~z~~ input $v[n] = u[n]$ has z-transform

$$u[n] \longleftrightarrow \frac{z}{z-1}.$$

So the output from the step input has transform

$$Y(z) = H(z)U(z) = \left(\frac{2z-1}{z^2+1}\right)\left(\frac{z}{z-1}\right)$$

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(c) So $\frac{Y(z)}{z} = \frac{2z-1}{(z^2+1)(z-1)}$

with poles at $p = \pm j, 1$.

Thus

$$\frac{Y(z)}{z} = \frac{c_1}{z-1} + \frac{c_2}{z-j} + \frac{\bar{c}_2}{z+j}$$

where $c_1 = \left.\frac{2z-1}{z^2+1}\right|_{z=1}$

$$= \frac{1}{2}$$

Obtain $c_2 = \frac{-1}{4} - \frac{3j}{4}$, $\bar{c}_2 = -\frac{1}{4} + \frac{3j}{4}$.

Taking inverse z Transforms give.

$$Y(z) = \frac{c_1 z}{z-1} + \frac{c_2 z}{z-j} + \frac{\bar{c}_2 z}{z+j}$$

$$\Rightarrow y(n) = c_1 u[n] + c_2 j^n + \bar{c}_2 (-j)^n$$

Letting $j = \sigma e^{j\Omega}$ for $\sigma = 1$, $\Omega = \frac{\pi}{2}$

we obtain

$$y(t) = \frac{1}{2} u[n] + 2|c| \cos\left(\frac{\pi n}{2} + \angle c\right)$$

$$= \frac{1}{2} u[n] + \frac{1}{2}\sqrt{10} \cos\left(\frac{\pi n}{2} - 1.893^c\right)$$

$$= \frac{1}{2} u[n] + \frac{3}{2} \sin\left(\frac{\pi n}{2}\right) - \frac{1}{2} \cos\left(\frac{\pi n}{2}\right)$$

Observe that $y_{ss}[n] = \frac{1}{2} u[n] = H(1)$

(116) By definition

$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n}$$

$$= h[1] z^{-1} + \sum_{n=2}^{\infty} [h[n-2] + h[n-1]] z^{-n}$$

$$= z^{-1} + \sum_{n=2}^{\infty} h[n-2] z^{-n} + \sum_{n=2}^{\infty} h[n-1] z^{-n}$$

$$= z^{-1} + z^{-2} \sum_{n=2}^{+\infty} h[n-2] z^{-(n-2)}$$
$$+ z^{-1} \sum_{n=2}^{\infty} h[n-1] z^{-(n-1)}$$

Let $m = n-2 \Rightarrow n=2 \Rightarrow m=0$
$p = n-1 \Rightarrow n=2 \Rightarrow p=1$.

Then

$$H(z) = z^{-1} + z^{-2} \sum_{m=0}^{\infty} h[m] z^{-m}$$
$$+ z^{-1} \sum_{p=1}^{+\infty} h[p] z^{-p}$$

$$= z^{-1} + z^{-2} \sum_{m=0}^{+\infty} h[m] z^{-m}$$
$$+ z^{-1} \sum_{p=0}^{\infty} h[p] z^{-p}$$

(as $h[0] = 0$.)

$$= z^{-1} + z^{-2} H(z) + z^{-1} H(z)$$

So

$$H(z) [1 - z^{-2} - z^{-1}] = z^{-1}$$

So $$H(z) = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$

(11b) So $H(z) = \dfrac{z}{z^2 - z - 1}$

(b) Since $Y(z) = H(z)\,U(z)$

we have
$$Y(z)\,(z^2 - z - 1) = z\,U(z)$$
$$\Longrightarrow\ y[n+2] - y[n+1] - y[n] = u[n+1].$$

for $n \geq -2$.

(117) Let $H(s) = \dfrac{1}{s^2 - 3s - 4} = \dfrac{Y(s)}{V(s)} = \dfrac{C(s)}{A(s)}$

(a) We have $C(s) = d s^2 + c_1 s + c_0 = 1$

$A(s) = s^2 + a_1 s + a_0 = s^2 - 3s - 4$.

Also $A(s) = (s-4)(s+1)$

with roots $p_1 = 4$, $p_2 = -1$

So the system is unstable as it has a pole in the right hand complex plane.

(b) We introduce variable w such that

$$\boxed{C(s) W(s) = Y(s)}$$

Hence $w(t) = y(t)$

Also introduce $x_1(t) = w(t)$

$x_2(t) = \dfrac{dw}{dt} = \dfrac{dx_1}{dt}$.

Then $y(t) = x_1(t)$

and

$$\boxed{V(s) = A(s) W(s)}$$

giving

$$v(t) = -4x_1 - 3x_2 + \frac{dx_2}{dt}$$

So we obtain state representation

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix}(t) = \begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Here $A = \begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = 0$.

(117)

(c) To obtain a diagonal state representation we need to diagonalize A

Simple calculations give eigenvalues/vectors of

$$\lambda_1 = -1, \quad v_1 = [-1, 1]^T$$
$$\lambda_2 = 4, \quad v_2 = [1, 4]^T$$

So we introduce

$$P = [v_1 \; v_2]^{-1} = \frac{1}{5}\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix}$$

and new state matrices

$$\bar{A} = PAP^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\bar{B} = PB = \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \end{bmatrix}$$

$$\bar{C} = CP^{-1} = [-1 \;\; 1]$$

$$\bar{D} = D = 0$$

Hence the diagonal representation is

$$\dot{\bar{x}}(t) = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix} \bar{x}(t) + \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \end{bmatrix} u(t)$$

$$y(t) = [-1 \;\; 1]\, \bar{x}(t) +$$

(d) We have

$$H(s) = \bar{C}(sI - \bar{A})\bar{B} + \bar{D}$$
$$= [-1 \;\; 1] \begin{bmatrix} s+1 & 0 \\ 0 & s-4 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \end{bmatrix}$$

$$= [-1 \;\; 1] \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s-4} \end{bmatrix} \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \end{bmatrix}$$

where $\cancel{\Delta = \det(sI - A) = (s+1)(s-4)}$

(d) So $H(s) = \frac{1}{5}\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s-4} \end{bmatrix}$

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$$= \frac{1}{5}\left[\frac{-1}{s+1} + \frac{1}{s-4} \right]$$

$$= \frac{1}{5}\left[\frac{-(s-4) + (s+1)}{(s+1)(s-4)} \right]$$

$$= \frac{1}{5}\left[\frac{5}{(s+1)(s-4)} \right]$$

$$= \frac{1}{(s+1)(s-4)}$$

$$= \frac{1}{s^2 - 3s - 4}$$

as expected.

(e) The stability of the system is given by the eigenvalues of A (or $\bar{A}$) and these are $\lambda_1 = -1$, $\lambda_2 = 4$. Hence one eigenvalue is in the Right hand plane and the system is unstable, as expected.

(11.9) $H(z) = \dfrac{z+2}{z^2+3z}$

(a) $H(z) = \dfrac{z+2}{z(z+3)} = \dfrac{C(z)}{A(z)} = \dfrac{Y(z)}{V(z)}$

So poles of the system are at

$p_1 = 0, \quad p_2 = -3$

As $|p_2| > 1$ it lies outside the unit disk and hence the system is unstable.

(b) Introduce variable w such that

$$\boxed{C(z)\,W(z) = Y(z)}$$

So $w[n+1] + 2w[n] = y[n]$

Then introduce variables

$$x_1[n] = w[n]$$
$$x_2[n] = w[n+1] = x_1[n+1]$$

So

Since $\boxed{V(z) = A(z)\,W(z)}$

$$\begin{aligned} v[n] &= w[n+2] + 3w[n+1] \\ &= x_2[n+1] + 3x_1[n+1] \\ &= x_2[n+1] + 3x_2[n] \end{aligned}$$

So we obtain

$$x_1[n+1] = x_2[n]$$
$$x_2[n+1] = -3x_2[n] + v[n]$$
$$y[n] = 2x_1[n] + x_2[n]$$

IN MATRIX FORM

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v[n]$$

$$y[n] = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix}$$