Week 3 – Data Link Layer Contd.

COMP90007 Internet Technologies

Error Bounds: Hamming distance

A code turns data of *n* bits into codewords of *n*+*k* bits Hamming distance is the minimum bit flips to turn one valid codeword into any other valid one:

- Example with 4 codewords of 10 bits (n=2, k=8):
 - **0000000000**,

Hamming distance is 5 here!

- **0000011111**.
- **1111100000**,
- 111111111111

Bounds for a code with a given distance D is then:

- □ D=2d+1 can correct d errors (e.g., 2 errors above)
- D=d+1 can detect d errors (e.g., 4 errors above)

Error Bounds

Q: Why can a code with distance 2d+1 can correct up to d errors only?

- Errors are corrected by <u>mapping a received invalid</u> <u>codeword to the nearest valid codeword</u>, i.e., the one that can be reached with the fewest bit flips
- If there are more than d bit flips, then the <u>received</u> <u>codeword may be closer to another valid codeword</u> than the codeword that was sent

Good Case: Sending 0000000000 with 2 flips might give 1100000000 which is closest to 0000000000, correcting the error

Bad Case: But with 3 flips 1110000000 might be received, which is closest to 1111100000, which is still an error

A More Advanced Method: Hamming Codes

- n=2^k-k-1 (n: number of data, k: check bits)
- Put <u>check bits in positions p that are power of 2</u>, starting with position 1

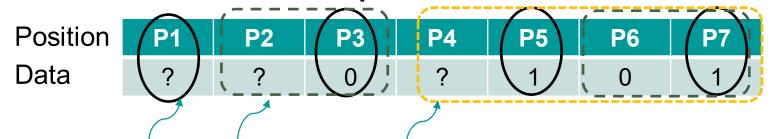
Hamming Codes Example

Example: Data: 0101 - > requires 3 check bits

$$4 = (2^3) - 3 - 1$$

Example Contd

■ Data: 0101 → requires 3 check bits



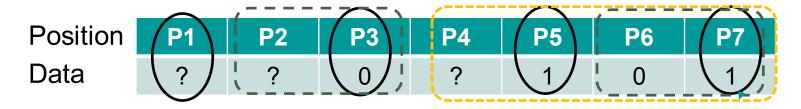
Put check bits in positions p that are power of 2, starting with position 1

Then: Calculate the **parity** for P1, P2, P4

P1 + P3 + P5 + P7 =
$$?+0+1+1=0$$
 (even OK)
P2 + P3 + P6 + P7 = $?+0+0+1=1$ (odd ADD)
P4 + P5 + P6 + P7 = $?+1+0+1=0$ (even OK)

7 is 111
so
appears
in all
formula

Example Contd



Calculating the parity bits for P1, P2, P4

Data sent: 0100101 error error

Example 1: At the receiver: 0100100

$$1 P2 + P3 + P6 + P7 = 1+0+0+0= 1 \times$$

$$\P4 + P5 + P6 + P7 = 0 + 1 + 0 + 0 = 1 \times$$

Error bit =
$$P1+P2+P4 = P7$$

Example 2: At the receiver: 0000101

$$P1 + P3 + P5 + P7 = 0 + 0 + 1 + 1 = 0$$

Error bit = P2

Example Contd

- The example we saw is in the category of error correction
- And the example design could correct only single bit errors