Lecture 8: Feature Selection and Analysis

COMP90049
Introduction to Machine Learning

Semester 2, 2021

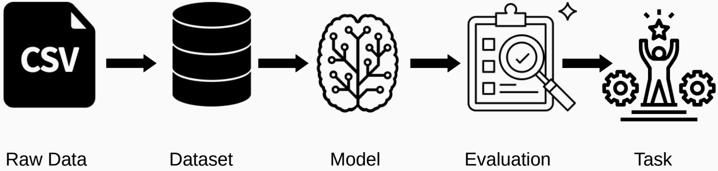
Lida Rashidi, CIS

Acknowledgement: Jeremy Nicholson, Tim Baldwin, Karin Verspoor & Lea Frermann

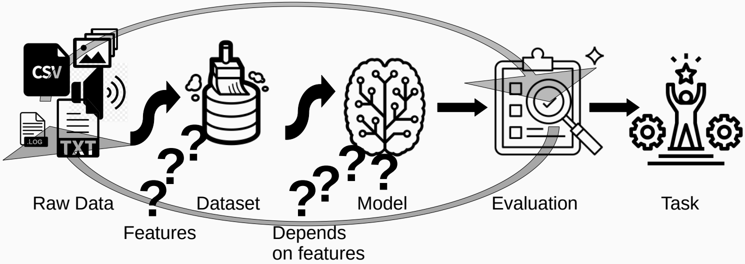
Features in Machine Learning

Machine Learning Workflow

The Dream



Reality



Data Preparation vs Feature Selection

GIGO: Garbage In, Garbage Out

Data Preparation and Cleansing (discussed before)

- Data Cleaning
- Data Aggregation
- Dealing with missing values
- Transformation (e.g., log transform)
- Binarization
- Binning
- Scaling or Normalization

Feature Selection (this lecture)

- Wrapper methods (aka recursive elimination)
- Filtering (aka univariate filtering)
- Glance into some other common approaches

Data Preparation vs Feature Selection

Our job as Machine Learning experts:

- Inspect / clean the data
- Choose a model suitable for classifying the data according to the attributes
- Choose attributes suitable for classifying the data according to the model

    - Inspection
    - Intuition

Data Preparation vs Feature Selection

Our job as Machine Learning experts:

- Inspect / clean the data
- Choose a model suitable for classifying the data according to the attributes
- Choose attributes suitable for classifying the data according to the model

  - ~~Inspection~~
  - ~~Intuition~~
  - Neither possible in practice

Feature Selection

What makes features good?

Lead to better models

- Better performance according to some evaluation metric

Side-goal 1

- Seeing important features can suggest other important features
- Tell us interesting things about the problem

Side-goal 2

- Fewer features $\rightarrow$ smaller models $\rightarrow$ faster answer
  - More accurate answer $>>$ faster answer

Iterative feature selection: Wrappers

Choosing a good feature set

"Wrapper" methods

- Choose subset of attributes that give best performance on the development data

- For example: for the Weather data set:

    - Train model on $\{\texttt{Outlook}\}$
    - Train model on $\{\texttt{Temperature}\}$
    - ...
    - Train model on $\{\texttt{Outlook, Temperature}\}$
    - ...
    - Train model on $\{\texttt{Outlook, Temperature, Humidity}\}$
    - ...
    - Train model on $\{\texttt{Outlook, Temperature, Humidity, Windy}\}$

Choosing a good feature set

“Wrapper” methods

- Choose subset of attributes that give best performance on the development data

- For example: for the Weather data set:

  - Evaluate model on $\{\texttt{Outlook}\}$
  - Evaluate model on $\{\texttt{Temperature}\}$
  - ...
  - Evaluate model on $\{\texttt{Outlook, Temperature}\}$
  - ...
  - Evaluate model on $\{\texttt{Outlook, Temperature, Humidity}\}$
  - ...
  - Evaluate model on $\{\texttt{Outlook, Temperature, Humidity, Windy}\}$

- Best performance on data set $\rightarrow$ best feature set

Choosing a good feature set

"Wrapper" methods

- Choose subset of attributes that give best performance on the development data
- Advantages:
  - Feature set with **optimal** performance on development data
- Disadvantages:
  - Takes a **long** time

Aside: how long does the full wrapper method take?

Assume we have a fast method (e.g. Naive Bayes) over a data set of non-trivial size ($\sim$10K instances):

- Assume: train–evaluate cycle takes 10 sec to complete

How many cycles? For m features:

- 2^m subsets $= \frac{2^m}{6}$ minutes
- $m = 10 \rightarrow 3$ hours
- $m = 60 \rightarrow$ heat death of universe

Only practical for very small data sets.

More practical wrapper methods: Greedy search

Greedy approach

- Train and evaluate model on each single attribute
- Choose best attribute
- Until convergence:
  - Train and evaluate model on best attribute(s), plus each remaining single attribute
  - Choose best attribute out of the remaining set
- Iterate until performance (e.g. accuracy) stops increasing

More practical wrapper methods: Greedy search

Greedy approach

- Bad news:
  - Takes $\frac{1}{2}m^2$ cycles, for m attributes
  - In theory, 386 attributes $\rightarrow$ days
- Good news:
  - In practice, converges much more quickly than this
- Bad news again:
  - Convergences to a sub-optimal (and often very bad) solution

More practical wrapper methods: Ablation

“Ablation” approach

- Start with all attributes
- Remove one attribute, train and evaluate model
- Until divergence:
    - From remaining attributes, remove each attribute, train and evaluate model
    - Remove attribute that causes least performance degradation
- Termination condition usually: performance (e.g. accuracy) starts to degrade by more than ϵ

More practical wrapper methods: Ablation

“Ablation” approach

for example:

- Start with all features
  - Train, evaluate model on {`Outlook`, `Temperature`, `Humidity`, `Windy`}
- Consider feature subsets of size 3:
  - Train, evaluate model on {`Outlook`, `Temperature`, `Humidity`}
  - Train, evaluate model on {`Outlook`, `Temperature`, `Windy`}
  - Train, evaluate model on {`Outlook`, `Humidity`, `Windy`}
  - Train, evaluate model on {`Temperature`, `Humidity`, `Windy`}
- Choose best of previous five (let’s say THW):
- Consider feature subsets of size 2:
  - Train, evaluate model on {`Temperature`, `Humidity`}
  - Train, evaluate model on {`Temperature`, `Windy`}
  - Train, evaluate model on {`Humidity`, `Windy`}
- etc...

More practical wrapper methods: Ablation

"Ablation" approach

- Good news:
  - Mostly removes irrelevant attributes (at the start)
- Bad news:
  - Assumes independence of attributes
    (Actually, both approaches do this)
  - Takes $O(m^2)$ time; cycles are slower with more attributes
  - Not feasible on non-trivial data sets.

Feature Filtering

Feature filtering

Intuition: Evaluate the "goodness" of each feature, separate from other features

- Consider each feature separately: linear time in number of attributes
- Possible (but difficult) to control for inter-dependence of features
- Typically most popular strategy

Feature “goodness”

What makes a ~~feature set~~ single feature good?

Toy example

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

Which of a_1, a_2 is good?

Toy example

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

Toy example

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

Pointwise Mutual Information

Discrepancy between the **observed joint probability** of two random variables A and C and the expected joint probability **if A and C were independent.**

Recall independence: $P(C|A) = P(C)$

Pointwise Mutual Information

Discrepancy between the **observed joint probability** of two random variables A and C and the expected joint probability **if A and C were independent.**

Recall independence: $P(C|A) = P(C)$

PMI is defined as

$$PMI(A, C) = \log_2 \frac{P(A, C)}{P(A)P(C)}$$

We want to find attributes that are **not** independent of the class.

- If PMI $>>$ 0, attribute and class occur together much more often than randomly.
- If RHS $\sim$ 0, attribute and class occur together as often as we would expect from random chance
- If RHS $<<$ 0, attribute and class are negatively correlated. (More on that later!)

Attributes with greatest PMI: best attributes

Toy example, revisited

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

Calculate PMI of a_1, a_2 with respect to c

Toy example, revisited

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

$$P(a_1) =$$

$$P(c) =$$

$$P(a_1, c) =$$

$$PMI(a_1, c) =$$

Toy example, revisited

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

$$P(a_1) =$$

$$P(c) =$$

$$P(a_1, c) =$$

$$PMI(a_1, c) =$$

Toy example, revisited

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

$$P(a_2) = \frac{2}{4}$$
$$P(c) = \frac{2}{4}$$
$$P(a_2, c) = \frac{1}{4}$$

Toy example, revisited

a_1	a_2	c
Y	Y	Y
Y	N	Y
N	Y	N
N	N	N

$$P(a_2) = \frac{2}{4}$$

$$P(c) = \frac{2}{4}$$

$$P(a_2, c) = \frac{1}{4}$$

$$PMI(a_2, c) = \log_2 \frac{\frac{1}{4}}{\frac{1}{2} \cdot \frac{1}{2}}$$

$$= \log_2(1) = 0$$

Feature “goodness”, revisited

What makes a single feature good?

- Well correlated with class
    - Knowing a lets us predict c with more confidence

- Reverse correlated with class
    - Knowing $\bar{a}$ lets us predict c with more confidence

- Well correlated (or reverse correlated) with not class
    - Knowing a lets us predict $\bar{c}$ with more confidence
    - Usually not quite as good, but still useful

Mutual Information

- Expected value of PMI over all possible events
- For our example: Combine PMI of all possible combinations: a, $\bar{a}$, c, $\bar{c}$

Aside: Contingency tables

Contingency tables: compact representation of these frequency counts

	a	$\bar{a}$	Total
c	$\sigma(a, c)$	$\sigma(\bar{a}, c)$	$\sigma(c)$
$\bar{c}$	$\sigma(a, \bar{c})$	$\sigma(\bar{a}, \bar{c})$	$\sigma(\bar{c})$
Total	$\sigma(a)$	$\sigma(\bar{a})$	N

$P(a, c) = \frac{\sigma(a,c)}{N}$, etc.

Aside: Contingency tables

Contingency tables for toy example:

a_1	$a =$Y	$a =$N	Total
$c =$Y	2	0	2
$c =$N	0	2	2
Total	2	2	4

a_2	$a =$Y	$a =$N	Total
$c =$Y	1	1	2
$c =$N	1	1	2
Total	2	2	4

Mutual Information

Combine PMI of all possible combinations: a, $\bar{a}$, c, $\bar{c}$

$$\begin{aligned} MI(A,C) = & P(a,c)PMI(a,c) + P(\bar{a},c)PMI(\bar{a},c) + \\ & P(a,\bar{c})PMI(a,\bar{c}) + P(\bar{a},\bar{c})PMI(\bar{a},\bar{c}) \end{aligned}$$

$$\begin{aligned} MI(A,C) = & P(a,c)\log_2 \frac{P(a,c)}{P(a)P(c)} + P(\bar{a},c)\log_2 \frac{P(\bar{a},c)}{P(\bar{a})P(c)} + \\ & P(a,\bar{c})\log_2 \frac{P(a,\bar{c})}{P(a)P(\bar{c})} + P(\bar{a},\bar{c})\log_2 \frac{P(\bar{a},\bar{c})}{P(\bar{a})P(\bar{c})} \end{aligned}$$

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Mutual Information

Combine PMI of all possible combinations: a, $\bar{a}$, c, $\bar{c}$

$$\begin{aligned} MI(A,C) = &P(a,c)PMI(a,c) + P(\bar{a},c)PMI(\bar{a},c) + \\ &P(a,\bar{c})PMI(a,\bar{c}) + P(\bar{a},\bar{c})PMI(\bar{a},\bar{c}) \end{aligned}$$

$$\begin{aligned} MI(A,C) = &P(a,c)\log_2 \frac{P(a,c)}{P(a)P(c)} + P(\bar{a},c)\log_2 \frac{P(\bar{a},c)}{P(\bar{a})P(c)} + \\ &P(a,\bar{c})\log_2 \frac{P(a,\bar{c})}{P(a)P(\bar{c})} + P(\bar{a},\bar{c})\log_2 \frac{P(\bar{a},\bar{c})}{P(\bar{a})P(\bar{c})} \end{aligned}$$

Often written more compactly as:

$$MI(A,C) = \sum_{i \in \{a,\bar{a}\}} \sum_{j \in \{c,\bar{c}\}} P(i,j) \log_2 \frac{P(i,j)}{P(i)P(j)}$$

We define that $0 \log 0 \equiv 0$.

Mutual Information Example

Contingency Table for attribute a_1

a_1	$a =$Y	$a =$N	Total
$c =$Y	2	0	2
$c =$N	0	2	2
Total	2	2	4

Mutual Information Example

Contingency Table for attribute a_1

a_1	$a =$Y	$a =$N	Total
$c =$Y	2	0	2
$c =$N	0	2	2
Total	2	2	4

$$P(a,c) = \tfrac{2}{4};\quad P(a) = \tfrac{2}{4};\quad P(c) = \tfrac{2}{4};\quad P(a,\bar{c}) = 0$$
$$P(\bar{a},\bar{c}) = \tfrac{2}{4};\quad P(\bar{a}) = \tfrac{2}{4};\quad P(\bar{c}) = \tfrac{2}{4};\quad P(\bar{a},c) = 0$$

Mutual Information Example

Contingency Table for attribute a_1

a_1	$a=$Y	$a=$N	Total
$c=$Y	2	0	2
$c=$N	0	2	2
Total	2	2	4

$P(a,c)=\frac{2}{4};\quad P(a)=\frac{2}{4};\quad P(c)=\frac{2}{4};\quad P(a,\bar{c})=0$
$P(\bar{a},\bar{c})=\frac{2}{4};\quad P(\bar{a})=\frac{2}{4};\quad P(\bar{c})=\frac{2}{4};\quad P(\bar{a},c)=0$

$$\begin{aligned} MI(A_1,C) &= P(a_1,c)\log_2\frac{P(a_1,c)}{P(a_1)P(c)} + P(\bar{a}_1,c)\log_2\frac{P(\bar{a}_1,c)}{P(\bar{a}_1)P(c)} + \\ & P(a_1,\bar{c})\log_2\frac{P(a_1,\bar{c})}{P(a_1)P(\bar{c})} + P(\bar{a}_1,\bar{c})\log_2\frac{P(\bar{a}_1,\bar{c})}{P(\bar{a}_1)P(\bar{c})} \end{aligned}$$

Mutual Information Example

Contingency Table for attribute a_1

a_1	$a=$Y	$a=$N	Total
$c=$Y	2	0	2
$c=$N	0	2	2
Total	2	2	4

$P(a,c)=\frac{2}{4}$; $P(a)=\frac{2}{4}$; $P(c)=\frac{2}{4}$; $P(a,\bar{c})=0$
$P(\bar{a},\bar{c})=\frac{2}{4}$; $P(\bar{a})=\frac{2}{4}$; $P(\bar{c})=\frac{2}{4}$; $P(\bar{a},c)=0$

$$\begin{aligned}
MI(A_1,C) &= P(a_1,c)\log_2\frac{P(a_1,c)}{P(a_1)P(c)} + P(\bar{a}_1,c)\log_2\frac{P(\bar{a}_1,c)}{P(\bar{a}_1)P(c)} + \\
&\quad P(a_1,\bar{c})\log_2\frac{P(a_1,\bar{c})}{P(a_1)P(\bar{c})} + P(\bar{a}_1,\bar{c})\log_2\frac{P(\bar{a}_1,\bar{c})}{P(\bar{a}_1)P(\bar{c})} \\
&= \frac{1}{2}\log_2\frac{\frac{1}{2}}{\frac{1}{2}\frac{1}{2}} + 0\log_2\frac{0}{\frac{1}{2}\frac{1}{2}} + 0\log_2\frac{0}{\frac{1}{2}\frac{1}{2}} + \frac{1}{2}\log_2\frac{\frac{1}{2}}{\frac{1}{2}\frac{1}{2}} \\
&= \frac{1}{2}(1)+0+0+\frac{1}{2}(1)=1
\end{aligned}$$

Mutual Information Example continued

Contingency Table for attribute a_2

a_2	a =Y	a =N	Total
c =Y	1	1	2
c =N	1	1	2
Total	2	2	4

Mutual Information Example continued

Contingency Table for attribute a_2

a_2	$a =$Y	$a =$N	Total
$c =$Y	1	1	2
$c =$N	1	1	2
Total	2	2	4

$$P(a,c) = \tfrac{1}{4}; \quad P(a) = \tfrac{2}{4}; \quad P(c) = \tfrac{2}{4}; \quad P(\bar{a},c) = \tfrac{1}{4}$$
$$P(\bar{a},\bar{c}) = \tfrac{1}{4}; \quad P(\bar{a}) = \tfrac{2}{4}; \quad P(\bar{c}) = \tfrac{2}{4}; \quad P(a,\bar{c}) = \tfrac{1}{4}$$

Mutual Information Example continued

Contingency Table for attribute a_2

a_2	$a =$Y	$a =$N	Total
$c =$Y	1	1	2
$c =$N	1	1	2
Total	2	2	4

$$P(a,c) = \tfrac{1}{4}; \quad P(a) = \tfrac{2}{4}; \quad P(c) = \tfrac{2}{4}; \quad P(\bar{a},c) = \tfrac{1}{4}$$
$$P(\bar{a},\bar{c}) = \tfrac{1}{4}; \quad P(\bar{a}) = \tfrac{2}{4}; \quad P(\bar{c}) = \tfrac{2}{4}; \quad P(a,\bar{c}) = \tfrac{1}{4}$$

$$\begin{aligned} MI(A_2, C) &= P(a_2, c)\log_2 \frac{P(a_2, c)}{P(a_2)P(c)} + P(\bar{a}_2, c)\log_2 \frac{P(\bar{a}_2, c)}{P(\bar{a}_2)P(c)} + \\ & P(a_2, \bar{c})\log_2 \frac{P(a_2, \bar{c})}{P(a_2)P(\bar{c})} + P(\bar{a}_2, \bar{c})\log_2 \frac{P(\bar{a}_2, \bar{c})}{P(\bar{a}_2)P(\bar{c})} \end{aligned}$$

Mutual Information Example continued

Contingency Table for attribute a_2

a_2	a =Y	a =N	Total
c =Y	1	1	2
c =N	1	1	2
Total	2	2	4

$P(a, c) = \frac{1}{4}$; $P(a) = \frac{2}{4}$; $P(c) = \frac{2}{4}$; $P(\bar{a}, c) = \frac{1}{4}$
$P(\bar{a}, \bar{c}) = \frac{1}{4}$; $P(\bar{a}) = \frac{2}{4}$; $P(\bar{c}) = \frac{2}{4}$; $P(a, \bar{c}) = \frac{1}{4}$

$$\begin{aligned} MI(A_2, C) &= P(a_2, c) \log_2 \frac{P(a_2, c)}{P(a_2)P(c)} + P(\bar{a}_2, c) \log_2 \frac{P(\bar{a}_2, c)}{P(\bar{a}_2)P(c)} + \\ & \quad P(a_2, \bar{c}) \log_2 \frac{P(a_2, \bar{c})}{P(a_2)P(\bar{c})} + P(\bar{a}_2, \bar{c}) \log_2 \frac{P(\bar{a}_2, \bar{c})}{P(\bar{a}_2)P(\bar{c})} \\ &= \frac{1}{4} \log_2 \frac{\frac{1}{4}}{\frac{1}{2}\frac{1}{2}} + \frac{1}{4} \log_2 \frac{\frac{1}{4}}{\frac{1}{2}\frac{1}{2}} + \frac{1}{4} \log_2 \frac{\frac{1}{4}}{\frac{1}{2}\frac{1}{2}} + \frac{1}{4} \log_2 \frac{\frac{1}{4}}{\frac{1}{2}\frac{1}{2}} \\ &= \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(0) = 0 \end{aligned}$$

Chi-square

Similar idea, different solution:

	a	$\bar{a}$	Total
c	$\sigma(a, c)$	$\sigma(\bar{a}, c)$	$\sigma(c)$
$\bar{c}$	$\sigma(a, \bar{c})$	$\sigma(\bar{a}, \bar{c})$	$\sigma(\bar{c})$
Total	$\sigma(a)$	$\sigma(\bar{a})$	N

Contingency table (shorthand):

	a	$\bar{a}$	Total
c	W	X	$W + X$
$\bar{c}$	Y	Z	$Y + Z$
Total	$W + Y$	$X + Z$	$N = W + X + Y + Z$

If a, c were independent (uncorrelated), what value would you expect in W?

Denote the expected value as E(W).

Chi-square

If a, c were independent, then $P(a, c) = P(a)P(c)$

$$
\begin{aligned}
P(a, c) &= P(a)P(c) \\
\frac{\sigma(a, c)}{N} &= \frac{\sigma(a)}{N}\frac{\sigma(c)}{N} \\
\sigma(a, c) &= \frac{\sigma(a)\sigma(c)}{N} \\
E(W) &= \frac{(W+Y)(W+X)}{W+X+Y+Z}
\end{aligned}
$$

Chi-square

Compare the value we actually observed $O(W)$ **with the expected value** $E(W)$:

- If the **observed value is much greater than the expected value**, a occurs more often with c than we would expect at random — **predictive**
- If the observed value is **much smaller than the expected value**, a occurs less often with c than we would expect at random — **predictive**
- If the **observed value is close to the expected value**, a occurs as often with c as we would expect randomly — **not predictive**

Similarly with X, Y, Z

Chi-square

Actual calculation (to fit to a chi-square distribution)

$$\begin{aligned}\chi^2 &= \frac{(O(W)-E(W))^2}{E(W)} + \frac{(O(X)-E(X))^2}{E(X)} + \\ &\quad \frac{(O(Y)-E(Y))^2}{E(Y)} + \frac{(O(Z)-E(Z))^2}{E(Z)} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{i,j}-E_{i,j})^2}{E_{i,j}}\end{aligned}$$

- i sums over rows and j sums over columns.
- Because the values are squared, χ^2 becomes much greater when $\mid O - E \mid$ is large, even if E is also large.

Chi-square Example

Contingency table for toy example (observed values):

a_1	$a =$Y	$a =$N	Total
$c =$Y	2	0	2
$c =$N	0	2	2
Total	2	2	4

Contingency table for toy example (expected values):

a_1	$a =$Y	$a =$N	Total
$c =$Y	1	1	2
$c =$N	1	1	2
Total	2	2	4

Chi-square Example

$$
\begin{aligned}
\chi^2(A_1, C) &= \frac{(O_{a,c} - E_{a,c})^2}{E_{a,c}} + \frac{(O_{\bar{a},c} - E_{\bar{a},c})^2}{E_{\bar{a},c}} + \\
&\quad \frac{(O_{a,\bar{c}} - E_{a,\bar{c}})^2}{E_{a,\bar{c}}} + \frac{(O_{\bar{a},\bar{c}} - E_{\bar{a},\bar{c}})^2}{E_{\bar{a},\bar{c}}} \\
&= \frac{(2-1)^2}{1} + \frac{(0-1)^2}{1} + \frac{(0-1)^2}{1} + \frac{(2-1)^2}{1} \\
&= 1 + 1 + 1 + 1 = 4
\end{aligned}
$$

$\chi^2(A_2, C)$ is obviously 0, because all observed values are equal to expected values.

Common Issues

Types of Attribute

So far, we've only looked at binary (Y/N) attributes:

- Nominal attributes
- Continuous attributes
- Ordinal attributes

Types of Attributes: Nominal

Two common strategies

1. Treat as multiple binary attributes:
   - e.g. `sunny`=Y, `overcast`=N, `rainy`=N, etc.
   - Can just use the formulae as given
   - Results sometimes difficult to interpret
     - For example, `Outlook`=sunny is useful, but `Outlook`=overcast and `Outlook`=rainy are not useful... Should we use `Outlook`?

2. Modify contingency tables (and formulae)

O	s	o	r
$c=$Y	U	V	W
$c=$N	X	Y	Z

Types of Attributes: Nominal

Modified MI:

$$\begin{aligned}
MI(O, C) &= \sum_{i \in \{s,o,r\}} \sum_{j \in \{c,\bar{c}\}} P(i,j) \log_2 \frac{P(i,j)}{P(i)P(j)} \\
&= P(s,c) \log_2 \frac{P(s,c)}{P(s)P(c)} + P(s,\bar{c}) \log_2 \frac{P(s,\bar{c})}{P(s)P(\bar{c})} + \\
&\quad P(o,c) \log_2 \frac{P(o,c)}{P(o)P(c)} + P(o,\bar{c}) \log_2 \frac{P(o,\bar{c})}{P(o)P(\bar{c})} + \\
&\quad P(r,c) \log_2 \frac{P(r,c)}{P(r)P(c)} + P(r,\bar{c}) \log_2 \frac{P(r,\bar{c})}{P(r)P(\bar{c})}
\end{aligned}$$

- Biased towards attributes with many values.

Chi-square can be used as normal, with 6 observed/expected values.

- To control for score inflation, we need to consider "number of degrees of freedom", and then use the significance test explicitly (beyond the scope of this subject)

Types of Attributes: Continuous

Continuous attributes

- Usually dealt with by estimating probability based on a Gaussian (normal) distribution
- With a large number of values, most random variables are normally distributed due to the **Central Limit Theorem**
- For small data sets or pathological features, we may need to use binomial/multinomial distributions

All of this is beyond the scope of this subject

Types of Attributes: Ordinal

Three possibilities, roughly in order of popularity:

1. Treat as binary
   - Particularly appropriate for frequency counts where events are low-frequency (e.g. words in tweets)
2. Treat as continuous
   - The fact that we haven't *seen* any intermediate values is usually not important
   - Does have all of the technical downsides of continuous attributes, however
3. Treat as nominal (i.e. throw away ordering)

Multi-class problems

So far, we've only looked at binary (Y/N) classification tasks.

Multiclass (e.g. LA, NY, C, At, SF) classification tasks are usually much more difficult.

Multi-class problems

What makes a single feature good?

- Highly correlated with class
- Highly reverse correlated with class
- Highly correlated (or reverse correlated) with not class

... What if there are many classes?

Multi-class problems

What makes a single feature good?

- Highly correlated with class
- Highly reverse correlated with class
- Highly correlated (or reverse correlated) with not class

... What if there are many classes?

What makes a feature bad?

- Irrelevant
- Correlated with other features
- Good at only predicting one class (but is this truly bad?)

Multi-class problems

Consider multi-class problem over LA, NY, C, At, SF:

- PMI, MI, χ^2 are all calculated *per-class*
- (Some other feature selection metrics, e.g. Information Gain, work for all classes at once)
- Need to make a point of selecting (hopefully uncorrelated) features for *each* class to give our classifier the best chance of predicting everything correctly.

Actual example (MI):

LA	NY	C	At	SF
la	nyc	chicago	atlanta	sf
angeles	york	bears	atl	httpdealnaycom
los	ny	il	ga	francisco
chicago	chicago	httpbitlyczmk	lol	san
hollywood	atlanta	cubs	u	u
atlanta	yankees	la	georgia	lol
lakers	sf	chi	chicago	save

Multi-class problems

Intuitive features:

LA	NY	C	At	SF
la	**nyc**	**chicago**	**atlanta**	**sf**
angeles	**york**	**bears**	**atl**	`httpdealnaycom`
los	**ny**	**il**	**ga**	**francisco**
`chicago`	`chicago`	`httpbitlyczmk`	`lol`	**san**
hollywood	`atlanta`	**cubs**	`u`	`u`
`atlanta`	**yankees**	`la`	**georgia**	`lol`
lakers	`sf`	**chi**	`chicago`	`save`

Multi-class problems

Features for predicting not class (MI):

LA	NY	C	At	SF
la	nyc	chicago	atlanta	sf
angeles	york	bears	atl	httpdealnaycom
los	ny	il	ga	francisco
chicago	**chicago**	httpbitlyczmk	lol	san
hollywood	**atlanta**	cubs	u	u
atlanta	yankees	**la**	georgia	lol
lakers	**sf**	chi	**chicago**	save

Multi-class problems

Unintuitive features:

LA	NY	C	At	SF
la	nyc	chicago	atlanta	sf
angeles	york	bears	atl	**httpdealnaycom**
los	ny	il	ga	francisco
chicago	chicago	**httpbitlyczmk**	**lol**	san
hollywood	atlanta	cubs	**u**	**u**
atlanta	yankees	la	georgia	**lol**
lakers	sf	chi	chicago	**save**

What's going on with MI?

Mutual Information is biased toward rare, uninformative features

- All probabilities: no notion of the raw frequency of events
- If a feature is seen rarely, but always with a given class, it will be seen as "good"
- Best features in the Twitter dataset only had MI of about 0.01 bits; 100th best for a given class had MI of about 0.002 bits

Glance into a few other common approaches to feature selection

A common (unsupervised) alternative

Term Frequency Inverse Document Frequency (TFIDF)

- Detect important words / Natural Language Processing
- Find words that are relevant to a document in a given document collection
- To be relevant, a word should be
  - Frequent enough in the corpus (TF). A word that occurs only 5 times in a corpus of 5,000,000 words is probably not too interesting
  - Special enough (IDF). A word that is very general and occurs in (almost) every document is probably not too interesting

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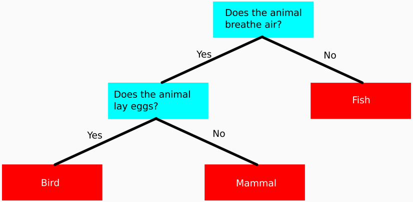
$$tfidf(d, t, D) = tf + idf$$
$$tf = log(1 + freq(t, d))$$
$$idf = log\left(\frac{|D|}{count(d \in D : t \in d)}\right)$$

d=document, t=term, D=document collection;
$|D|$=number of documents in D

Embedded Methods:

Some ML models include feature selection inherently

1. **Decision trees**: Generalization of 1-R



2. **Regression models with regularization**

$$\text{house\_price} = \beta_0 + \beta_1 \times \text{size} + \beta_2 \times \text{location} + \beta_3 \times \text{age}$$

Regularization (or 'penalty') nudges the weight β of unimportant features towards zero

Image:

https://towardsdatascience.com/a-beginners-guide-to-decision-tree-classification-6d3209353ea?gi=e0ee0b2b622e

And there are many more strategies

https://scikit-learn.org/stable/modules/classes.html#module-sklearn.feature_selection

sklearn.feature_selection: Feature Selection

The sklearn.feature_selection module implements feature selection algorithms. It currently includes univariate filter selection methods and the recursive feature elimination algorithm.

User guide: See the Feature selection section for further details.

feature_selection.GenericUnivariateSelect([...])	Univariate feature selector with configurable strategy.
feature_selection.SelectPercentile([...])	Select features according to a percentile of the highest scores.
feature_selection.SelectKBest([score_func, k])	Select features according to the k highest scores.
feature_selection.SelectFpr([score_func, alpha])	Filter: Select the pvalues below alpha based on a FPR test.
feature_selection.SelectFdr([score_func, alpha])	Filter: Select the p-values for an estimated false discovery rate
feature_selection.SelectFromModel(estimator, *)	Meta-transformer for selecting features based on importance weights.
feature_selection.SelectFwe([score_func, alpha])	Filter: Select the p-values corresponding to Family-wise error rate
feature_selection.SequentialFeatureSelector(...)	Transformer that performs Sequential Feature Selection.
feature_selection.RFE(estimator, *[, ...])	Feature ranking with recursive feature elimination.
feature_selection.RFECV(estimator, *[, ...])	Feature ranking with recursive feature elimination and cross-validated selection of the best number of features.
feature_selection.VarianceThreshold([threshold])	Feature selector that removes all low-variance features.

feature_selection.chi2(X, y)	Compute chi-squared stats between each non-negative feature and class.
feature_selection.f_classif(X, y)	Compute the ANOVA F-value for the provided sample.
feature_selection.f_regression(X, y, *[, center])	Univariate linear regression tests.
feature_selection.mutual_info_classif(X, y, *)	Estimate mutual information for a discrete target variable.
feature_selection.mutual_info_regression(X, y, *)	Estimate mutual information for a continuous target variable.

So ... is feature selection worth it?

Absolutely!

- Even marginally relevant features usually a vast improvement on an unfiltered data set
- Some models **need** feature selection
  - k-Nearest Neighbors, hugely
  - Naive Bayes, to a lesser extent
- Machine learning experts (us!) need to think about the data!

Summary

Today

- Wrappers vs. Filters
- Popular filters: PMI, MI, χ^2, how should we use them and what are the results going to look like
- Importance of feature selection for different methods (even though it sometimes isn't the solution we were hoping for)

Next week(s):

- Logistic regression
- Perceptron and neural networks
- ...and their respective learning algorithms (iterative optimization)

The lectures will be pre-recorded, as sequences of shorter videos. We will have only one Q&A session on Thursdays.

References

Guyon, Isabelle, and Andre Elisseeff. 2003. An introduction to variable and feature selection. *The Journal of Machine Learning Research*. Vol 3, 1157–1182.

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Yang, Yiming and Jan Pedersen (1997). A Comparative Study on Feature Selection in Text Categorization.

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