ELEN 3000 MID-TEST 2018

1 $n=-2$: $h[0] + 1.5\,h[-1] + 0.5\,h[-2] = \delta[-2]$

$\Rightarrow h[0] = 0$ (1)

(a) $n=-1$: $h[1] + 1.5\,h[0] + 0.5\,h[-1] = \delta[-1]$

$\Rightarrow h[1] = 0$ (1)

$n=0$: $h[2] + 1.5\,h[1] + 0.5\,h[0] = \delta[0]$

$\Rightarrow h[2] = 1$ (1)

$n=1$: $h[3] + 1.5\,h[2] + 0.5\,h[0] = \delta[1]$

$\Rightarrow h[3] = -1.5\,h[2]$

$= -1.5$ (1)

$\boxed{4}$

(b) $\cos(\pi t)$ has $w_1 = \pi \Rightarrow T_1 = \frac{2\pi}{w} = 2$ $(\frac{1}{2})$

$\cos(\frac{5t}{2})$ has $w_2 = \frac{5}{2} \Rightarrow T_2 = \frac{2\pi}{5/2} = \frac{4\pi}{5}$ $(\frac{1}{2})$

So $\frac{T_1}{T_2} = \frac{2}{4\pi/5}$

$= \frac{5}{2\pi}$ (1)

This is not a rational number, (1) hence x is not periodic. (1) $\boxed{4}$

(c) $y[n] = v^5[n+1]$

is not linear

Let $v[n] = \delta[n]$, $y[n] = \delta^5[n+1]$

and let $\alpha = 2$. Then let

$v_1[n] = \alpha\, v[n] = 2\,\delta[n]$

The output $y_1[n] = v_1^5[n+1]$

$= (2\,\delta[n+1])^5$

$= 2^5\,\delta^5[n+1]$

$\neq 2\,y[n]$

Hence the homogeneity property is not satisfied, and the system is nonlinear. $\boxed{5}$

1 (d) Yes the system is time-invariant
Let (v, y) be any trajectory of the system, then

$$y(t) = v(t-1)$$

Let $T \in \mathbb{R}$ and let

$$v_1(t) = v(t-T)$$

Let y_1 be the output from v_1,
so

$$\begin{aligned} y_1(t) &= v_1(t-1) \\ &= v(t-T-1) \\ &= y(t-T) \end{aligned}$$

So $(v(t-T), y(t-T))$ is also a trajectory and the system is time-invariant. $\boxed{5}$

(e) Yes the system has memory because the output at time t depends on the input at earlier time $t-1$. $\boxed{2}$

(f) Yes the system is causal because the output at time t does not depend on the input at future time $t_0 > t$. $\boxed{2}$

2 Let $x_1(t) = x(t)\cos(w_0 t)$.
Then by Euler's Identities

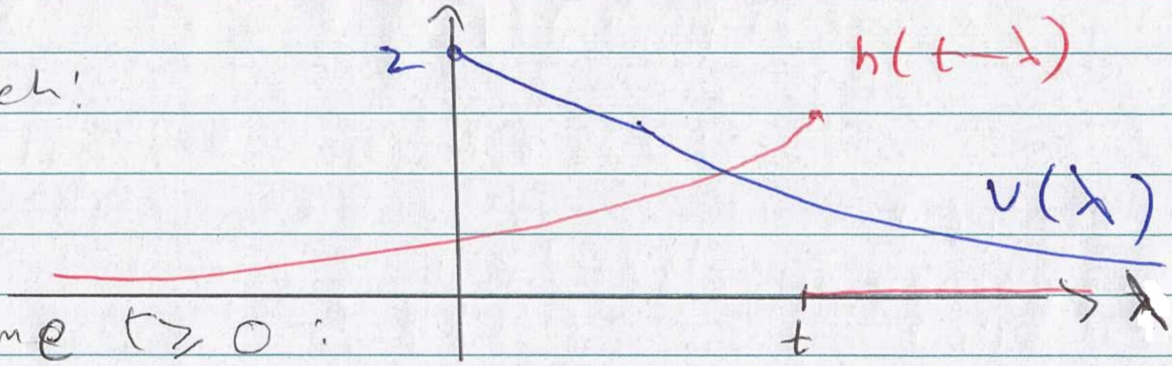
$$\begin{aligned} X_1(w) &= \int_{-\infty}^{\infty} x_1(t) e^{-jwt}\, dt \\ &= \int_{-\infty}^{\infty} x(t) \left(\tfrac{1}{2}\right)\left[e^{jw_0 t} + e^{-jw_0 t}\right] e^{-jwt}\, dt \\ &= \frac{1}{2}\int_{-\infty}^{\infty} x(t) e^{-j(w-w_0)t} + x(t) e^{-j(w+w_0)t}\, dt \\ &= \frac{1}{2}\left[X(w_0 - w_0) + X(w+w_0)\right] \end{aligned}$$

as required. $\boxed{6}$

(3) The output is given by

$$y(t) = (v * h)(t)$$
$$= \int_{-\infty}^{\infty} v(\lambda)\, h(t-\lambda)\, d\lambda \quad (2)$$

Sketch:



Assume $t \geq 0$:

$$y(t) = \int_0^t 2e^{-2\lambda} e^{-(t-\lambda)}\, d\lambda \quad (2)$$

$$= 2\int_0^t e^{-(\lambda+t)}\, d\lambda \quad (2)$$

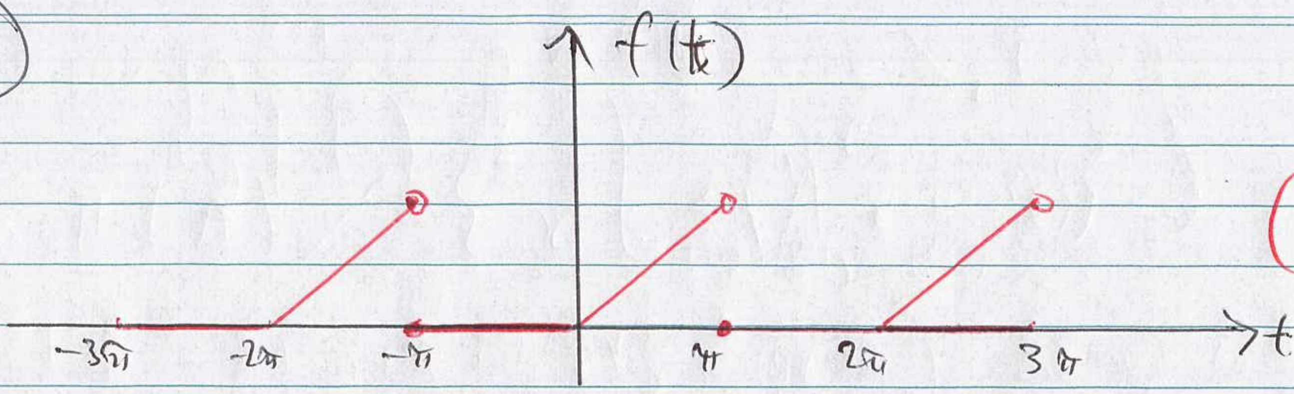
$$= -2\left[e^{-(\lambda+t)}\right]_0^t \quad (2)$$

$$= -2\left(e^{-2t} - e^{-t}\right) \quad (2)$$

$$= 2\left(e^{-t} - e^{-2t}\right) \qquad \boxed{10}$$

$$= \begin{cases} 2e^{-t}\left(1 - e^{-t}\right), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

(4)



(3)

(1) Dirichlet conditions:
As f is bounded on the interval [-π, π] hence it is absolutely integrable. (1)
(2) f has a local maximum at π, and one minima at [-π, 0], so it has finitely many maxima and minima on [-π, π] (1)
(3) f is discontinuous at t = π, hence it has finitely many discontinuities on [-π, π] (1)

$\boxed{6}$

(b) $T = 2\pi \Rightarrow w_0 = 1$

So $c_1 = \frac{1}{T}\int_0^T x(t)e^{-jt}\,dt$

$= \frac{1}{2\pi}\int_0^{\pi} t e^{-jt}\,dt$ (1)

Let $u = t$, $u' = 1$, $v' = e^{-jt}$, $v = je^{-jt}$

So $c_1 = \frac{1}{2\pi}\left[tje^{-jt}\right]_0^{\pi} - \int_0^{\pi} je^{-jt}\,dt$ (1)

$= \frac{1}{2\pi}\left[tje^{-jt} + e^{-jt}\right]_0^{\pi}$ (1)

$= \frac{1}{2\pi}\left[\left(j\pi e^{-j\pi} + e^{-j\pi}\right) - \left(0 + e^{0}\right)\right]$ (1)

$= \frac{1}{2\pi}\left[-\pi j - 2\right]$

$= \frac{1}{2\pi}\left[-2 - j\pi\right]$ (1)

and

$c_{-1} = \overline{c_1} = \frac{1}{2\pi}\left[-2 + j\pi\right]$ (1)

$\boxed{6}$