

THE UNIVERSITY OF MELBOURNE

Semester 2 Assessment

August 2018

Department of Electrical and Electronic Engineering
ELEN30012 SIGNALS AND SYSTEMS

Time allowed: 50 minutes

This paper has 3 pages

Authorised materials:

Only the following calculators may be used:

- Casio FX82 (any suffix)
- Casio FX100 (any suffix)

Students may bring **ONE** sheet of A4 paper containing **their own** notes into the examination room.

Instructions to invigilators:

Script books are to be collected at the end of the Test.

Instruction to students:

Attempt **ALL** questions.

The questions carry weight in proportion to the marks in brackets after the question numbers. These marks total 50 marks. You must show your work in order to receive credit!

Answer all questions and show all working in the script book provided.

Question 1 (22 marks)

- (a) Compute the unit-pulse response $h[n]$ for $n = 0, 1, 2, 3$ for the following discrete-time system with input-output trajectories (v, y) . Assume the system has zero initial conditions, i.e. $y[n] = 0$ for $n < 0$.

$$y[n+2] + 1.5y[n+1] + 0.5y[n] = v[n]$$

- (b) Is following continuous-time signal periodic? If it is, what is its fundamental period?

$$x(t) = \cos(\pi t) + \cos\left(\frac{5t}{2}\right)$$

- (c) Is the following discrete-time system linear in the input-output trajectories (v, y) ? Justify your answer with either a proof or a counterexample.

$$y[n] = v^5[n+1]$$

- (d) Is the following continuous-time system time-invariant in the input-output trajectories (v, y) ? Justify your answer with either a proof or a counterexample.

$$y(t) = v(t-1)$$

- (e) Does the system in part (d) have memory? Give a reason for your answer.

- (f) Is the system in part (d) causal? Give a reason for your answer.

Question 2 (6 marks)

Prove the Modulation Theorem for Fourier transforms: If

$$x(t) \longleftrightarrow X(\omega)$$

is a Fourier Transform pair, then for any $\omega_0 \in \mathbf{R}$,

$$x(t) \cos(\omega_0 t) \longleftrightarrow \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$$

is a Fourier Transform pair.

Question 3 (10 marks)

The continuous-time system with input-output trajectories (v, y) defined by the first-order differential equation

$$\frac{dy}{dt} + y(t) = v(t)$$

has impulse response h given by

$$h(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

Assuming zero initial conditions, use h to find the output y when the input is

$$v(t) = \begin{cases} 2e^{-2t}, & t \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

Question 4 (12 marks)

Consider the periodic continuous-time signal

$$f(t) = \begin{cases} 0, & -\pi \leq t < 0 \\ t, & 0 \leq t < \pi \end{cases} \quad \text{and } f(t) = f(t + 2\pi)$$

- .
- (a) Sketch f on the interval $[-3\pi, 3\pi]$ and explain briefly why it satisfies the Dirichlet conditions.
 - (b) Compute c_1 and c_{-1} , the first pair of coefficients in the complex Fourier series for f .