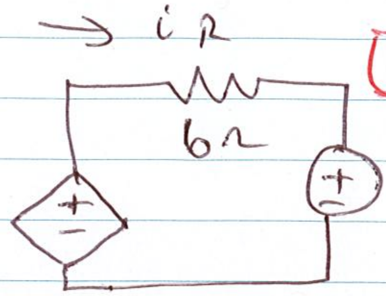
ELEN 20005 MID-TEST 2018 SOLUTIONS

(1) (a)



$(2\Omega)i_R$

(3)

KVL (clockwise)

$0 = 6i_R + 10 - 2i_R$

$= 4i_R + 10V$

$\Rightarrow i_R = -2.5\,A.$

(b)

Power (resistor) $= \cancel{(-2.5)(-2.5)}\ i_R^2 R$

$= (-2.5)^2(6)$

$= 37.5\ W.$ Sink (1)

Power (10 V) $= vi$ (PSL applied)

$= (-2.5A)(10V)$

$= -25\ W.$ Source (1)

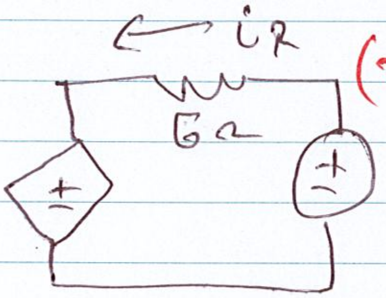
Power (CCVS) $= -vi$ (ASL applied)

$= -2i_R^2$

$= -12.5\ W$ Source (1)

$\boxed{3}$

(c)



$2\Omega i_R$

(2)

KVL (clockwise)

$0 = -6i_R + 10V - 2i_R$

$= -8i_R + 10V$

$\Rightarrow i_R = 1.25\ A.$

Power (resistor) $= i_R^2 R$

$= 9.375\ W.$ Sink (1)

Power (10 V) $= -vi$ (ASL applies)

$= -(10v)(1.25A)$

$= -12.5\ W$ Source (1)

Power (CCVS) $= vi$ (PSL applies)

$= 2i_R^2$

$= 3.125\ W$ Sink (1)

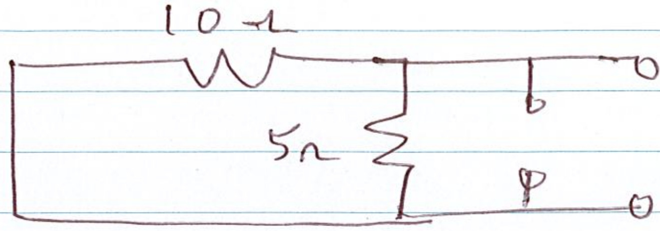
$\boxed{5}$

(d) $R = 560 \times 10^1 \pm 5\%$

$= 5600\ \Omega \pm 5\%.$ $\boxed{1}$

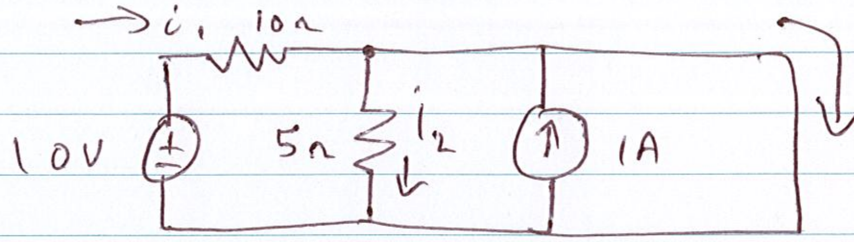
TOTAL $\boxed{12}$

(2) Circuit has no dependent sources so we can zero the independent sources to find R_N:



$R_N = 10 \| 5\,\Omega$
$= 3\frac{1}{3}\ \Omega$. (2)

To find i_{sc} we add a short circuit



(1)

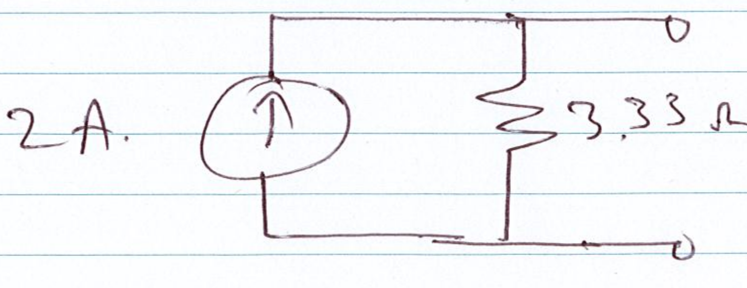
We see that $i_2 = 0$ due to the short circuit.

$i_1 = \frac{10V}{10\Omega} = 1\ A.$ (1)

So by KCL,

$i_{sc} = i_1 + 1A$
$= 2\ A.$ (1)

The Norton circuit is



(2)

TOTAL : $\boxed{7}$

3 (a) Before $t=0$, the capacitor acts as an open circuit, and all current goes through the $4k\Omega$ resistor. (1)

By voltage division

$$v_c(0^-) = \left(\frac{4k\Omega}{4k\Omega + 8k\Omega}\right)(30V) = 10V \quad (1)$$

$\boxed{2}$

(b) The capacitor resists instantaneous changes in voltage, so (1)

$$V_c(0^+) = v_c(0^-) = 10\ V. \quad (1)$$

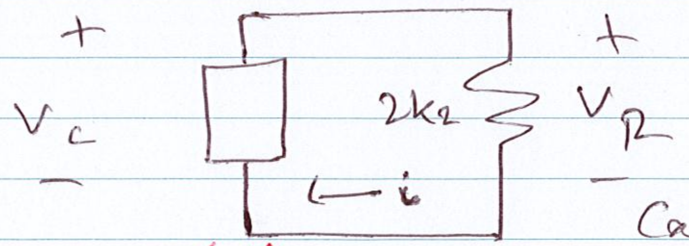
$\boxed{2}$

(c) At $t=\infty$, the capacitor will have discharged completely through the (1) $2k\Omega$ resistor, so

$$V_c(\infty) = 0\ V. \quad (1)$$

$\boxed{2}$

(d) After $t \geq 0$, the circuit is



KVL loop:

$$0 = V_R - V_c. \quad (1)$$

Capacitor equation $(\tfrac{1}{2})$

$$i = -C\frac{dv_c}{dt} \quad \text{(ASC applies)} \ (\tfrac{1}{2})$$

Resistor equation: $V_R = iR$ (PSC applies) $(\tfrac{1}{2})$

So we obtain the o.d.e.

$$\begin{aligned} 0 &= iR - V_c \\ &= -RC\frac{dV_c}{dt} - V_c \\ &= V_c + RC\frac{dV_c}{dt}, \quad V_c(0) = 10V. \end{aligned}$$

$(\tfrac{1}{2})$

(3) (d) Solution has the form

$$v_C(t) = K_1 e^{st} + K_2.$$

$$\frac{dv_C}{dt} = sK_1 e^{st}$$

Substitute these into the o.d.e.

$$0 = K_1 e^{st} + K_2 + RCsK_1 e^{st}$$

$$= K_1(1 + RCs)e^{st} + K_2.$$

Equating coefficients:

$$0 = 1 + RCs \Rightarrow s = -1/RC \quad (\tfrac{1}{2})$$

$$0 = K_2 \quad (\tfrac{1}{2})$$

So $v_C(t) = K_1 e^{-t/RC}$

Also

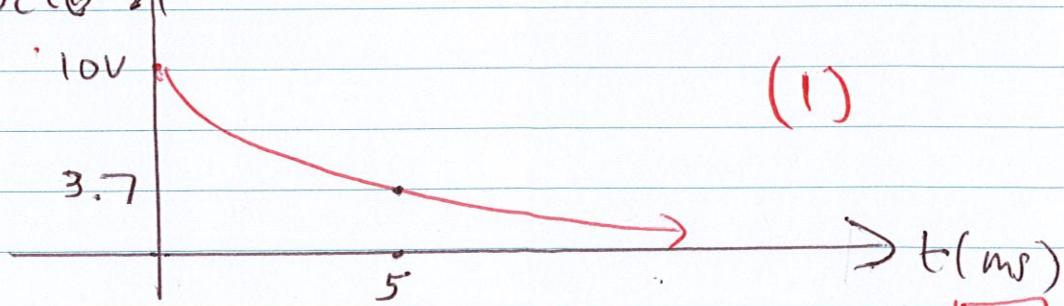
$$v_C(0) = K_1 = 10\text{ V}. \quad (\tfrac{1}{2})$$

So finally $v_C(t) = 10 e^{-t/RC}$

$$= 10 e^{-200t}\text{ V}. \quad (\tfrac{1}{2})$$

as $RC = 0.005$.

Sketch $v_C(t)$



$\boxed{6}$

(e) We want $v_R(t_0) = v_C(t_0) = 2\text{V}$ (1)

t_0 such that $\Rightarrow e^{-200t_0} = 0.2$ (1)

$$\Rightarrow -200t_0 = \ln(0.2)$$

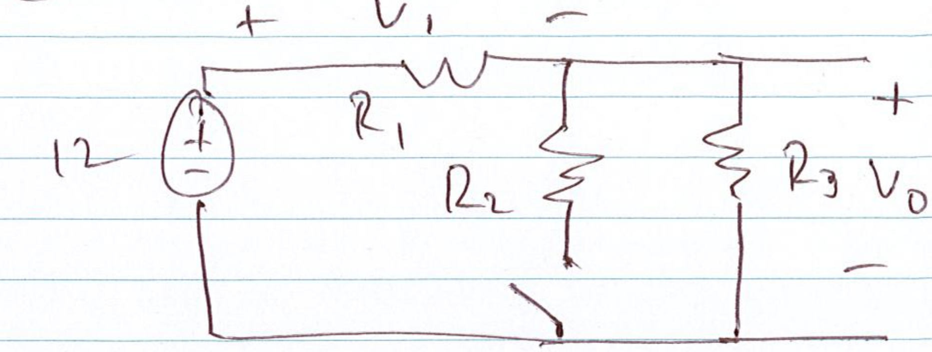
$$\Rightarrow t_0 = \frac{-\ln(0.2)}{200}$$

$$= 8\text{ ms}. \quad (2)$$

$\boxed{4}$

TOTAL: $\boxed{16}$

(4) (a) We use the voltage-divider circuit and we seek R_1, R_2, R_3 to satisfy the desired output voltages.



Switch OFF: $V_{OUT} = \dfrac{12R_3}{R_1+R_3} = 6V$ if $R_1 = R_3$

Switch ON: $V_{OUT} = \dfrac{12R_{eq}}{R_1+R_{eq}}$, where R_{eq} is $R_2 \parallel R_3$

$= 4V$ if $R_{eq} = \frac{1}{2}R$,

So we may try $R_1 = 100\,\Omega$ (1), $R_2 = 100\,\Omega$ (1), $R_3 = 100\,\Omega \Rightarrow R_{eq} = 50\,\Omega$ as required, (1)

Check Power rating: With Switch ON.
V_1 across R_1 is $8V$.
$\rightarrow$ Power$(R_1) = \dfrac{V^2}{R} = 0.36\,W$
$> 0.25\,W$.

So we must increase all resistors and use $R_1 = 200\,\Omega = R_2 = R_3$. (2)
Then V_1 is split across two $100\,\Omega$ resistors each with voltage drop $4V$,
So the $P = \dfrac{V^2}{R}$
$= 0.16\,W$ which meets the (1) power rating.
Resistors R_2 and R_3 have smaller voltages (1) so these are also satisfactory.

$\boxed{7}$

Question 4 (15 marks)

Assume that you have a supply voltage of 12 V. You have also have a single-pole single-throw (SPST) switch and a supply of 100 Ω resistors, with power ratings of 0.25 W.

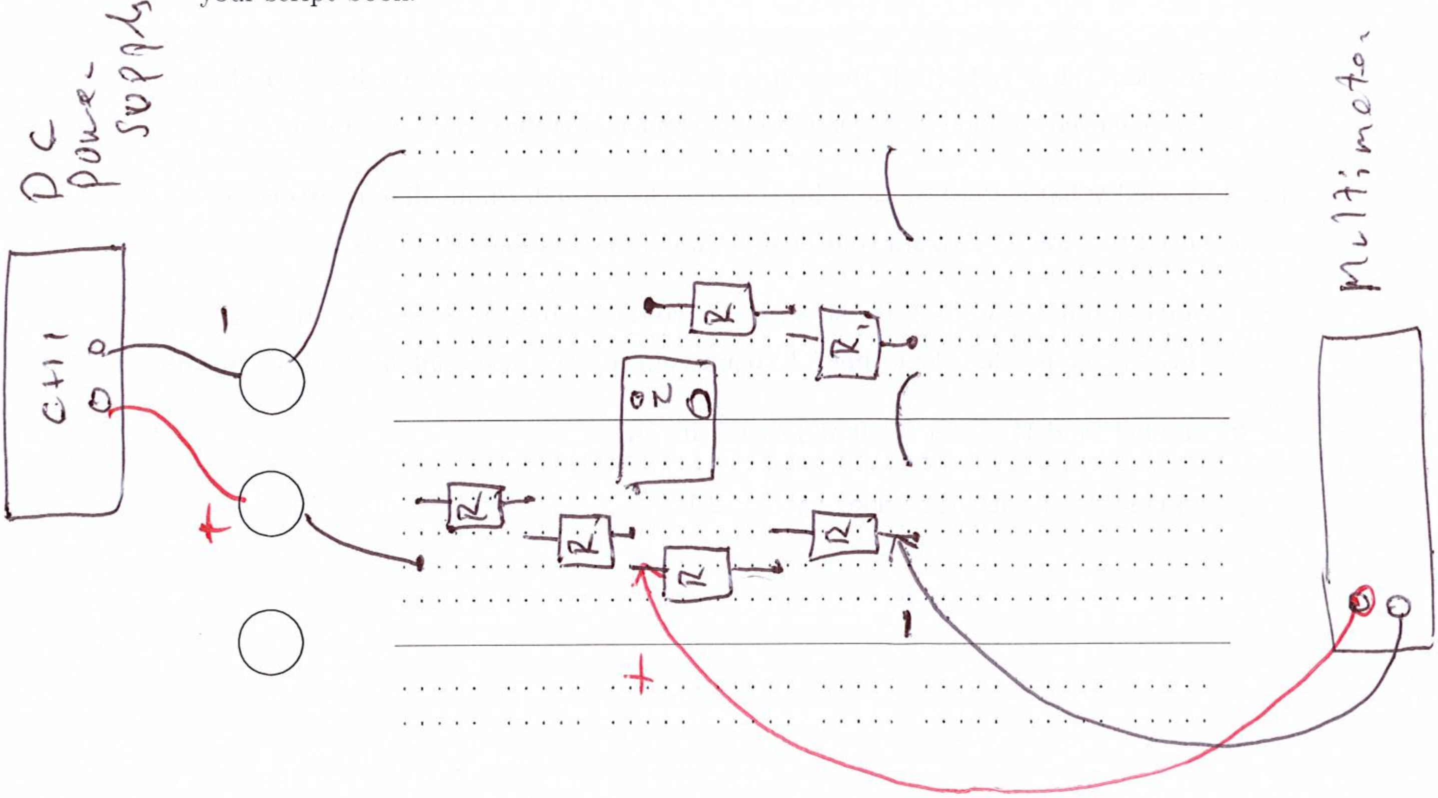
(a) **[7 marks]** Design a two-terminal circuit that will produce 6 V at the outputs when the switch is OFF, and 4 V when the switch is ON. Show that each resistor does not exceed its power rating.

(b) **[8 marks]** Your laboratory kit contains the following equipment:

- GDM 8135 digital multimeter and a GPS 3303 DC Power Supply;
- A breadboard, connecting wires and power leads;
- One SPST (single-pole, single throw) switch and a supply of 100 Ω resistors.

Use the breadboard template below to show how you to build your circuit:

Then explain how you would test the circuit you have built to show that is does meet the desired output voltages. Use a maximum of 80 words, and write your explanation in your script book.



R=100Ω

(5)

(4) (b) Build the circuit as shown on the breadboard.
Set up the multimeter in voltmeter mode
Attach voltage probes across resistors as shown. (1)
With SWITCH OFF, voltmeter should show 6V. (1)
With SWITCH ON, voltmeter should show 4V. (1)

TOTAL : $\boxed{8}$

TOTAL MID-TEST : $\boxed{50}$.