THE UNIVERSITY OF MELBOURNE

Semester 2, 2018 Assessment

Department of Electrical and Electronic Engineering

ELEN30012 SIGNALS AND SYSTEMS

Time allowed: 180 minutes

Reading time: 15 minutes

This paper has 6 pages

Authorised materials:

Only the following calculators may be used:
Casio FX82
Casio FX100
Students may bring **TWO** sheets of A4 paper containing their own notes into the examination room.

Instruction to students:

Attempt **ALL** questions. You may attempt the questions in any order.
The marks given for each question are shown in brackets after the question numbers. The examination paper has a total of 100 marks.

You must show your work in order to receive credit.

Ensure your student number is written on all your script books. No annotating of script books is allowed during reading time or after the end of writing time.

Answer all questions and show all work in your script books.

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Question 1 (16 marks)

Consider the discrete-time system with trajectories (v, y) described by the difference equation

$$y[n+2] - 3y[n+1] + y[n] = -v[n+1]$$

(a) **[3 marks]** Is this system linear? Justify your answer with a proof or counterexample.

(b) **[3 marks]** Is this a time-invariant system? Justify your answer with a proof or counterexample.

(c) **[3 marks]** Obtain H, the input-output transfer function of the system. Is the system stable? Give a reason for your answer (use about 10 to 15 words).

(d) **[2 marks]** Is the system causal? Does the system have memory? Give reasons for your answers (use about 15 to 20 words).

(e) **[5 marks]** Assume that (δ, h) is a trajectory of the system, where δ is the unit pulse function and h is given by

$$h[n] = \frac{1}{\sqrt{5}} \left(\lambda_1^n - \lambda_2^n\right) u[n]$$

where

$$\lambda_1 = \frac{3-\sqrt{5}}{2}, \qquad \lambda_2 = \frac{3+\sqrt{5}}{2}$$

Further assume that $y[1] = y[2] = 0$. Show that (v, y) is a trajectory of the system when

$$v[n] = \delta[n-3] - \delta[n-4], \qquad y[n] = \frac{1}{2\sqrt{5}} \left[(1-\sqrt{5})\lambda_1^{n-4} - (1+\sqrt{5})\lambda_2^{n-4}\right] u[n-4]$$

Question 2 (9 marks)

Let x_1 be the continuous-time periodic signal given by

$$x_1(t) = \cos(10\pi t) + \cos(20\pi t)$$

(a) **[2 marks]** What is the fundamental frequency f_1 of x_1?

(b) **[2 marks]** Let y_1 be the discrete-time signal obtained by sampling x_1 with sampling frequency $f_s = 8\ Hz$. Write down an expression for $y_1[n]$.

(c) **[2 marks]** Find the period of y_1.

(d) **[3 marks]** Show that aliasing of x_1 occurs at this sampling frequency by considering the continuous-time periodic signal x_2 with

$$x_2(t) = \cos(6\pi t) + \cos(12\pi t)$$

Question 3 (19 marks)

The periodic continuous-time signal v has cosine-with-phase Fourier series

$$v(t) = a_0^v + \sum_{k=1}^{\infty} A_k^v \cos(2kt + \theta_k^v)$$

where

$$a_0^v = 2,\ A_k^v = \frac{1}{k^2} \ \text{ and } \ \theta_k^v = \pi \text{ for all integer } k \geq 1$$

Let the complex Fourier series of v be given by

$$v(t) = \sum_{k=-\infty}^{\infty} c_k^v e^{j2kt}$$

The signal v is applied as an input to a linear time-invariant system with frequency response H given by

$$H(\omega) = \begin{cases} -5e^{-j\omega}, & |\omega| \geq \pi \\ 0, & |\omega| < \pi \end{cases}$$

and the resulting output y has cosine-with-phase Fourier series

$$y(t) = a_0^y + \sum_{k=1}^{\infty} A_k^y \cos(2kt + \theta_k^y)$$

(a) [**1 mark**] What is the fundamental period T_0 of v?

(b) [**2 marks**] Find expressions for c_k^v for all $k \in \mathbf{Z}$.

(c) [**6 marks**] Obtain expressions for the amplitude and phase spectra of H and plot their graphs for $-5 \leq \omega \leq 5$.

(d) [**4 marks**] Obtain expressions for a_0^y, A_k^y and θ_k^y, for all integer $k \geq 1$.

(e) Consider the continuous-time signal f given by

$$f(t) = v(t - 3)$$

(i) [**2 marks**] Is f also periodic? Justify your answer.

(ii) [**4 marks**] Suppose f has complex Fourier series

$$f(t) = \sum_{k=-\infty}^{\infty} c_k^f e^{j2kt}$$

Obtain expressions for c_k^f, for all $k \in \mathbf{Z}$.

Question 4 (8 marks)

(a) [**2 marks**] Find the Discrete-time Fourier transform (DTFT) of

$$x[n] = \delta[n+4]$$

where δ is the Kronecker delta function.

(b) [**3 marks**] Find the inverse DTFT of

$$Y(\Omega) = \cos(4\Omega)$$

(c) [**3 marks**] Find the inverse DTFT of

$$G(\Omega) = \sin(4\Omega)\cos(4\Omega)$$

Question 5 (14 marks)

Let x be a continuous-time signal, and let v, w and z be the signals given by

$$v(t) = x(3t), \qquad w(t) = tx(t), \qquad y(t) = \cos(2t)x(t), \qquad z(t) = x(3t-4)u(3t-4)$$

where u is the unit step function. Assume that x has Laplace transform

$$X(s) = \frac{3}{s^2+9}$$

(a) [**4 marks**] Use partial fractions to find $x(t)$.

(b) [**1 mark**] Find V, the Laplace transform of v.

(c) [**1 mark**] Find W, the Laplace transform of w.

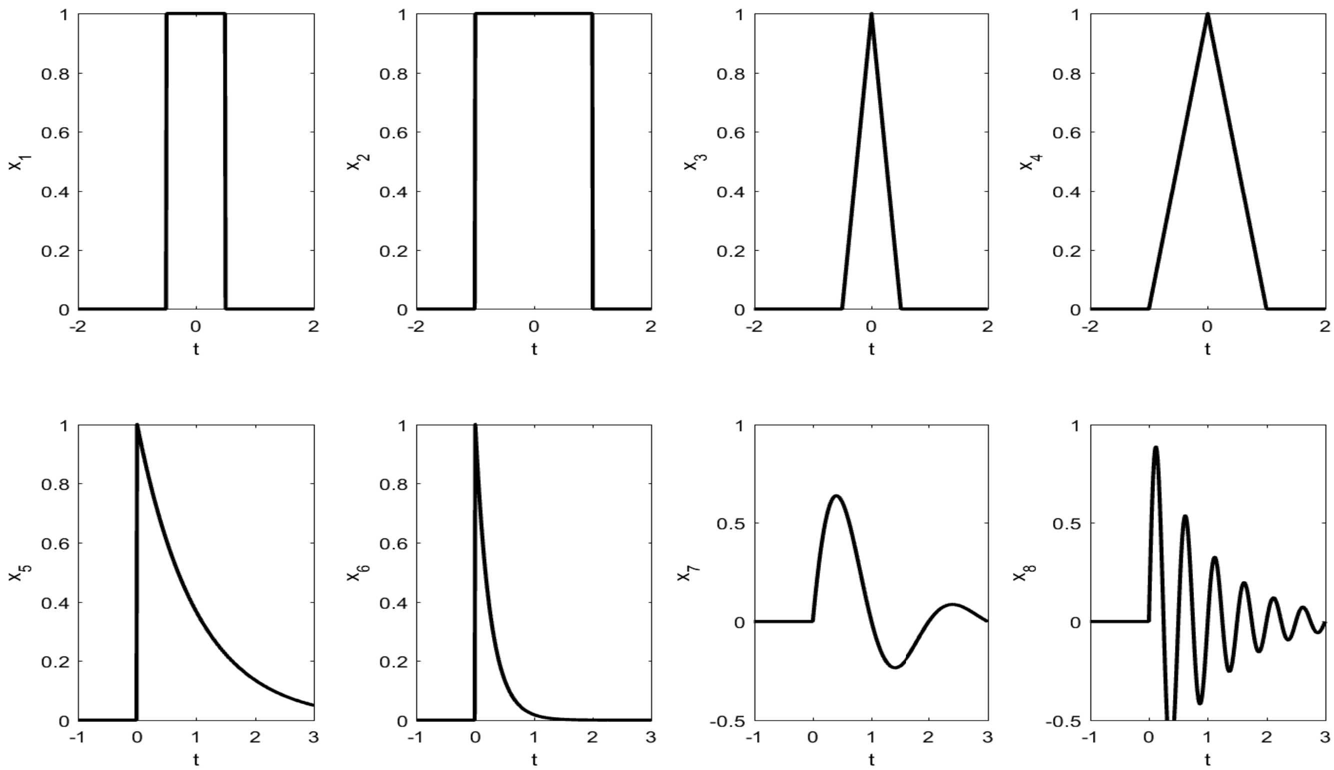
(d) [**2 marks**] Find Y, the Laplace transform of y.

(e) [**2 marks**] Show that $u(t) = u(3t)$, for all $t \in \mathbf{R}$.

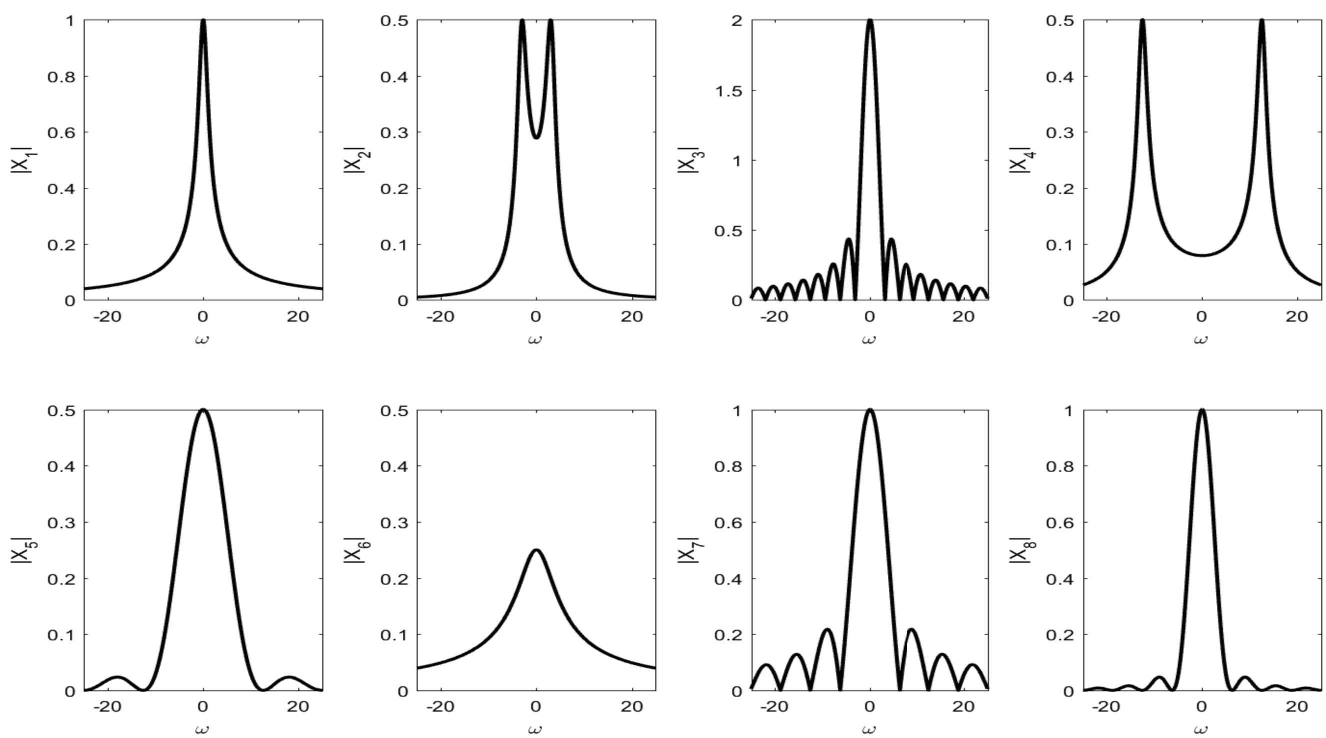
(f) [**4 marks**] Express z in terms of v and u, and hence find Z, the Laplace transform of z.

Question 6 (12 marks)

Consider the eight continuous-time signals $x_1,\ x_2,\ ..., x_8$ shown below:



Match these signals to the appropriate amplitude spectra $|X_1|,\ |X_2|,\ ..., |X_8|$ shown below. For each signal, give a brief explanation using about 10 to 15 words.



Question 7 (14 marks)

A linear time-invariant continuous-time system with input-output trajectories (v, y) is defined by the differential equation

$$\frac{d^2y}{dt^2} + 4y(t) = \frac{dv}{dt} - 2v(t)$$

(a) **[5 marks]** Obtain the input-output transfer function H for this system, and use it to introduce suitable state variables to obtain a state representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) \\ y(t) &= Cx(t) \end{aligned}$$

for this system in controller canonical form.

(b) **[2 marks]** Draw the integrator realization for the system (A, B, C).

(c) **[4 marks]** Let the invertible matrix P be given by

$$P = \begin{bmatrix} \frac{1}{2} & \frac{-j}{4} \\ \frac{1}{2} & \frac{j}{4} \end{bmatrix}$$

Use P to introduce a suitable coordinate change to obtain an equivalent state representation $(\bar{A}, \bar{B}, \bar{C})$ for the system such that $\bar{A}$ is a diagonal matrix. You may assume the following matrix equation

$$\begin{bmatrix} j2 & 0 \\ 0 & -j2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{-j}{4} \\ \frac{1}{2} & \frac{j}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-j}{4} \\ \frac{1}{2} & \frac{j}{4} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$$

(d) **[3 marks]** Show that when $x(0) = [0\ 2]^T$, the zero-input response for the system is

$$y_{zi}(t) = 2\cos(2t) - 2\sin(2t)$$

Question 8 (8 marks)

Let x be a discrete-time signal such that $x[n] = 0$ for $n \leq 0$, and let v be the sum of x, defined by

$$v[n] = \sum_{i=1}^{n} x[i]$$

Let X and V be the z-transforms of x and v, respectively. Express V in terms of X.

END OF EXAMINATION