

School of Computing and Information Systems
COMP90038 Algorithms and Complexity Tutorial Week 12

- Recall the definition of the knapsack problem. Given a set of items $S = \{i_1, i_2, \dots, i_n\}$ with
 - weights: $w(i_1), w(i_2), \dots, w(i_n)$
 - values: $v(i_1), v(i_2), \dots, v(i_n)$

and a knapsack of capacity W , find the most valuable selection of items that will fit in the knapsack. That is, find a set $I \subseteq S$ such that $\sum_{i \in I} w(i) \leq W$ and so that $\sum_{i \in I} v(i)$ is maximised.

Define the *benefit* of an item i to be the rational number $v(i)/w(i)$. Consider the following greedy approach to the problem:

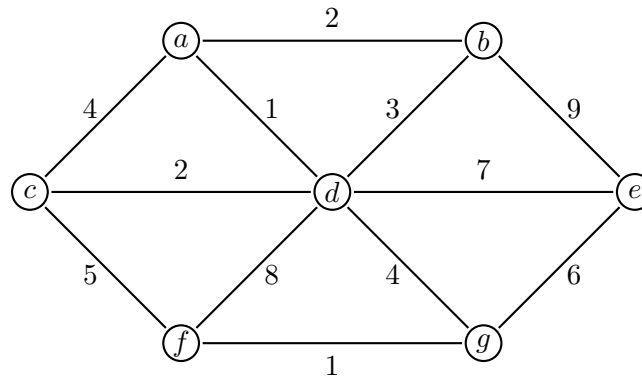
```

Let  $A[1] \dots A[n]$  hold the items from  $S$ , in decreasing order of benefit
 $val \leftarrow 0$ 
 $weight \leftarrow 0$ 
 $k \leftarrow 1$ 
while  $k \leq n \wedge weight + w(A[k]) \leq W$  do
  select  $A[k]$ 
   $val \leftarrow val + v(A[k])$ 
   $weight \leftarrow weight + w(A[k])$ 
   $k \leftarrow k + 1$ 

```

That is, at each step, from the remaining items we simply pick the one that has the greatest benefit. Give a simple example to show that this greedy algorithm does not solve the knapsack problem.

- Work through Prim's algorithm for the graph below. Assume the algorithm starts by selecting node a . Which edges are selected for the minimum spanning tree, and in which order?



- Use Dijkstra's algorithm to find the shortest paths for node e in the previous question's graph. That is, run the algorithm to determine the length of the shortest path from e to v , for all seven nodes v . Is the shortest path from e to b part of the graph's minimum spanning tree?
- Lemuel Gulliver wishes to compress the string "all big endians and all small endians". Help him by building a Huffman tree for the string (there may be several valid trees) and assign a binary code accordingly, to each of the eleven characters involved (we have used `␣` to make each space character visible). The frequencies are:

a	b	d	e	g	i	l	m	n	s	␣
6	1	3	2	1	3	6	1	5	3	6

How many bits are required for the encoded string?