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A Mathematical Model and Two-Stage Heuristic for the Container Stowage Planning Problem With Stability Parameters

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ABSTRACT Owing to the significant increase in the volume of world trade, mega-container vessels are being used to meet transportation demands. As the size of the vessels increases, the loading sequence of containers onto the vessels presents an important challenge for planners. In this study, we consider the container stowage planning problem with stability constraints (e.g. shear force, bending moment, trim) and develop a mixed integer linear programming (MILP) formulation which generates load plans by minimizing total cost associated with the over-stows and trimming moments. Our study adopts a holistic perspective which encompasses several real-world features such as different container specifications, a round-robin tour of multiple ports, technical limitations related to stack weight, stress, and ballast tanks. We also propose a two-stage heuristic solution methodology that employs an integer programming (IP) formulation and then a swapping heuristic (SH) algorithm. This approach first acquires a lower bound on the total over-stow cost with the IP model, thereby creating an initial bay plan. Then, it applies the SH algorithm to this initial bay plan to minimize cost resulting from trimming moments. The efficiency of the MILP formulation and heuristic algorithm is investigated through numerical examples. The results have shown that the heuristic has greatly improved the solution times as well as the size of the solvable problems compared to the MILP formulation. In particular, the two-stage heuristic can solve all size problem instances within an average optimality gap of 0-25% in less than 8 minutes, whereas the MILP can only achieve an approximate optimality gap of 55-80% in 2 hours.

INDEX TERMS Container stowage plan, mixed integer linear programming, over-stow, stability.

I. INTRODUCTION

Over the past two decades, there has been a continuous increase in the demand for cost-efficient containerized transportation. Container traffic has increased from nearly 85 million twenty-foot equivalent units (TEUs) in 1990 to 651 million TEUs in 2013, with an annual growth rate of 9.3% [1]. To meet this demand, shipping companies have deployed larger container vessels, which can transport up to 20,000 TEUs [2]. These vessels sail from port to port, loading and unloading thousands of containers. Processing these large quantities of containers requires well-organized port design and operations, and efficient container stowage plans (CSPs) [3].

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The efficiency of port design and operations determines the duration between ship arrivals and departures. Port authorities need to reduce loading and unloading periods to serve as many ships as possible to achieve higher profits. Additionally, liner companies seek to minimize the duration of port visits to reduce the associated costs. However, the length of these periods is strongly correlated with the container specifications. Generally, the properties of containers to be processed become known only prior to ship arrival. This factor transforms the problem into both real-time and time-constrained. Consequently, port authorities and liner companies are generally left with only a couple of hours to attain an efficient CSP followed by other activities such as yard operations and crane utilization. Therefore, a well-planned CSP is crucial for the effective implementation of port operations.

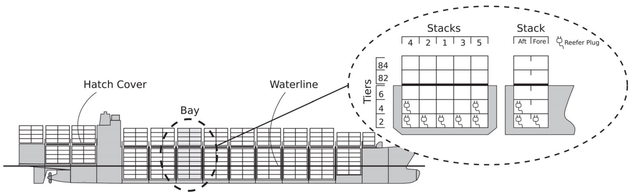

FIGURE 1. Layout of container vessel (source: [3]).

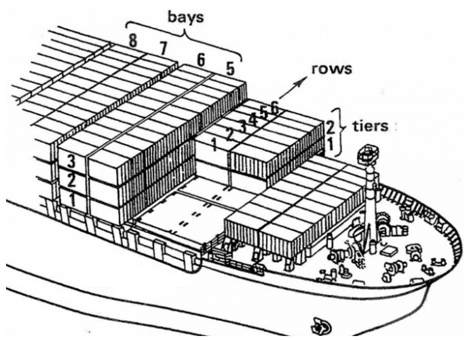

FIGURE 2. View of container positions on a vessel (source: www.bestshippingnews.com).

An exemplary layout of a container vessel is given in Figure 1. The figure shows how containers are arranged in storage areas called *bays* along the length of the vessel. Bays are the crosswise areas that divide the vessels into sections from bow to stern. A bay is composed of a number of cells, each indicating a possible stowage position. Cells usually have a capacity of two TEUs, meaning they can stow either two 20 ft. containers or one 40 ft. (or 45 ft) container. Each TEU position within a cell is referred to as a slot. Given a bay, its slots toward the bow of a vessel are called fore slots, and those toward the stern are called aft slots. Slots are identified by a stack or row number, indicating the slot's horizontal position within a bay, and a tier number indicating the slot's vertical position. These concepts are shown in Figure 2. Each container in the corresponding slot is attributed to a specific 2-digit bay, row, and stack or tier number. For example, a container with number 111412 refers to the location in the 11^{th} bay, 14^{th} row, and 12^{th} tier.

The CSP can be formulated as an assignment problem where a set of containers with a given port destination must be assigned to slots in a vessel with the aim of minimizing the transportation costs. Unnecessary handling operations, unstable ship conditions, and long port durations increase the total cost. Thus, the objective of the CSP includes the minimization of over-stows. This entails stowing a cargo on top of another cargo, which causes temporary removal and placement of containers. Containers are stored in stacks on a ship, and thus, if a container at the bottom needs to be removed, all of the containers on top of it should be removed. Containers that must be removed and replaced at a particular port in order to reach containers below them are known as *over-stows*. Figure 3 displays an example over-stow situation. In this example port A precedes port B. Therefore, when the ship arrives port A, the two containers going to port B (upper

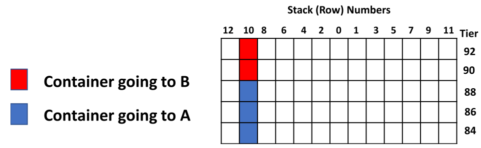

FIGURE 3. Representation of an over-stow.

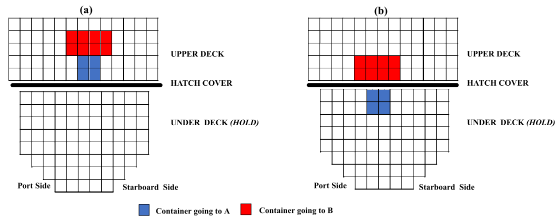

FIGURE 4. Illustration of an over-stow: (a) an over-stow on a single deck and (b) an over-stow between upper deck and under deck containers.

slot containers) should be removed first for the discharge of port A-containers.

Although only two over-stows occur in a single hold in this example (Figure 3), there are some cases in which over-stows exist between the upper deck and the under deck bays. In such cases, not only the containers causing over-stows but also all upper deck containers must be removed to reach the under deck containers. Under deck slots are usually called *holds*, and are protected by hatch covers. The purpose of hatch covers on container ships is to carry deck cargo, split the container stack between hold and deck, prevent excessive water ingress into the cargo hold to protect the goods from dampness and damage. Figures 4(a) and 4(b) represent over-stows in a single hold and between upper and under deck containers (separated by a hatch cover), respectively. In case (a), removal of only four port B-containers gives access to port A-containers. On the other hand, in case (b), all port B-containers along with the hatch cover must be removed for easy discharge of port A-containers. Therefore, over-stows occurring between upper and under deck containers are more critical in terms of transportation costs. Over-stows require extra crane operations and lengthen port visit duration and thus increase loading/unloading costs [4].

The CSP inherently has an impact on ship stability, which should be considered by planners. When ship stability is disregarded in CSP, the loading condition of containers might make the ship unsafe and unseaworthy. Among these parameters, the most significant one is the trim of the ship, which is the difference between the aft and fore drafts. An excessive trim that is caused by excess weight forward or aft, reduces the freeboard forward or aft, changes the vessel condition from the designed seagoing trim and may affect the seakeeping ability of the vessel. In this respect, planners must either relocate the containers or utilize ballast tanks to adjust the ship trim. In the former case, the optimality of a given CSP might be violated by creating extra over-stows, which increases loading/unloading costs. In the latter case, fuel consumption (and associated transportation cost) might increase owing to an increase in drafts. The other important

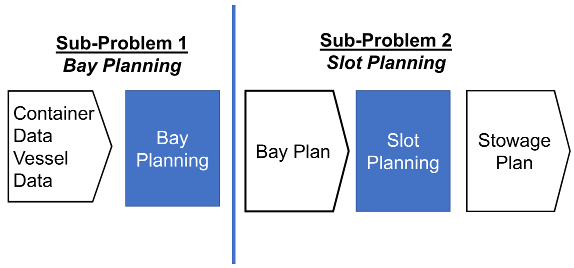

FIGURE 5. Sub-problems of the CSP.

stability parameter is the longitudinal weight distribution that determines the level of stresses occurring in the ship structure. As an uneven weight distribution causes excessive stress in the structure of the ship, which may lead to catastrophic failures, the distribution of containers must be planned very carefully. Consequently, an algorithm aiming to generate an efficient stowage plan must incorporate stability parameters in its models and consider additional fuel costs resulting from vessel instability.

The CSP problem, as an allocation of containers to slots, is an NP-complete combinatorial optimization problem [5]. For example, the number of possible stowage configurations for a 2000 TEU containership is approximately 3.3^{105775} [6]. Many researchers attempted to solve the CSP by dividing the problem into sub-problems as visualized in Figure 5 due to its NP-completeness. The first problem generates a bay plan in which the containers are assigned to specific bays. The second problem determines the exact slot assignments of the containers at the corresponding bay.

In this study, we consider the bay planning phase of the CSP and develop a mixed integer linear programming (MILP) formulation. Our MILP formulation minimizes the total costs associated with the number of over-stows and trimming moments while keeping stress factors within allowable limits during a multi-port voyage. We also propose a two-stage heuristic methodology for solving the CSP problem. In the first stage, we employ an integer programming (IP) formulation which seeks to generate an initial bay plan yielding the lowest total over-stow costs, i.e. a lower bound for over-stow cost. In the second stage, we implement a swapping heuristic (SH) algorithm that takes the initial bay plan from stage 1 as an input, and improves the plan by minimizing the costs associated with the trimming moment without deteriorating the over-stow cost performance. We show that the heuristic approach can rapidly generate a feasible bay plan by minimizing over-stows and can improve ship stability by minimizing the trim of the ship on a multi-port voyage while maintaining stress levels below allowable limits.

As will be discussed in the next section in more detail, some of the studies (e.g. [7]–[9]) existing in the literature tackle the CSP problem under simplistic assumptions that prevent them to be applied to real-world large-size containership loading problems. For example, most of these studies did not consider stability parameters, different types of containers, and loading and unloading operations at multiple ports. Thus, in our study, we attempt to incorporate several realistic features and assumptions that have been explored separately to some degree but not together. In particular, our modeling framework encompasses: (1) ship stability parameters that are vital for safe navigation, (2) different container specifications, (3) demand to and from multiple ports, (4) a number of technical limitations (e.g., limits on stack weight, stress factors, ballast tanks, pre-loaded ships at the first port, hatch covers between upper and under decks, etc.), (5) different types of over-stows, (6) multiple holds within bays, i.e. two upper and two under decks, and (7) costs associated with over-stows and fuel consumption.

The remainder of this paper is structured as follows. In Section II, we give a review of related literature. Next, in Section III, we first provide preliminaries on a number of concepts, such as containers, trim, stack weight, shear force, bending moments, buoyancy force, and define our problem in more detail. The details of our assumptions, MILP formulation and heuristic solution methodology are explained in Section IV. We demonstrate the performance of our proposed solution approach on a number of test instances in Section V. Section VI concludes the paper with a few remarks and possible future work ideas.

II. RELATED WORK

The problem of finding efficient procedures for container stowage planning operations has attracted the attention of shipping companies and many researchers since the 1980s. As one of the first papers that study the CSP problem, [10] proposed a combined use of Monte Carlo simulation and human interaction to generate vessel plans.Reference [11] further investigated the CSP problem and proposed a methodology for minimizing the number of undesired shifting operations, but did not incorporate ship stability factors. Reference [12] proposed a heuristic algorithm known as the suspensory heuristic procedure. This procedure is basically a dynamic slot-assignment scheme which seeks to minimize the number of over-stows. Reference [13] has later improved the suspensory heuristic algorithm. Although this heuristic is only applicable when loading and unloading information for all ports is available, the algorithm proposed in [13] requires such information only for the current port. However, container weights and stability parameters were not considered in [13]. The first attempt to derive some rules for determining good CSPs was reported in [14]. The authors used a constraint satisfaction approach to define and characterize the space of feasible solutions.

References [5] and [15] later showed that the CSP problem is *NP*-complete by stating that it is related to a known *NP*-hard problem. Additionally, a systematic classification scheme for the CSP problems was introduced in [16]. Due to its *NP*-hard nature, [17], [18] and [19] presented a more realistic model by decomposing the planning process into two phases. In the first phase, they created a vague CSP based on grouping containers with the same characteristics in terms factors such as size and destination, and assigning them to blocks of stowage space in the ship. In the second phase, individual containers were assigned to specific locations, resulting in a detailed stowage plan. The authors achieved feasible solutions for small-size problem instances. This methodology was also adopted by [20]. Similarly, they proposed a two-phase algorithm. The entire problem was later described by [21] in the form of an optimization problem. The authors referred to the problem as a ''master bay plan problem (MBPP)'', and developed a basic 0/1 linear programming (LP) model to minimize total stowage time. Reference [22] later extended the work in [21] by proposing a three-phase algorithm for the MBPP. The algorithm is based on a partitioning procedure that splits the ship into different portions and assigns containers based on their destinations. However, they assumed that only unloading operations are allowed at the ports visited, which implies that the loading problem can only be considered at the first port. Considering several structural and operational constraints, [7] proposed a mixed integer programming (MIP) based heuristic for the multi-port MBPP with the objective of minimizing berthing time of ships. Later, [23] proposed a MIP model that generates a CSP for ships that need to visit a given number of ports in a circular route. They considered standard, reefer, and open -top containers with three different weight categories. They also assumed that the considered route had six ports, and for each origin port, they considered the transport demand for the next three ports along the route. Their objective function minimized the weighted sum of re-handles and crane unbalance. In a more recent study, [24] solved the CSP problem in the presence of hazardous containers with specific stowage principles.

In another study, [4] proposed a multi-objective IP formulation which incorporates four objectives as the minimization of three ship stability terms and the number of re-handles. They formulate the problem as a single objective function with the weighting method and determined a number of non-inferior solutions for different problem instances. However, their reported solutions were acceptable only for practical purposes if no re-handles occur during unloading. [25] developed a multi-stage heuristic for the CSP problem by decomposing the problem into four stages. In the first stage the number of over-stows was minimized. In the subsequent stages, the stowage plan was generated. On the other hand, the first stage of the model proposed by [25] used the following simplifying assumptions: (*i*) all containers have the same size of 20 ft., (*ii*) container weights, hatch cover constraint and ship stability are not considered, (*iii*) bays are treated as a single dimension, (*iv*) the vessel is assumed to be empty at the initial port and (*v*) all containers are unloaded at the final port. Moreover, the generated bay plan sets forth the percentage of a bay being allocated to a set of containers and does not directly generate the (integer) number of containers assigned to a bay. In other words, the bay percentage produces fractional capacity values that cannot be implemented in the bay assignment problem.

Another two-phase approach was proposed by [26]. First, an IP model for assigning groups of containers to storage areas of the vessel over multiple ports was presented. Second, a constraint programming (CP) approach and a local search procedure for stowing individual containers (slot planning) were used. Reference [27] extended their previous work by incorporating ballast tanks, linearizing the center of gravity calculation and incorporating the hydrostatic data tables of the vessel to formulate stability and stress moment constraints that can handle variable displacements. Reference [28] focused on the slot-planning phase of this approach and presented CP and IP models for stowing a set of containers in a single bay section. The authors presented a model for stowing standard 20 and 40 ft. containers with the objective of minimizing over-stows. In their study, [8] investigated a multi-objective CSP problem that seeks to optimize ship stability and the number of re-handled containers simultaneously. In particular, they considered three stability parameters (i.e., metacentric height, list value (heeling to either starboard or port), and trim value) and three rehandle types (i.e., rehandles at yard cranes, quay cranes and future ports) in their model. They solved the problem using a variant of the Nondominated Sorting Genetic Algorithm-III (NSGA-III). In another study, [3] presented a greedy randomized adaptive search procedure (GRASP) for a container stowage slot-planning problem. The model employs a given master bay plan and implements the GRASP algorithm to determine an efficient slot plan. Similarly, [29] studied the multi-port CSP problem. They proposed an IP model and a GRASP algorithm that is capable of generating stowage plans with minimum over-stow for large scale problems. The authors considered that all containers are of the same weight and did not include ship stability constraints. [30] developed a metaheuristic algorithm based on a two-phase procedure. The first phase is the master bay planning phase, which allocates containers to be stowed at each bay. The second phase is the slot planning phase, which allocates a specific slot for each container within each bay. [31] implemented two metaheuristics, i.e., the genetic algorithm (GA) and simulated annealing (SA), for solving the CSP. Their results implied that SA can reach the near-optimum result at a much faster rate than the GA. The objective was to reduce the number of container movements during the vessel journey due to the additional cost of unnecessary container movements. However, the model assumed that only the same size of containers (i.e. only 40 ft.) can be loaded and unloaded, and it did not consider the container weights and heights during multi-port voyages; thus, it is not practical to implement the model in real cases.

Ship stability plays a crucial role not only during sailing but also at the port, especially in loading and unloading operations known as the quay crane scheduling problem (QCSP). Despite the presence of various studies on the QCSP, only a couple of proposed models have accounted for ship stability. Among them, [37], [38] and [33] included ship stability in their models. In particular, ensuring the longitudinal stability of the vessel through its journey, [37] minimized the makespan of the port operations with trim-related

TABLE 1. A summary of literature with respect modelling methodology, solution approach, and presence of stability constraints.

		Modeling Methodology		Solution Approach			Stability Constraints		
Year	Ref.	Single-Phase	Multi-Phase	Mathematical Modelling	Heuristic Algorithm	Combined Approach	Explicitly	Implicitly	Not Considered
1993	[11]	+				+			+
1998	[12]	+			+				+
2000	[18]		+			+			+
2002	[20]		+			+		+	
2004	[21]	+				+			+
2006	[22]		+			+		+	
2006	[4]	+				+	+		
2008	[25]		+			+	+		
2012	[27]	+		+			+		
2012	[28]	+		+			+		
2015	[13]	+			+				+
2015	[7]	+		+				+	
2015	[8]	+			+		+		
2017	[23]	+		+			+		
2017	[30]		+			+			+
2017	[31]	+			+				+
2018	[24]		+			+		+	
2018	[9]		+			+			+
2018	[32]	+				+			+
2018	[33]	+				+		+	
2019	[34]		+			+		+	
2019	[35]		+			+	+		
2019	[29]	+			+				+
2020	[36]		+			+			+

feasibility being checked at every unit time of the handling process. Moreover, the proposed model captures the practical constraints of the QCSP, such as non-crossing and the safety margin and crane traveling time constraints. Since their mathematical model was capable of producing satisfactory results for small-sized problems, a GA is designed to solve medium and large-sized problems. Reference [38] formulated the QCSP for a single ship, with the objective of further extending it to the multi-ship case with stability constraints. The multi-ship case is solved with the help of Lagrangian relaxation, whereby the problem is decomposed by ship, and each sub-problem is solved efficiently as a single ship case. In both studies, ship stability was investigated thoroughly. Reference [33] proposed a model which integrates the 3D Stowage Planning (3DSP) and Scheduling of Quay Crane (SQC) problems. To solve large scale problems, they also proposed a combined solution methodology which incorporates mathematical programming and simulation optimization methodologies with a GA meta-heuristic. Similarly, in this work, the authors adopted some simplifying assumptions such as same-size and weight containers, empty ship at the first port and evacuation of all containers at the last port.

Recently, a few other studies that tackle the CSP problem have been published. Among them [9] addressed the basic container stowage planning problem (CSPP) originally defined by [11]. They considered the ship as a grid where each column represents a stack and each row represents a tier. The CSPP problem is a simplified version of the CSP problems encountered in practice. Reference [39] investigated the effectiveness of the stowage planning methods on a number of real-life scenarios. They presented an empirical study by testing these cases in terms of navigational safety and the inter-dependent relationship between terminal storage operations and containership cargo handling operations. Reference [32] presented the flexible containership loading problem for seaport container terminals. They integrated the assignment and scheduling of transfer vehicles and container load sequencing with the assignment of specific containers to the vessel positions. Reference [36] proposed a matheuristic algorithm for the slot planning phase of the CSP problem. The authors built their algorithm with respect to an initial master bay plan as input (also adopted by [26], [27] and [28]). Reference [34] studied the CSP problem in inland container liner shipping of the Yangtze River to create stowage plans for multiple ports over the full route. Another study for inland shipping was proposed by [35]. The author developed two mathematical models. The first model seeks to maximize the number of containers onboard by ensuring stability, whereas the second attempts to develop feasible container handling sequences to guarantee stability during loading and unloading operations.

Among the recent studies which tackle the CSP problem, a few of them include the stability constraints into their modeling framework. For example, [7] proposed a model which only includes the trim and list values as stability parameters. In another study, [8] incorporated the metacentric height, list value, and trim value into their model. However, they only considered 20 ft. containers and did not incorporate container unloading operations. References [3] and [36] addressed the slot planning phase with stack weight limits.

We provide a detailed comparison of the above-mentioned studies in Table 1. In this table the reader can compare the existing works in terms of modeling methodology, solution approach implemented, and the presence of stability constraints. We further categorize the modeling methodology as: (*i*) single-phase, and (*ii*) multi-phase, solution approach as: (*i*) mathematical model, (*ii*) heuristic, and (*iii*) combined mathematical and heuristic model applications, and finally the presence of stability constraints as: (*i*) explicitly, (*ii*) implicitly, and (*iii*) not considered. Our review of the literature reveals that none of the existing studies, to the best of our knowledge, explored the CSP problem with the all realistic assumptions and features as we do in this study. Different from previous works, our study adopts a holistic perspective which encompasses several ship stability parameters, different container specifications in terms of size and weight, round-robin tour with a demand to and from multiple ports, several technical limitations related to stack weight, stress, ballast tanks, etc.,

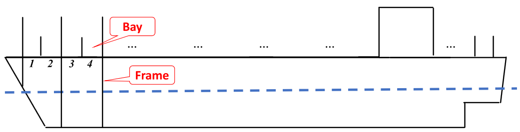

FIGURE 6. Frames and container bay layout for 20 and 40 ft. containers.

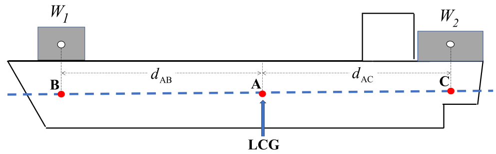

FIGURE 7. Trim and trimming moment on a ship.

TABLE 2. Calculation of shear forces for the example force distributions given in Figure 8.

Points	Shear Force (ton)	
	(a) Single Loaded Bar	(b) Uniformly Loaded Bar
A	10	$0(\Delta - H) = 0$
B	$10 - 20 = -10$	$\|AB\|(\Delta - H) = 15$
C	$10 + 40 - 20 - 15 = 15$	$\|AC\|(\Delta - H) = 35$
D	$10 + 40 + 35 - 20 - 15 - 30 = 20$	$\|AD\|(\Delta - H) = 55$
E	$10 + 40 + 35 - 20 - 15 - 30 - 20 = 0$	$\|AE\|(\Delta - H) = 65$

different types of over-stows, and costs associated with over-stows and fuel consumption.

III. PRELIMINARIES

In this section, we first present preliminaries on the container types and specifications, shipping operations and related stability concepts, e.g., trim, stack weight, shear force, bending moment, buoyancy force and weight distribution. Next, we provide a problem definition.

A. CONTAINERS

Different types of containers, such as standard, reefer, out-of-gauge, and hazardous, can usually be stowed in a containership, but their stowage rules for some types are constrained. The location of reefer containers, for example, is defined by the ship coordinator so that their exact positions are known, this is generally near power-points to maintain the required temperature during transportation. Additionally, hazardous containers are assigned in line with IMO policy and must be stowed into pre-planned slots.

In this study, the size of the containers is regarded as either 20 and 40 ft. Standard locations are generally considered for dry 20 ft. containers, which are referred to as one TEU location. Containers with a size of 40 ft. require two adjacent locations of 20 ft. (see Figure 6, which displays the general layout of bays and frames for a containership). Note from the figure that a 40 ft. container can be stowed on either adjacent bays (1–2 or 3–4), but not on bays 2–3. Consequently, if a 40 ft. container is stowed in a bay (for instance, bay 1), the locations of the same row and tier corresponding to an adjacent bay (for instance, bay 2) is not subsequently available for stowing containers with a size of 20 ft. Also note within the diagram, another important component of a ship structure, the frame, which is defined as a rib (bolted or welded to the keel) that supports the ship's hull to give shape and strength. Figure 6 displays a general layout of bays and frames for a containership.

The standard weight of an empty container ranges from 2 to 3.5 tons, whereas the maximum weight of a full container to be stowed in a containership ranges from 20 to 32 tons. The distribution of containers of various weights has a direct effect on ship stability. Thus, there exists a strong correlation between CSP and stability. CSP must ensure proper stowage of containers so that the ship will be stable and, most importantly seaworthy. Improper stowage, on the other hand, can jeopardize seaworthiness through its potential to cause heavy damage and, at the very least, high operating costs.

B. TRIM

We refer to the trim of a ship as one of the most important parameters in this study. A ship is supported by fluid pressure and because of this, it will incline in any direction according to the position of the weights placed on it. The trim is the angle that a ship makes, *fore* and *aft*, with the water. More formally, the trim is defined as the difference between a ship's aft and fore drafts resulting from the difference between the stern moment and bow moment. It is calculated as $M_d/MTC \times 100$, where MTC is the moment required to produce a 1 cm change in trim and M_d is the longitudinal moment. Since MTC is a constant, the trim is determined by the difference in the longitudinal moment between the stern and bow sides.

The longitudinal moment occurs at the point where the longitudinal center of gravity (LCG) is designed. The LCG point is unique for each ship. Both the position of the LCG and the distance of bays to the LCG point can be easily found from the ship stability book. Here, M_d can take a positive or negative value depending on which side the decisive moment occurs and calculated as $M_d = M_s - M_b$, where M_s and M_b are the trimming moments occurring at the aft and fore of the LCG, respectively. If $M_s > M_b$, then $M_d > 0$ and the ship is trimmed by the stern. Conversely, if $M_b > M_s$, then $M_d < 0$ and the ship is trimmed by the bow. And, if $M_b = M_s$, then $M_d = 0$ and the ship is in a stable condition. Figure 7 illustrates the trim and trimming moment concepts on a ship with two containers of weights W_1 and W_2 located above points B and C, respectively. Considering the ship and its LCG point A in the figure, the moments of the loading configuration derived from CSP are $M_b = W_1 d_{AB}$ and $M_s = W_2 d_{AC}$ where d_{AB} and d_{AC} are the distances of containers to the LCG point. Therefore, the resulting trimming moment will simply be $M_d = W_2 d_{AC} - W_1 d_{AB}$. The sign of M_d indicates the side by which the ship is trimmed [40].

C. STACK WEIGHT

The distribution of the containers not only affects the ship stability, but also the stack weight capacity. Stack weight, as the name suggests, is the maximum weight of containers that can be stacked upon (vertical pile) a row. Prior to loading a cargo, the stacking weights of each row must be checked against the allowable stack weights on board the vessel. Neglecting

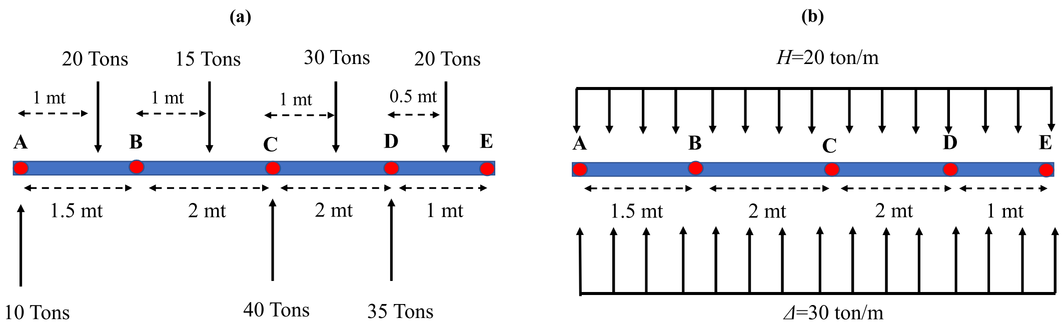

FIGURE 8. **A beam subject to (a) single forces, (b) uniformly distributed forces.**

this may cause serious damage to the ship structure and hull. Eventually, the overall stability of the ship could be affected.

D. SHEAR FORCE AND BENDING MOMENTS

All ships are designed with limitations imposed on their operability to ensure that their structural integrity is maintained. Exceeding these limitations may result in over-stressing of the ship's structure, which could lead to catastrophic failure. The ship's approved loading manual provides a description of the operational loading conditions upon which the design of the hull structure is based. The stresses that may occur in the ship's structural parts (such as frames) are also related to the ship's weight distribution. Ships should be loaded with cargo such that stresses in the ship structure remain within allowable levels. Two main causes of stress are the shear force (SF) and bending moments (BM) resulting from the weight distribution along the ship. The SF and BM can be minimized by evenly distributing the cargo and by homogeneous loading. The loading instrument provides a means to readily calculate the still water SF and BM, in any load or ballast condition, and to assess these values against the design limits.

SF, measured in tons, represents two external parallel forces that act in opposite directions on any part of a structure to break or shear it. This force is considered by taking the difference between the buoyancy and the weight force acting on it. Wherever one of these forces exceeds the other, shearing stresses are likely to occur. Consider a simple example of a beam subjected to loads as shown in Figure 8(a). To determine the SF at any point (i.e., A, B, C, D, and E), we simply cover everything to the right of that point and add up the external forces left of that point. Even when the load is uniformly distributed on the beam as shown on Figure 8(b), the same rule applies. In the figure, H and Δ refer to the weight force and buoyancy force, respectively. The calculation steps of the values of SFs at points A, B, C, D, and E for the single and uniformly distributed forces are shown in Table 2.

BM is the amount of bending caused to the ship's hull by the external forces that cause shearing stresses. The forces mentioned in Figure 8 also create a bending moment that must be kept within allowable limits for the safe voyage of the vessel. The BMs for each point on the bar illustrated in Figures 8(a) and 8(b) are obtained as shown in Table 3.

E. BUOYANCY FORCE AND WEIGHT DISTRIBUTION

In water, only two forces are exerted on ships: (1) the weight of the ship, which acts downwards and is distributed over the entire length of the ship, and (2) the buoyancy force that the water exerts on the ship's underwater body; it acts upwards and is distributed over the length of the underwater portion of ship. The buoyancy force distribution depends on the underwater profile of the ship, i.e., it is determined by the shape of the underwater hull and is the weight of the water displaced by the underwater hull. In other words, the buoyancy distribution is the same as the volume distribution of the underwater portion of the hull. If the underwater volume is divided into sections, then the volume of the underwater hull is the sum of the areas of these sections along the ship's length. Thus, the buoyancy distribution is a plot of section areas along the length of the underwater body of the hull. The buoyancy force distribution is determined in the design process for different drafts and can be obtained from Bonjean curves in stability books [41].

The total weight of the ship is composed of two different types of weight. First is the self-weight, which is also called the lightweight. It comprises the ship hull's structural weight, the weight of the ship's machinery, and the weight of outfitting (basically, all the items that are unchanging are part of the ship's lightweight). The second type of weight is called the dead-weight. It is the weight of all the changeable items, like the cargo, fuel, ballast water, fresh water, and all the other items in the ship's tanks. Together, the lightweight and deadweight add up to the total weight of the ship, which is called the ship's displacement.

The weight distribution of the lightweight of a ship is determined in the design process and might be obtained from stability books. However, the changeable weights, basically the cargo, which is the most important component of deadweight, become clear prior to ship departure. In that respect, the only instrument that the planner may use to keep the SF and BM under allowable levels is the weight distribution of the cargo. Figure 9(a) illustrates all the forces acting on a ship. The difference between the upward and downward forces is the stress to which the ship is exposed.

Let $b \in B$ denote the set of bays of a containership where b is a specific bay. Consider the structural frame intersecting

TABLE 3. Calculation of bending moments for the example force distributions given in Figure 8.

Points	Bending Moment (ton meter)	
	(a) Single Loaded Bar	(b) Uniformly Loaded Bar
A	0	0
B	$10(1.5) - 20(0.5) = 5$	$(\Delta - H)\lvert AB\rvert^2/2 = 22.50$
C	$10(3.5) + 40(0) - 20(2.5) - 15(1) = -30$	$(\Delta - H)\lvert AC\rvert^2/2 = 122.50$
D	$10(5.5) + 40(2) + 35(0) - 20(4.5) - 15(3) - 30(1) = -30$	$(\Delta - H)\lvert AD\rvert^2/2 = 302.50$
E	$10(6.5) + 40(3) + 35(1) - 20(5.5) - 15(4) - 30(2) - 20(0.5) = -20$	$(\Delta - H)\lvert AE\rvert^2/2 = 422.50$

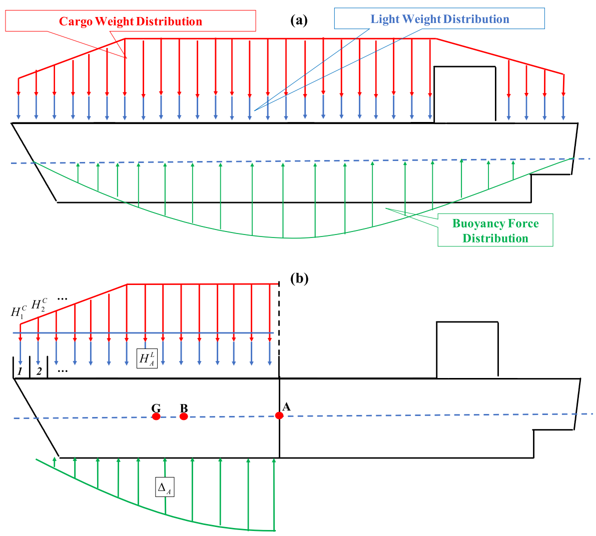

FIGURE 9. An illustration of the forces acting on a ship. (a) The ship weight and buoyancy forces (b) Forces on a part of a ship.

point A as sketched in Figure 9(b). With an abuse of notation, we call this frame as frame A, and the distance between frame A and bay b as d_{A}^{b}. In other words, $d_{\mathrm{A}}^{b}, \forall b \in B$ represents the distance of each downward lightweight distribution force acting on Frame A. Let points B and G represent the center of the buoyancy force and center of lightweight gravity from frame A, respectively. Similarly, d_{A}^{B} and d_{A}^{G} denote the distances between frame A to point B and to point G, respectively. Δ_{A} denotes the buoyancy force being exerted on the illustrated section of the hull. H_{A}^{L} is the lightweight of the ship up to frame A, whereas H_b^C is the cargo weight on bay b. Additionally, let $B_A \subset B$ denote the subset of bays starting from 1 up to bay corresponding to frame A. Therefore, in this illustration, the SF is expressed as:

$$SF_{\mathrm{A}} = \Delta_{\mathrm{A}} - H_{\mathrm{A}}^{L} - \sum_{b \in B_A} H_b^C \tag{1}$$

The BM is formulated as follows:

$$BM_{\mathrm{A}} = \Delta_{\mathrm{A}} d_{\mathrm{A}}^{B} - H_{\mathrm{A}}^{L} d_{\mathrm{A}}^{G} - \sum_{b \in B_A} H_b^C d_{\mathrm{A}}^{b} \tag{2}$$

Since for a given point A, the values Δ_A, H_{A}^{L}, d_{A}^{B}, and d_{A}^{G} are constant (and can be obtained from stability books), we define new parameters $\alpha_{\mathrm{A}} = \Delta_{\mathrm{A}} - H_{\mathrm{A}}^{L}$ and $\beta_{\mathrm{A}} = \Delta_{\mathrm{A}} d_{\mathrm{A}}^{B} - H_{\mathrm{A}}^{L} d_{\mathrm{A}}^{G}$. Next, we rewrite equations (1) and (2) as:

$$SF_{\mathrm{A}} = \alpha_{\mathrm{A}} - \sum_{b \in B_A} H_b^C \tag{3}$$

$$BM_{\mathrm{A}} = \beta_{\mathrm{A}} - \sum_{b \in B_A} H_b^C d_{\mathrm{A}}^{b} \tag{4}$$

When the resulting SF and BM values are within allowable limits, then we may conclude that the current weight distribution is safe for the ship to sail.

F. PROBLEM DEFINITION

Formally, the stowage-planning problem consists of determining how to stow different types of a given set of containers into a set of available container ship locations with minimum loading time. Therefore, the objective of a CSP problem is the minimization of the loading time of all containers, thereby reducing the operational costs. While the turnaround time of a ship includes the time for berthing, unloading, loading, and departure, the major activities affecting the turnaround time are the unloading and loading processes. In particular, loading is the most difficult and sensitive activity affecting the efficiency of the operation [4]. Thus, the main stowing rule is to load containers with destinations for nearer ports in a way that allows unloading of these containers first. When this rule is violated, over-stows can occur resulting in higher costs.

Trim, is another important factor that must be considered since it has a direct impact on the draft of a ship and its ability to sail efficiently. Container ships consume a certain amount of fuel per nautical mile under a specified trim condition, in connection with the draft. Since trim has a substantial impact on the resistance of the ship through water, minimizing it can deliver significant fuel cost savings. Therefore, an effective and efficient loading plan should minimize both unnecessary re-handling of containers and excessive fuel costs due to vessel instability and trim.

In this paper, considering stability constraints (e.g. shear force, bending moment, trim), we seek to determine optimal container stowage plans which yield minimum total costs associated with the over-stows and trimming moments. Our problem definition addresses the bay planning phase of the CSP problem. Each bay normally consists of several holds. The holds between under deck and upper deck are separated by hatch covers. We treat each hold as a single bay and incorporate the hatch in our model. Our formulation seeks to allocate containers to the holds of a container ship such that the number of over-stows and resulting trimming moment is minimized. The number of over-stows directly affects the ship duration at a port and its resulting trim. We further assume that the container ship follows a circular route among multiple ports by unloading all containers destined to one port and loading containers to be transported to the subsequent ports. Since ships navigate in a round-robin tour, some containers will be present on-board at any port, meaning that it never unloads all containers nor moves empty towards the next port. Given this fact, the loading plan for the next port should take the current load configuration of the incoming ship into consideration.

IV. SOLUTION METHODOLOGY

A. ASSUMPTIONS

The stowage-planning problem studied in this paper seeks to determine how to stow different types of a given set of containers into a set of available container ship locations such that the total cost of over-stow and fuel consumption (resulting from vessel instability) is minimized. We denote the set of bays and frames as $b \in B$ and $f \in F$ respectively. Containers, in general, are available in three principal sizes, i.e. 20, 40, and 45 ft., and there are storage sections associated with the size of a container. In our model, we include 20 and 40 ft. dry containers. Since a 40 ft. container occupies two subsequent bays, we model those containers as two 20 ft. containers loaded on adjacent bays. Hence, the model ensures the integrity of 40 ft. containers such that, if the first half of a 40 ft. container is stowed to bay 1, for example, the other half is stowed to bay 2. The weights of 40 ft. containers, on the other hand, are also adjusted in transportation matrices and distributed to a suitable weight category.

In bay assignment problems, considering bays as one dimension (one undivided hold), may give rise to some difficulties in stowing containers to slots in further phases. To prevent the occurrence of this problematic situation, we divided all bays into four parts, i.e., holds, as upper and under decks, assuming the presence of two hatch covers in each bay between these decks. Therefore, we assign containers to either on deck or under deck positions depending on their effects on over-stow and stability. Our model, in this sense, treats each hold as a bay.

A containership transports its cargo between a number of ports denoted by $i, j \in I$ in a round-robin tour, calling at each port. Assuming that a ship starts its tour from port 1, all containers bound for the next ports are loaded. At the second port, it discharges all containers bound for this port and loads containers to be transported to subsequent ports. Delivering containers in a round-robin tour fashion creates a dynamic nature for the problem. To overcome this difficulty, in our study, as also proposed by [23], we assume that the route includes six port visits and for each origin port the planning horizon includes the next three ports along the route. For example, the transport demand originating from port 1 in Figure 10 has destination ports 2, 3 and 4, while the transport demand originated in port 6 has destination ports 1′, 2′ and 3′, and so on. Moreover, we considered that the ship has containers onboard already (loaded at previous ports) when it arrives at the first port. In other words, the ship capacity is not fully available and there are containers onboard bound for port 2 and 3 when it arrives at the first port. To account for this realistic assumption, we denoted previous ports as port 0 and assumed that all pre-loaded containers are placed on the under-deck bays.

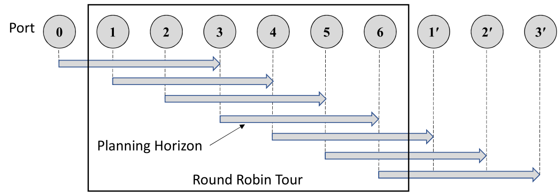

FIGURE 10. Origin and destination ports in a round-robin tour.

TABLE 4. List of basic assumptions.

Model Entity	Basic Assumptions
Containers	- Containers can exist in 20 or 40 ft. sizes
	- Containers are categorized with respect to weight groups
Bays	- Each bay is separated into 4 holds
	- 2 upper and 2 under decks exist
	- 2 hatch covers exist between upper and lower decks
Route	- 6 port round-robin tour is considered
	- Ship starts the journey from port 1
Demand	- Transport demand is available for the next 3 ports
Load	- The ship has containers onboard already at port 1

Container weights are of vital importance to minimize the trimming moments of a ship at any port before departure. An uneven distribution of containers of different weights is the main cause of trim. Since the weights of 20 ft. and 40 ft. containers vary between 2 and 32 tons, including the actual weight of each container makes the problem intractable. To address this issue, we first define set $k \in K$ which denotes the set of weight groups of containers. We represent the weight of a container in group k, by w_k. We further assume that the number of containers for each weight group $k \in K$ to be transported from port i to j (denoted as $i-j$ container) is given by a transportation matrix.

In Table 4 we provide a summary of our basic assumptions grouped with respect to the model entities; containers, bays, ship route, demand and load.

B. MILP FORMULATION

In this section, we describe the MILP formulation for the above mentioned CSP. The sets and indices, parameters, decision variables, objective function and constraints of our MILP formulation are explained below. It should be noted that, for ease of readability, we categorize parameters, decision variables, and constraints with respect to the part of the problem they are related to.

Sets and Indices:

$b \in B$: set of bays
$f \in F$: set of frames

$i, j \in I$: set of ports
$k \in K$: set of container weight groups

Parameters:

Over-stow parameters

C^{os} = Cost of stowing a container [\$/unit]

E^{20}_{bjk} = Number of 20 ft. containers in weight group k loaded before port 1 on bay b and transported to port j [unit]

E^{40}_{bjk} = Number of 40 ft. containers in weight group k loaded before port 1 on bay b and transported to port j [unit]

Stability parameters

C^{tm} = Additional fuel cost per mile resulting from ship stability maintenance [\$/(ton meter $\times$ nm)]. This cost is associated with the change in the draft. If all containers are loaded evenly, that is with 0 trimming moment, then a mean draft value occurs in the center. However, if the distribution is not even, fore and/or aft drafts can be beyond allowable ranges due to large trimming moment values. In such cases, engineers fill the ballast tanks to keep the drafts within limits and ensure stability. This implies that the ship takes extra load and attains a higher draft value. This increased draft actually contributes to higher fuel consumption.

SF^{max}_f = Allowable SF at frame f [ton]

BM^{max}_f = Allowable BM at frame f [ton meter]

d_b = Distance from bay b to the LCG of the ship [meter]

d^L_f = Distance from frame f to the LCG of the lightweight ship [meters]

d^C_f = Distance from frame f to the LCG of cargo up to frame f [meters]

d^B_{fi} = Distance from frame f to the longitudinal center of buoyancy of the hull at port i [meters]

D = Number of bays at the fore of the LCG [unit]

H^L_f = The light weight of the ship up to frame f [ton]

M^{fore} = The maximum moment that ballast tanks at the fore of the LCG can create [ton meter]

M^{aft} = The maximum moment that ballast tanks at the aft of the LCG can create [ton meter]

U = Number of upper-deck or under-deck bays [unit] (we assume that there are the same number of lower-deck and upper-deck bays)

w_k = Weight of a container in group k [ton]

W_b = Weight capacity of bay b [ton]

α_{fi} = The difference between buoyancy force and lightship forming at frame f and port i. That is, $\alpha_{fi} = \Delta_{fi} - H^L_f, \forall f \in F, i \in I$. The value of this parameter can be obtained from ship stability books by taking into account the draft value that can occur at port i with the containers described in the transportation matrix.

β_{fi} = The difference between moments created by buoyancy force and lightship up to frame f. That is, $\beta_{fi} = \Delta_{fi} d^B_{fi} - H^L_f d^L_f, \forall f \in F, i \in I$. The value of this parameter can be obtained from ship stability books.

Δ_{fi} = Displacement (or buoyancy) force up to frame f from the bow of the ship at port i [ton]

s_{ij} = Distance between ports i and j [nm]

Other Parameters

N^{20}_i = Number of 20 ft. containers onboard after loading operation at port i [unit]

N^{40}_i = Number of 40 ft. containers onboard after loading operation at port i [unit]

Q_b = Container capacity of bay b [unit]

T^{20}_{ijk} = Number of 20 ft. containers transported from port i to j in weight group k (for simplicity, containers transported from port i to j are called as $i-j$ containers) [unit]

T^{40}_{ijk} = Number of 40 ft. containers transported from port i to j in weight group k [unit]

Decision Variables:

Decision variables related to over-stows

X^{20}_{bijk} = Number of 20 ft. $i-j$ containers in weight group k assigned to bay b [unit]

X^{40}_{bijk} = Number of 40 ft. $i-j$ containers in weight group k assigned to bay b [unit]

Y_{bij} = 1, if there is any container being transported from port i to j in bay b; 0, otherwise

π_{bi} = Number of over-stows that might occur on a single bay b (either upper deck or under deck bays) at port i [unit]

λ_{bi} = Number of over-stows that might occur between upper deck and under deck bays at port i [unit]

ρ_{bi} = Number of replicating over-stows that might occur between the upper deck and the under deck bays at port i [unit]

Decision variables related to stability

H^C_{fi} = The cargo weight of the ship up to frame f at port i [ton]

M_i = Actual trimming moment that occurs at port i [ton meter]

$\bar{M}_i$ = Absolute value of the trimming moment that occurs at port i [ton meter]

Z_{bi} = Total weight of containers loaded on bay b at port i [ton]

Objective function:

$$\min_{\pi,\lambda,\rho,\mathbf{M},\mathbf{X^{20}},\mathbf{X^{40}},\mathbf{Y},\mathbf{H^C},\mathbf{Z}} \sum_{i=0}^{|I|-1} \sum_{b=1}^{|B|} C^{os}(\pi_{bi} + \lambda_{bi} + \rho_{bi}) + \sum_{i=1}^{|I|-2} s_{i(i+1)} C^{tm} \bar{M}_i \quad (5)$$

Constraints:

Over-Stow Constraints

$$\sum_{i'=0}^{i} \sum_{j=i+1}^{|I|-1} \sum_{k=1}^{|K|} X^{20}_{bi'jk} + \sum_{i'=0}^{i} \sum_{j=i+1}^{|I|-1} \sum_{k=1}^{|K|} X^{40}_{bi'jk} \le Q_b,$$

$$\forall i \in I, b \in B \tag{6}$$

$$X^{40}_{bijk} = X^{40}_{(b+1)ijk}, \quad \forall \{b \in B \,|\, \mathrm{mod}(b,2) = 1\}, \quad \{i,j \in I \,|\, i < j\},\ k \in K \tag{7}$$

$$X^{20}_{b0jk} = E^{20}_{bjk}, \quad \forall b \in B, j \in I,\ k \in K \tag{8}$$

$$X^{40}_{b0jk} = E^{40}_{bjk}, \quad \forall b \in B, j \in I,\ k \in K \tag{9}$$

$$\sum_{b=1}^{|B|} X^{20}_{bijk} = T^{20}_{ijk}, \quad \forall i,j \in I \,|\, i < j,\ k \in K \tag{10}$$

$$\sum_{b=1}^{|B|} X^{40}_{bijk} = T^{40}_{ijk}, \quad \forall i,j \in I \,|\, i < j,\ k \in K \tag{11}$$

$$\sum_{i'=i+1}^{j-1} \sum_{i''=j+1}^{|I|-1} \sum_{k=1}^{|K|} (X^{20}_{bi'i''k} + X^{40}_{bi'i''k}) + (Y_{bij} - 1)Q_b \leq \pi_{bj}, \quad \forall b \in B,\ \{i,j \in I | i < j\} \tag{12}$$

$$(Y_{bij} - 1)Q_{(b+U)} + \sum_{i'=i+1}^{j-1} \sum_{i''=j+1}^{|I|-1} \sum_{k=1}^{|K|} (X^{20}_{(b+U)i'i''k} + X^{40}_{(b+U)i'i''k}) \leq \lambda_{(b+U)j}, \quad \forall b \in B, \{i,j \in I \,|\, i < j\} \tag{13}$$

$$(Y_{bi'i''} - 1)Q_{(b+U)} + \sum_{k=1}^{|K|} (X^{20}_{(b+U)ijk} + X^{40}_{(b+U)ijk}) \leq \rho_{(b+U)i'}, \quad \forall b \in B,\ \{i, i', i'', j \in I \,|\, i < j, i+1 \leq i' \leq j-1, i+2 \leq i'' \leq |I|-1\} \tag{14}$$

$$\sum_{k=1}^{|K|} (X^{20}_{bijk} + X^{40}_{bijk}) \leq Q_b Y_{bij}, \quad \forall b \in B,\ \{i,j \in I \,|\, i < j\} \tag{15}$$

Stability Constraints

$$\sum_{i'=0}^{i} \sum_{k=1}^{|K|} \sum_{j=i+1}^{|I|-1} w_k (X^{20}_{bi'jk} + X^{40}_{bi'jk}) = Z_{bi}, \quad \forall b \in B, i \in I \tag{16}$$

$$\sum_{b=D+1}^{|B|} Z_{bi} d_b - \sum_{b=1}^{D} Z_{bi} d_b = M_i, \quad \forall i \in I \tag{17}$$

$$Z_{bi} \leq W_b, \quad \forall b \in B,\ i \in I \tag{18}$$

$$M_i \leq M^{fore}, \quad \forall i \in I \tag{19}$$

$$-M_i \leq M^{aft}, \quad \forall i \in I \tag{20}$$

$$\alpha_{fi} - H^C_{fi} \leq SF^{\max}_f, \quad \forall f \in F,\ i \in I \tag{21}$$

$$\beta_{fi} - H^C_{fi} d^C_f \leq BM^{\max}_f, \quad \forall f \in F,\ i \in I \tag{22}$$

$$H^C_{fi} = \sum_{b=1}^{2f} Z_{bi} + \sum_{b=1+(U/2)}^{2f+(U/2)} Z_{bi} + \sum_{b=1+U}^{2f+U} Z_{bi} + \sum_{b=1+U+(U/2)}^{2f+U+(U/2)} Z_{bi}, \quad \forall f \in F,\ i \in I \tag{23}$$

Other Constraints

$$\sum_{i'=0}^{i} \sum_{j=i+1}^{|I|-1} \sum_{k=1}^{|K|} \sum_{b=1}^{|B|} X^{20}_{bi'jk} = {}^{20}_{i}, \quad \forall i \in I \tag{24}$$

$$\frac{1}{2} \sum_{i'=0}^{i} \sum_{j=i+1}^{|I|-1} \sum_{k=1}^{|K|} \sum_{b=1}^{|B|} X^{40}_{bi'jk} = N^{40}_i, \quad \forall i \in I \tag{25}$$

Variable types

$$\bar{M}_i \geq M_i, \quad \forall i \in I \tag{26}$$

$$\bar{M}_i \geq -M_i, \quad \forall i \in I \tag{27}$$

$$X^{20}_{bijk}, X^{40}_{bijk} \geq 0, \quad \forall b \in B, i,j \in I,\ k \in K \tag{28}$$

$$Y_{bij} \in \{0,1\}, \quad ; \forall b \in B, i,j \in I \tag{29}$$

$$\pi_{bi},\ \lambda_{bi},\ \rho_{bi}, Z_{bi} \geq 0,\ \ b \in B,\ \ i \in I \tag{30}$$

$$M_i \text{ is unrestricted in sign}, \quad \forall i \in I \tag{31}$$

$$\bar{M}_i \geq 0, \quad \forall i \in I \tag{32}$$

$$H^C_{fi} \geq 0, \quad \forall f \in F,\ i \in I \tag{33}$$

The objective function (5) has two terms. The first term seeks to minimize the total cost associated with the number of over-stows. We assume that the stowage cost of a container, C^{os}, is fixed and known. To clarify how decision variables π, λ and ρ determine different types of over-stows, refer to the example in Figure 11. In this example, each figure represents a single bay b and the containers transported from port 1 to 4 ($1-4$ containers) have already been loaded. In this single bay if $2-5$, $2-6$, $3-5$, and $3-6$ containers are to be loaded on top of the $1-4$ containers, over-stows occur (see Figure 11(a)). Decision variable π_{bi} corresponds to over-stow instances for bay $b \in B$ at port $i \in I$ that might be either an upper deck or under deck bay, which does not require lifting the hatch cover as depicted in Figure 11(a). Figure 11(b), on the other hand, shows two overlaid bays, i.e., upper and under deck bays separated by a hatch-cover. Note that $1-4$ containers have been placed on the under-deck bay. Stowing the 1-5, 1-6, 2-5, 2-6, 3-5, and $3-6$ containers on the upper-deck bays creates inter-bay over-stows that are represented by $\lambda_{bi}, \forall b \in B, i \in I$. Lastly, in Figure 11(c), the $1-4$ containers have been placed on the upper deck bays. Stowing the $2-3$, $2-4$, $2-5$, $2-6$, $3-4$, $3-5$, and $3-6$ containers on the under deck bays creates replicating over-stows, i.e. the over-stow replicates two times at ports 2 and 3, and such over-stows are denoted by the decision variable $\rho_{bi}, \forall b \in B, i \in I$.

To be more clear, the decision variable π is used to count the containers loaded on top of the $i-j$ containers between ports $(i+1)$ and $(j-1)$ bound for ports between $(j+1)$ and $(|I|-1)$. For example, consider the bay shown in Figure 12. Assume that there is only one $1-4$ container already loaded at port 1. If the model places three more $2-5$ containers on this bay at port 2, it perceives that three over-stows occur as depicted in Figure 12(a). Although three over-stows occur technically on this bay, these three over-stows can be eliminated in the slot planning phase by placing the $2-5$ containers onto adjacent slots as shown in Figure 12(b).

To illustrate how π_{bj} functions, refer to the exemplary network representation of the bay plan (similar to the approach implemented in [25]) in Figure 13. The figure illustrates the transportation scheme for 20 ft. containers of weight group

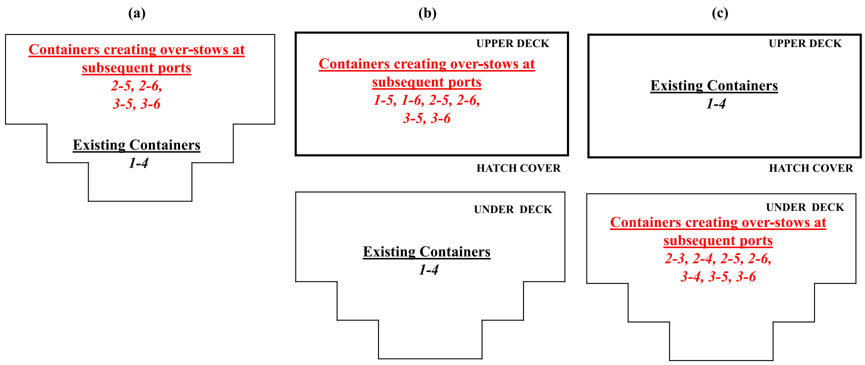

FIGURE 11. Over-stow instances.

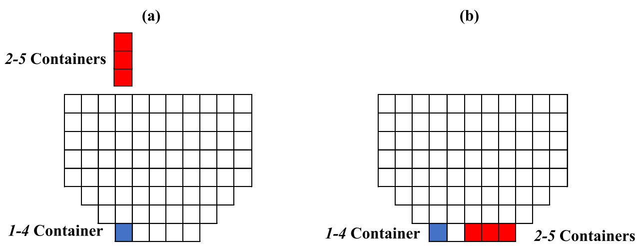

FIGURE 12. In-bay over-stow.

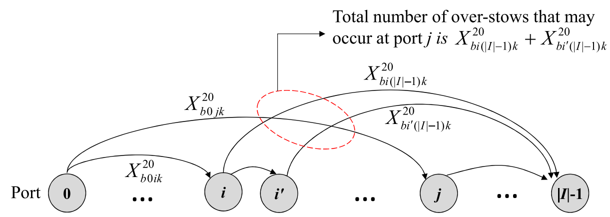

FIGURE 13. Network representation of the bay plan for bay b.

k at bay b. In particular starting from Port 0 to Port $|I|-1$, each arc in this network represents the presence of containers in weight group k being transported between corresponding ports on bay b. In the example, there are containers already loaded to be transported between ports $0-i$, $0-j$, $i-i'$, $i-(|I|-1)$, $i'-(|I|-1)$ and $j-(|I|-1)$. A closer look at the figure reveals that the maximum over-stow that can occur at port j is $X^{20}_{bi(|I|-1)k} + X^{20}_{bi'(|I|-1)k}$. There is an arc from 0 to j, which means that there are containers loaded at port 0 to be discharged at port j. The ship loads all these $0-j$ containers at port 0 and moves to subsequent ports. Once it arrives at port i, it first unloads $0-i$ containers and then loads the, $i-i'$ and $i-(|I|-1)$ containers on top those loaded at port 0, i.e., on top of $0-j$ containers. At port i', $i'-(|I|-1)$ containers are also loaded on top of $0-j$ containers following the discharge of $i-i'$ containers. Finally, when the ship arrives at port j, all $0-j$ containers must be unloaded. However, $i-(|I|-1)$ and $i'-(|I|-1)$ containers have been placed on top of the $0-j$ containers at previous ports. Therefore, $i-(|I|-1)$ and $i'-(|I|-1)$ containers must be temporarily removed first to freely discharge the $0-j$ containers and then placed back onto the ship. In this case, it is clear that the number of extra movements will be $X^{20}_{bi(|I|-1)k} + X^{20}_{bi'(|I|-1)k}$. However, from the definition, it is known that $X^{20}_{bi(|I|-1)k}$ and $X^{20}_{bi'(|I|-1)k}$ are the number of 20 ft. $i-(|I|-1)$ and $i'-(|I|-1)$ containers assigned to bay b. Hence, for this specific example, the exact number of the first type of over-stows for 20 ft. containers in weight group k occurring at port j for bay b is $\pi_{bj} = X^{20}_{bi(|I|-1)k} + X^{20}_{bi'(|I|-1)k}$.

In the second term of the objective function, we minimize the total additional fuel cost that will occur during the round-robin tour as a result of any ship instability. We can obtain the total weight of the containers onboard at port i from the transportation matrix. If containers are distributed evenly, that is with 0 trimming moment value, this weight results in an additional increase in the draft, which is unavoidable. This draft is the final and optimal draft in which the ship should sail. Therefore, the operating cost resulting from this draft is inevitable for the agencies. However, if containers are not evenly distributed, then a certain amount of trimming moment occurs. In order to rebalance the ship, engineers take ballast. This ballast operation brings extra weight to the ship, thereby yielding an increment in the existing draft. The model minimizes the cost incurred by this increment in the draft value. This cost is calculated for each port before departure. Similarly, we assume that the cost C^{tm} is fixed and known. Also note that if it is not equal to 0, M_i takes either a positive or negative value depending on the side of the decisive moment. Therefore, we consider its absolute value in the objective function and linearize it by introducing the decision variable $\bar{M}_i$ and constraint sets (26) and (27).

Constraint set (6) guarantees the total usage of any bay to be less than or equal to the capacity of that bay. In the first part of the constraint, the 20 ft. containers are considered, and in the latter part, the 40 ft. containers assigned to bay b are considered together. Starting from port 0, the constraint set takes into account the pre-loaded containers. In this sense, this constraint counts all the containers assigned to bay $b \in B$ at each port and ensures that the total number of containers will not exceed the capacity of the corresponding bay. As an example, suppose that a container ship travels among 6 ports and the ship is currently at port 2. For a particular bay b, the number of containers loaded at previous ports (i.e. port 0 and port 1) and at the current port (i.e. port 2) cannot exceed the capacity of the bay. In other words, the total number of containers loaded at previous ports (i.e. containers 0-3, 0-4, 0-5, 0-6, 1-3, 1-4, 1-5, 1-6) and at the current port (i.e. containers 2-3, 2-4, 2-5, 2-6) must be less than or equal to the capacity of bay b. It should be noted that, as discussed in Section III-A, if a 40 ft. container is stowed in a bay, its adjacent bay is not available for stowing containers with a size of 20 ft. Our formulation considers a 40 ft. container as two adjacent 20 ft. containers. Constraint set (7) ensures the integrity of 40 ft. containers in such a way that if the first half of a 40 ft. container is assigned to an odd-numbered bay, then the other half is assigned to the adjacent even-numbered bay. The modulo operation, denoted by the mod $(b, 2)$ function, returns the remainder after division of b by 2. Hence, only two adjacent bays, an odd-numbered bay and its even-numbered next bay can be assigned with a 40 ft. container.

Constraint sets (8) and (9) deal with the pre-loaded containers of size 20 ft. and 40 ft., respectively. These containers are the ones that were loaded at previous ports (denoted by port 0) and are already occupying some parts of the corresponding bays. As the locations of these containers were determined prior to the ship's arrival at the first port and their precise locations are known, our model sets the initial number of containers to parameters E^{20}_{bjk} and E^{40}_{bjk}. Constraint sets (10) and (11) ensure that all containers of size 20 ft. and 40 ft. in the transportation matrix are stowed. Constraint set (12) determines the maximum number of first type over-stows that may occur on a single bay $b \in B$ at port $j \in I$ and defines the upper bound for π_{bj}. Similarly, constraint set (13) determines the maximum number of over-stows that may occur between the upper and under deck bays (separated by a hatch-cover) and defines the upper limit for the second type of over-stow. In this constraint set, each bay is investigated with its upper neighbor deck bay, i.e., $b + U$ in the constraint set refers to the upper neighbor bay of bay $b \in B$. Constraint set (14) deals with replicating over-stows described in Figure 11(c). It indicates that, if there is any $i - j$ container available on the upper deck, then loading a container to the under deck at ports between $i + 1$ and $j - 1$ will create over-stows as many as the number of $i - j$ containers. Moreover, this over-stow case may replicate itself $j - (i + 1)$ times depending on the existence of $(i + 1) - (j - 1)$ containers.

Constraint set (15) interrelates variables X^{20}_{bijk} and X^{40}_{bijk} with the binary variable Y_{bij}. Moreover, this constraint set, together with constraint sets (12)-(14), determines the maximum number of over-stows that may occur on a bay $b \in B$. Constraint set (16) determines the total weight of each bay $b \in B$ after the loading operation at port $i \in I$. Constraint set (17) calculates the trimming moment at each port $i \in I$.

As mentioned in Section 1, each bay consists of tiers or stacks. These stacks have different weight capacities, which determine the maximum number of containers in various weights that they can hold. If the total weight of containers assigned to bay $b \in B$ is greater than the maximum weight capacity of all stacks in that bay, then the slot planning procedure may end up as infeasible. To avoid such a situation, we compute the bay weight capacity by summing all stack weight capacities in that bay based on their upper or under deck positions. In this sense, the constraint set (18) ensures that these bay capacities are not exceeded.

For a containership gaining a trim by fore, the trimming moment will be negative, and the planners must utilize aft ballast tanks to stabilize the ship. However, for the ballast condition to work, the moment created by aft ballast tanks must be greater than the current trimming moment. Therefore, constraint sets (19) and (20) ensure that the trimming moments are under allowable limits so that they can be balanced with fore and aft ballast tanks, respectively. Constraint sets (21) and (22) guarantee that the shear force and the bending moment at any frame are within their allowable limits. As we consider a single bay consisting of four holds, each weight in the holds must be taken into consideration for the shear force calculation. Hence, constraint sets (21) and (22) together ensure a homogenous weight distribution along the ship length and that the stress factors are under design levels. Constraint set (23) calculates the weight of all containers loaded onto the bays up to frame f. Constraint sets (24) and (25) count the number of 20 ft. and 40 ft. containers onboard just prior to leaving port $i \in I$, and are used for KPI purpose. Constraint sets (26) and (27) ensure that the trimming moment takes only positive values. Finally, constraint sets (28)- (33) declare variable domains.

C. TWO-STAGE HEURISTIC METHODOLOGY

The MILP formulation described in the previous section produces a CSP by minimizing the total costs associated with the over-stows and trimming moments together. For large-size problems, however, the MILP may not produce an optimal solution within a reasonable solution time. For such cases, several heuristics, meta-heuristics, evolutionary algorithms, and variants of linear programming approaches that are capable of generating efficient solutions within relatively short CPU times have been successfully applied by many researchers (e.g. [13], [29]–[31], [33]), as also summarized in Table 1. Similarly, we propose a two-stage heuristic solution methodology which includes an IP formulation in stage 1 and SH algorithm in stage 2. The IP model produces an initial CSP by minimizing the total cost of over-stows without considering the trimming moments, hence, it determines the lower bound for the over-stow cost within a short computing time. The SH algorithm of stage 2 adopts this initial CSP as input and minimizes the fuel costs without violating the optimality provided by the IP model in terms of over-stows.

1) STAGE 1: IP FORMULATION

Our IP formulation seeks to minimize only the total cost of over-stows and is formulated as follows:

$$\min_{\pi,\lambda,\mathbf{M},\mathbf{X^{20}},\mathbf{X^{40}},\mathbf{Y},\mathbf{H^{C}},\mathbf{Z}} \sum_{i=0}^{|I|-1} \sum_{b=1}^{|B|} c^{os}(\pi_{bi} + \lambda_{bi} + \rho_{bi})$$
$$\text{subject to Constraints (6)-(23) and (28)-(33)} \quad (34)$$

The objective function (34) of the IP model consists of solely over-stow variables and relaxes constraints related to trimming moments.

2) STAGE 2: SWAPPING HEURISTIC ALGORITHM

The IP model in stage 1 determines the initial stowage plan, i.e. number of $i-j$ containers of weight group $k \in K$ are assigned to bay $b \in B$. Our proposed heuristic algorithm minimizes the cost associated with the trimming moments by ensuring that the lower bound of the over-stow provided by the IP model is retained while maintaining the stress level below its allowable limits. To understand the structure of our heuristic approach, imagine a containership with bays numbered 1 to $|B|$, 1 at the fore and $|B|$ at the aft. Suppose also that $X^{20}_{1ij1} = m$ and $X^{20}_{|B|ij4} = n$ as a result of the initial stowage plan. These variables imply that m of 20 ft. $(i-j)$ containers of weight w_1 are placed on bay 1, whereas n of 20 ft. $(i-j)$ containers of weight w_4 are placed on bay $|B|$. Because both container groups are to be transported between ports i and j, swapping two containers in bay 1 and bay $|B|$ will not change the number of over-stows obtained by the IP model. After the swapping operation, $X^{20}_{1ij1} = m-1$, $X^{20}_{1ij4} = 1$, $X^{20}_{|B|ij1} = 1$ and $X^{20}_{|B|ij4} = n-1$. Furthermore, this swapping operation between $(i-j)$ containers in different weight groups may decrease the trimming moment, hence the additional fuel cost, as bay 1 and bay $|B|$ are at opposite ends of the ship according to the LCG point. The difference between the former and latter moments associated with this change can be accepted if the trimming moment is decreased while the SFs and BMs remain within allowable limits.

For a numerical example, consider the ship in Figure 14. Suppose that the current trim before loading the two containers (one is 20 and the other is 30 tons) bound for the same destination, is 0. In the first case (Figure 14(a)) the 20 ton container is placed on a bay 100 meters away from the LCG toward the aft and the other on a bay 150 meters away from the LCG toward the fore. With this configuration, the trimming moment created by these two containers is −2500 ton meters ($100 \times 20 - 150 \times 30$). This trimming moment will induce the ship to trim by the fore. Now assume that these containers are swapped as displayed in Figure 14(b). In this case, the trimming moment is reduced to 0 ton meters ($100 \times 30 - 150 \times 20$) without deteriorating the over-stow performance. Thus, even one swapping operation may reduce the trimming moment without allowing any extra over-stow and keep the stress parameters at acceptable levels. If the assignments of all containers to bays are examined in this sense, it might be possible to attain a significant improvement in the costs associated with the trimming moments. The essence of the heuristic model basically depends on this swapping operation between $(i-j)$ containers of the same length in different weight groups while maintaining the minimum number of over-stows and stress levels provided by the IP model.

The heuristic algorithm performs the swapping operation starting from the first port, hence, initially $i = 1$. The output of the IP model, which provides the initial bay plan, is used as an input for phase 1. Using this plan, starting from port 1, $(i-j)$ containers $\forall \{i, j \in I \mid i < j\}$ are considered one by one in ascending order of i. The pseudocode for the swapping heuristic is detailed in Algorithm 1.

Algorithm 1 Pseudocode for the Swapping Heuristic

```
Initialization Lock 0-containers i = 1 Tolerance = 0
while i < |I| − 1 do
    counter = 0;
    while counter ≤ iterlim or Trimming Moment ≤
    Tolerance do
        Calculate current trimming moment for port i;
        Randomly select two containers transported
        from port i to j of weight group k;
        Swap the containers;
        Calculate total bay weights for two bays;
        if bay weight capacities are not exceeded then
            Calculate new trimming moment for port i;
            if new trimming moment ≤ current
            trimming moment then
                Calculate SF and BM for corresponding
                frame;
                if SF and BM are below stress limits
                then
                    Save this change;
                    current trimming moment = new
                    trimming moment;
                else
                    Revert swapping operation;
                end
            else
                Revert swapping operation;
            end
        else
            Revert swapping operation;
            counter = counter + 1;
        end
        Lock i-containers;
        i = i + 1;
    end
end
```

One of the key issues that our heuristic model handles is checking the bay weight capacity before the swapping operation. If the existent swapping operation results in a capacity that exceeds the bay weight capacity, the algorithm abandons this swapping operation. The algorithm has two stopping criteria. The first criterion checks whether the resulting trimming moment after the swapping operation is less than the user-defined "tolerance" value (ton meters) indicated in the code. The second criterion of the algorithm is the maximum iteration number denoted by the term "iterlim". This number limits the number of swapping operations and can be chosen arbitrarily by the decision-maker.

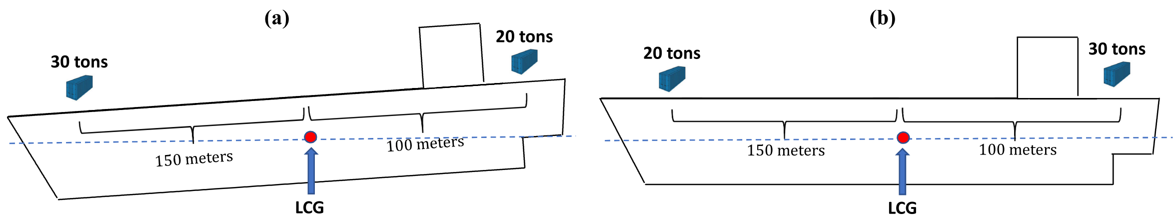

FIGURE 14. Numerical illustration of a containership.

TABLE 5. Transportation matrix for the example problem.

Port (i,j)	1						2						3					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
0	21	7	4	14	4	3	0	2	2	0	0	2	4	4	0	8	2	2
1	-	-	-	-	-	-	3	1	3	3	3	3	3	3	3	3	3	5
2	-	-	-	-	-	-	-	-	-	-	-	-	1	2	0	3	2	4

The algorithm performs swapping operations until either the trimming moment reaches a value less than the tolerance or iterlim times swapping operations are conducted.

V. EXPERIMENTS

In this section, we report the results of our numerical experiments conducted by both the MILP and proposed heuristic algorithm for different problem instances. All experiments were run by a Microsoft Windows 10 64-Bit operated PC with Intel Core i7 6500U CPU @2.50 GHz with 8 GB RAM. The MILP and IP models are implemented in General Algebraic Modeling System (GAMS©) environment and solved with CPLEX®12.5. The proposed SH algorithm is coded in Python 3.6 (PyCharm Community 2017.3).

A. SOLVING AN EXAMPLE PROBLEM

Before proceeding with the details of test problem designs and numerical results, we find it useful to provide a simple exemplary CSP problem and its solution obtained by the MILP.

In this example, we consider a vessel with a capacity of 80 containers divided equally into 4 bays, calling at 3 ports in a round-robin tour. The details of the demand is given in Table 5. In this table, rows represent the originating ports (denoted by index i) while columns represent the destination ports (denoted by index j). The demand is specified as the number of containers transported from port i to port j (simply denoted as $i-j$ containers). All $i-j$ containers are separated into 6 weight groups as {5,10,…,30} tons as shown in the second row. For instance; it can be inferred from the table that, there exists 21 containers of 30 tons to be transported from port 0 to port 1, and similarly, 4 containers of 5 tons to be transported from port 2 to port 3. Additionally, the CSP of port 0 yields a total of -500 ton meters of trimming moment. The trimming moment in a particular port is calculated by using the stability concepts explained in Section III.

Recall that, our model also encompasses pre-loaded containers, i.e. containers that are already loaded on-board before the vessel arrives at port 1. The originating port of these containers is denoted as port 0 and the demand from port 0 is given in the third row (row 0) of the transportation matrix. For example, the table reveals that (row 3 and column 17) there exist 8 containers of 15 tons loaded at any previous port to be transported to port 3. However, since these pre-loaded containers are already on-board, their locations are precise and known. Therefore, the model uses this information to generate the CSP for successive ports. This information is provided in Table 6. In particular, this table depicts the bay location and destination information of all pre-loaded containers along with their weight categories. It can be observed that, there exists 7 containers of different weights in bays 1,2 and 3 with 6 containers in bay 4. A visualization of the pre-loaded containers location scheme is depicted in Figure 15 (a).

Considering these planning factors, the MILP model takes the data presented in Tables 5 and 6 as input and generates a CSP by allocating containers to the available bays such that the objective function (5) is minimized. The details of

TABLE 6. The layout of pre-loaded containers for the example problem.

Bay	Bay capacity	# of containers	Port (i,j)	w_k
1	20	2	0-2	25
		4	0-3	15
		1	0-3	10
2	20	2	0-2	20
		4	0-3	15
		1	0-3	10
3	20	2	0-2	5
		4	0-3	25
		1	0-3	5
4	20	4	0-3	30
		2	0-3	5

TABLE 7. Stowage plan for the example problem.

Bay	Bay capacity	Port 1			Port 2		
		# of containers	Port (i,j)	w_k	# of containers	Port (i,j))	w_k
1	20	2	1-2	30	2	2-3	25
		2	1-3	30	2	2-3	15
		3	1-3	25	2	2-3	10
		1	1-3	20			
2	20	1	1-2	20	1	2-3	15
		3	1-2	15			
		1	1-2	10			
3	20	1	1-2	30	1	2-3	30
		1	1-2	25			
		2	1-2	20			
		2	1-2	10			
		3	1-2	5			
4	20	1	1-3	30			
		2	1-3	20			
		3	1-3	15			
		3	1-3	10			
		5	1-3	5			

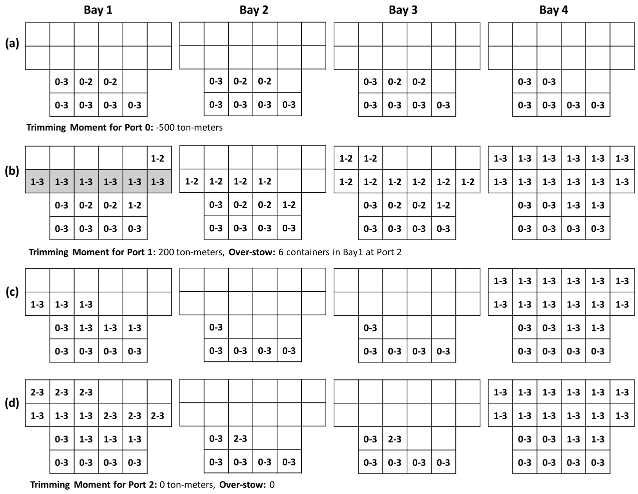

FIGURE 15. Container stowage plans for the example problem. (a) The layout of pre-loaded containers, (b) Port 1 plan after loading 1-*j* containers, (c) Port 2 plan after unloading *i*-2 containers and before loading 2-*j* containers, (d) Port 2 plan after loading 2-*j* containers.

the optimal plan suggested by the MILP is presented in Table 7. The third and fourth columns of the table indicate the number and weight group of containers to be loaded at port 1 and port 2, respectively. For example; in bay 1, there exists 7 pre-loaded containers of which 2 of them are bound for port 2 with the rest destined for port 3. Since the actual locations of these containers are fixed, the $1-2$ and $1-3$ containers need to be loaded on top of them at port 1. Once the vessel arrives at port 2, $0-2$ and $1-2$ containers must be discharged. However, the $1-3$ containers loaded at port 1 are blocking the easy discharge of $0-2$ containers. In that case, planners must first unload six $1-3$ and two $0-2$ containers, then place back the six $1-3$ containers to bay 1. In other words, this operation yields a total of 6 over-stows. These containers are highlighted in Figure 15(b) which shows the port 1 stowage plan after loading the $1-j$ containers. Using a similar approach, the MILP generates stowage plans for each port until the round-robin tour is completed. Figures 15(c) and 15(d) display the CSPs after the completion of unloading and loading operations at port 2, respectively. The resulting trimming moments for port 1 and port 2 are calculated as 200 ton meters and 0 ton meter, respectively.

B. TEST PROBLEMS

In order to evaluate the performance of the MILP and heuristic approaches, we generated seven problem settings of different sizes. The capacity of mid and large-size containerships operating worldwide vary between 6,000 and 20,000 TEUs and thus we have considered the CSP problem for 6,000, 8,000, 10,000, 12,000, 14,000 and 20,000 TEU ships traveling through six ports. For each configuration, we generate and solve 20 random replications, yielding a total of 120 instances.

The details of the vessel and container data for all ship capacities are reported in Table 8. We assume that ships are loaded with 20 and 40 ft. size containers. The first column is the ship capacity in TEUs. The second and third columns report the number of bays and holds, respectively. Note that each hold consists of 4 bays. Columns four and five report the number of bays at the fore of the LCG point and number of upper (or lower) deck bays, respectively. To generate realistic test instances, we assumed that the ship is already loaded with a number of containers (at previous ports denoted as port 0) when it arrives at the first port. Hence, the number of pre-loaded containers for each ship configuration is given in the sixth column of the table. Finally, the seventh and eight columns are the number of 20 ft. and 40 ft. containers to be loaded, respectively.

In all runs we use six different container weight groups, i.e. $|K| = 6$, and set the weight of containers in group $k \in K$ as $w_1 = 30$, $w_2 = 25$, $w_3 = 20$, $w_4 = 15$, $w_5 = 10$, and $w_6 = 5$ tons. Bay weight capacities (W_b) are applied between 750 and 3800 tons, depending on their container capacities. We set the values of cost parameters as $C^{os} =$ \$300 and $C^{tm} = 1.5$ \$/(ton meter $\times$ nm). The M^{fore} and M^{aft} values are fixed to 3×10^5 ton meters. The s_{ij} values are determined random uniformly between values 100 and 2000 nm. We generate the transportation matrices for a number of $i-j$ containers of 6 different weight categories that are produced randomly according to the problem size and limits. In Appendix, we provide the input data for a small size example problem.

For the MILP and IP models, we set the relative optimality gap to zero and maximal computational time to 1 hour for 6,000 to 10,000 TEU instances, and 2 hours for 12,000 to 20,000 TEU instances. Recall that the SH algorithm has two user-defined stopping criteria, i.e. tolerance value, and maximum iteration number (iterlim). These stopping criteria have an impact on both the quality of the solutions and solution time. Low tolerance and high iterlim value yield high-quality results at the cost of increased computation times. Our initial trials have revealed that the iterlim value should be at least twice the container capacity of the ship, and the tolerance value could be chosen with respect to the desired balance between the efficiency of the algorithm in terms of both the solution quality and computation time. In our experiments, the tolerance value is applied as 50 ton meters and the iterlim value is set as twice the container capacity of the ship. For example, if the ship capacity is 6,000 TEUs, then iterlim is 12,000, i.e. the algorithm performs at most 12,000 swapping operations. Eventually, whichever stopping criterion is reached first, the algorithm terminates the swapping operation and saves the results for that port. Then it increases the reference port by one and continues in that fashion until it reaches the final port.

C. RESULTS AND DISCUSSION

Our numerical experiments aim to compare the performance of the MILP formulation and the proposed heuristic in terms of solution quality and computation time. For each approach, Table 9 reports the solution time in seconds, total number

TABLE 8. Vessel and container configurations.

Ship capacity (TEUs)	# of bays	# of holds	D	U	# of pre-loaded containers	# of 20 ft. containers	# of 40 ft. containers
6,000	25	100 (25 × 4)	48	50	745	[2,000, 3,000]	750
8,000	28	112 (28 × 4)	56	56	1,000	[4,000, 4,500]	1,000
10,000	32	128 (32 × 4)	64	64	1,500	[4,500, 5,000]	1,250
12,000	35	140 (35 × 4)	68	70	2,000	[5,000, 6,000]	1,500
14,000	39	156 (39 × 4)	78	78	2,000	[6,000, 8,000]	2,000
20,000	48	192 (48 × 4)	96	96	3,000	[8,000, 10,000]	3,000

of over-stows (unit), total trimming moment (ton meter), optimal objective function value, and optimality gaps (%). In detail, the first two columns display the problem size (in TEUs) and instance number. Columns 3 to 6 report the solution times of the MILP, IP, SH, and the sum of IP and SH (total computation time of the heuristic), respectively. Columns 7 to 9 report the total number of over-stows obtained after solving the problem with MILP, IP, and SH. Note that, since the total number of over-stows obtained from the IP is retained by the SH, the values for IP and SH are the same in columns 8 and 9. Similarly, columns 10 to 12 report the total trim-moment obtained by the MILP, IP and SH. As expected, since the IP does not seek to minimize the trim-moment, its respective moment values are much larger than those of the MILP's. However, the moments are minimized by the SH at the second stage of the heuristic. Finally, columns 13 to 14 report the gaps. It should be noted that gaps for the MILP are the values reported by the CPLEX. Gaps for the SH are calculated with respect to the best bound obtained from the MILP. It is also worth mentioning that, to test whether our numerical results comply with the normal distribution, we have carried out a statistical analysis for the solution time, over-stow, trimming-moment values and gaps. Upon investigating parameters such as the skewness and kurtosis, and implementing the Shapiro–Wilk test, we observed that these results comply with the normal distribution. Hence, in Table 9 we have provided both the mean and standard deviation of the 20 realized instances for each problem size.

The results in Table 9 show that the MILP cannot achieve optimal solutions within the pre-specified time limit (1 hour for instances with 6,000 to 10,000 TEU instances, 2 hours for instances with 12,000 to 20,000 TEU instances) and in the overall, performs worse than the two-stage heuristic methodology in terms of both solution time and quality. The heuristic approach, on the other hand, yields higher-quality solutions within satisfactory solution times. In specific, the solution times of the individual IP and the SH vary between 2 to 4 minutes, and the total computation time for the heuristic is always less than 8 minutes. The average number of over-stows in the MILP solutions vary between 90 and 635. Recall that the IP model first acquires an initial bay plan in which only the over-stow costs are minimized. The table reveals that all instances can be solved to optimality (zero gap) within short computation times and the resulting average number of over-stows vary between 26 and 148. As expected, the number of over-stows tend to increase with the problem size in both MILP and heuristic solutions.

The average trimming moments for the MILP solutions vary between 1875 ton meters (with a standard deviation of 324 ton meters for 6,000 TEU problem size) and 4713 ton meters (with a standard deviation of 596 ton meters 20,000 TEU problem size). As expected in the initial plan obtained from the IP, trimming moments appear to be relatively high, i.e. between 972,783 and 1,457,668 ton meters. After generating the initial bay plan, the SH algorithm adopts this plan and processes it in a way that minimizes the trimming moments by sticking to the lower bounds of over-stows. For this reason, in all instances, the over-stow numbers for the SH are always equal to their lower bounds obtained by the IP. The results show that the SH algorithm is capable of decreasing the trimming moments significantly while keeping the number of over-stows at the minimum level. In particular, the average trimming moments at the end of the SH algorithm vary between 408 and 3,200 ton meters. In terms of average values, in 5 out of 6 problem sizes, the resulting moments for the heuristic is less than those reported by the MILP. For example, in the largest problem size with 20,000 TEUs, the MILP yielded an average of 635 over-stows and 4714 ton meters of trimming moment whereas the heuristic yielded 134 over-stows and only 407 ton meters of trimming moment.

Considering the optimality gaps, the results reveal that, the MILP is outperformed by the heuristic approach in all problem sizes. The gaps reported by CPLEX in MILP solutions vary between 54.32% (for problem size of 6,000 TEU) and 87.61% (for problem size of 10,000 TEU). It should also be noted that, although we set the time limit to 2 hours for the 12,000 to 14,000 TEU instances, the average gaps in those instances are approximately 80%. These values tend to increase slightly as the problem size increases. The gaps calculated for the heuristic results, on the other hand, vary between 1% and 24%. We observe that, for all problem sizes, the heuristic is capable of returning solutions with smaller gaps in shorter CPU times.

In addition to our numerical comparison, we have also carried out a non-parametric 2-related Wilcoxon test [42] (a statistical hypothesis test used to compare two related samples) to compare the performance of the MILP and heuristic algorithm. The Wilcoxon test returns a pooled ranking of all observed differences to determine if the sets are different in a statistically significant manner. A significance level less

TABLE 9. Computation results for the MILP and the two-stage heuristic.

#TEU	Result	Solution time (s) MILP	Heuristic IP	Heuristic SH	Heuristic IP+SH	Total number of over-stows (unit) MILP	Heuristic IP	Heuristic SH	Total trim-moment (ton meter) MILP	Heuristic IP	Heuristic SH	Gap(%) MILP	Gap(%) Heuristic
6,000	Average	3,600.00	151.22	135.18	286.40	89.47	26.40	26.40	1,874.70	1,124,589.00	5,356.00	54.32	0.69
	Std.Dev.	0.00	31.14	49.06	53.12	44.09	12.51	12.51	324.40	194,428.30	1,391.41	7.16	0.12
8,000	Average	3,600.00	217.75	200.50	418.25	195.55	49.20	49.20	3,516.15	1,457,668.00	3,200.80	74.86	12.65
	Std.Dev.	0.00	39.84	51.40	64.46	70.52	12.14	12.14	441.30	144,320.00	731.47	7.91	0.87
10,000	Average	3,600.00	262.70	208.70	471.40	314.65	103.60	103.60	3,999.50	1,520,544.00	424.30	87.61	13.62
	Std.Dev.	0.00	81.37	33.13	80.61	85.06	7.54	7.54	630.70	120,386.00	48.21	9.27	2.12
12,000	Average	7,200.00	171.10	201.10	372.20	480.25	96.10	96.10	3,624.30	972,783.00	429.75	81.82	10.67
	Std.Dev.	0.00	44.81	55.31	118.34	146.54	7.14	7.14	676.70	138,765.00	109.70	8.57	1.41
14,000	Average	7,200.00	193.75	232.20	425.95	603.80	147.40	147.40	2,997.90	1,325,516.00	1,277.85	83.66	24.10
	Std.Dev.	0.00	100.11	61.26	174.68	179.10	20.50	20.50	604.60	68,290.80	348.90	9.11	3.63
20,000	Average	7,200.00	213.05	240.45	453.50	635.10	133.90	133.90	4713.70	979,639.00	407.90	78.88	15.49
	Std.Dev.	0.00	14.78	34.61	31.30	119.03	25.15	25.15	595.84	252,947.00	166.46	8.04	1.73

than 0.05 indicates that there is significant difference between data. Since the heuristic outperforms the MILP in terms of solution time, number of over-stows and optimality gap for all 120 instances, we have only compared the performance of the two solution approaches in terms of the trimming moment values. Our test results for the trimming moment indicate that the MILP algorithm outperforms SH for small-size problem instances. In particular, for the 6,000 TEU, the mean rank of SH is considerably higher than the that of MILP with a significance level close to zero. For the 8,000 TEU instances, the mean ranks appear to be close (11.86 and 8.83, for the MILP and SH, respectively) with a significance level of 0.34. This result implies that, for the 8,000 TEU case, there is not enough evidence to assert that there exists statistical difference between these two solution approaches. For larger size problem instances, on the other hand, the test results show that there is statistical difference in favor of the SH. Since all significance levels computed for the 10,000, 12,000, 14,000 and 20,000 TEU instances are close to zero and the ranks returned for the MILP are considerably higher than those of the SH algorithm, we conclude that the SH outperforms the MILP.

Therefore, the overall results of numerical experiments and statistical analysis indicate that the proposed heuristic, compared to the MILP formulation, yields higher-quality bay plans within reasonable computation times. Moreover, the resulting moments for each instance are considered relatively low for real-world containerships operations. In that sense, our heuristic algorithm can be applied to real-world CSP cases even when the size of the containerships is very large. The next step is the slot assignment process. If the bay plan satisfies as many requirements as possible, it is likely to end up with a good slot assignment performance in terms of accuracy, CPU time, and solution quality. In the slot assignment phase, the most important parameter to be considered is the transversal stability of the ship. In that respect, our proposed algorithm provided good results that can be processed efficiently in the slot assignment phase.

Note that the running time and proximity of resulting trimming moments to their lower bounds also depend on the quality of the initial bay plan in terms of trimming moments, i.e., if the trimming moments obtained from the IP model are relatively low, the SH algorithm spends less time bringing them down to their lower bounds. This is provided by constraint sets (19) and (20) which ensure that the moments are always under a certain value, so the ship can be stabilized with ballast operation in case the SH algorithm fails to produce satisfactory results.

VI. CONCLUSION

As the size of containerships escalates, the number of transported containers increases considerably. The loading and unloading of large quantities of containers require efficient port design and port operations, and an efficient CSP. The CSP is the placement of containers onto the ship in such a way that container movement is minimized, i.e., minimum over-stow. The CSP must also retain the stability of ships while ensuring minimum over-stow. The CSP problem is proven to be NP-complete. Nevertheless, researchers have developed some algorithms to create an efficient solution to the CSP. However, these algorithms use simplistic assumptions that are impractical to implement. In particular, stability parameters are not addressed in most of the algorithms.

In this study, we considered the CSP problem with stability restrictions and proposed a MILP formulation which seeks to avoid costs due to over-stows and ship instability. We then proposed a two-stage heuristic algorithm which acquires a lower bound on the total over-stow cost by an IP formulation and, uses the SH algorithm to minimize additional fuel cost resulting from the ship instability. Our study achieves an important improvement in this area and makes significant contributions to the field, since we include stability parameters in our formulations as well as other realistic features and constraints. The cost of eliminating the trim caused by improper loading is very high compared to that of heeling and an odd trim poses a higher risk. Consequently, we consider the trim rather than heeling as the most important stability parameter in our models.

We tested the proposed MILP and heuristic algorithm on different size problem instances each having six port visits and for each origin port, the possible destinations correspond

TABLE 10. Bay capacities and distances to the LCG point of each bay for a containership with a capacity of 4,000 containers.

Bay number (bow of LCG)	Bay capacity	Distance to LCG (meter)	Bay number (stern of LCG)	Bay capacity	Distance to LCG (meter)
1	50	157	12	220	7
2	120	142	13	220	22
3	142	127	14	220	37
4	168	112	15	220	52
5	188	97	16	220	67
6	188	82	17	216	82
7	200	67	18	216	97
8	216	52	19	156	112
9	220	37	20	156	127
10	220	22	21	126	142
11	220	7	22	98	157

TABLE 11. Modified bay capacities and distances to the LCG point of each bay for a containership with a capacity of 4000 containers.

Bay location	Bay number (bow of LCG)		Bay capacity		Distance to LCG (meter)	Bay number (stern of LCG)		Bay capacity		Distance to LCG (meter)
Under deck	1	23	13	12	157	12	34	55	55	7
	2	24	30	30	142	13	35	55	55	22
	3	25	36	35	127	14	36	55	55	37
	4	26	42	42	112	15	37	55	55	52
	5	27	47	47	97	16	38	55	55	67
	6	28	47	47	82	17	39	54	54	82
	7	29	50	50	67	18	40	54	54	97
	8	30	54	54	52	19	41	39	39	112
	9	31	55	55	37	20	42	39	39	127
	10	32	55	55	22	21	43	32	31	142
	11	33	55	55	7	22	44	25	24	157
Upper deck	45	67	12	13	157	56	78	55	55	7
	46	68	30	30	142	57	79	55	55	22
	47	69	35	36	127	58	80	55	55	37
	48	70	42	42	112	59	81	55	55	52
	49	71	47	47	97	60	82	55	55	67
	50	72	47	47	82	61	83	54	54	82
	51	73	50	50	67	62	84	54	54	97
	52	74	54	54	52	63	85	39	39	112
	53	75	55	55	37	64	86	39	39	127
	54	76	55	55	22	65	87	31	32	142
	55	77	55	55	7	66	88	24	25	157

TABLE 12. Transportation matrix.

Port (i,j)	1						2						3					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
0	22	17	29	11	7	9	27	28	32	27	14	10	49	65	41	37	24	51
1	-	-	-	-	-	-	92	105	103	76	105	163	95	156	28	169	132	38
2	-	-	-	-	-	-	-	-	-	-	-	-	37	15	20	28	32	17
Port (i,j)	4						5						6					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
1	214	49	16	153	13	43	-	-	-	-	-	-	-	-	-	-	-	-
2	15	38	12	52	34	47	52	44	54	13	59	75	-	-	-	-	-	-
3	17	10	21	21	3	18	41	58	26	44	36	36	47	71	92	92	68	66
4	-	-	-	-	-	-	61	75	42	23	14	52	41	15	48	4	30	32
5	-	-	-	-	-	-	-	-	-	-	-	-	53	69	13	10	29	48
Port (i,j)	$1'$						$2'$						$3'$					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
4	64	35	64	76	78	22	-	-	-	-	-	-	-	-	-	-	-	-
5	104	24	113	35	52	41	65	13	6	36	58	36	-	-	-	-	-	-
6	46	38	4	127	43	64	49	20	41	51	47	49	30	46	40	65	35	33

to the next three ports along the route. All trials indicated that our heuristic algorithm performed well and was able to produce high-quality solutions in a short CPU time even for large ships with 20,000 TEU capacity. Having reached satisfactory results with the proposed two-stage heuristic methodology encouraged us to conclude that our algorithm can be applied on container ships of all sizes.

Nevertheless, there is one more step following the bay assignment, which is the placement of containers to the slots in bays (slot assignment). The bays consist of slots that have containers. However, considering the bays as one dimension might lead to the infeasibility of the slot assignment process. To prevent the occurrence of such cases and to facilitate slot assignment, our model divides each bay into four holds

TABLE 13. Transportation matrix for 20 ft. containers.

Port (i,j)	1						2						3					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
0	12	9	20	9	5	6	17	19	22	15	8	9	31	47	26	23	15	45
1	-	-	-	-	-	-	61	13	44	42	53	68	38	75	24	73	55	9
2	-	-	-	-	-	-	-	-	-	-	-	-	25	11	14	16	25	13
Port (i,j)	4						5						6					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
1	203	49	15	148	1	29	-	-	-	-	-	-	-	-	-	-	-	-
2	13	0	6	10	30	23	26	0	21	9	16	23	-	-	-	-	-	-
3	6	3	15	12	0	15	11	3	14	8	13	6	7	58	83	61	47	16
4	-	-	-	-	-	-	53	72	34	15	4	45	15	3	23	4	20	12
5	-	-	-	-	-	-	-	-	-	-	-	-	20	20	10	1	1	19
Port (i,j)	$1'$						$2'$						$3'$					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
4	53	19	59	65	78	4	-	-	-	-	-	-	-	-	-	-	-	-
5	86	1	91	31	43	24	0	4	5	3	7	7	-	-	-	-	-	-
6	12	38	3	28	29	4	36	6	38	40	24	21	4	28	21	57	19	12

TABLE 14. Transportation matrix for 40 ft. containers.

Port (i,j)	1						2						3					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
0	10	8	9	2	2	3	10	9	10	12	6	1	18	28	15	14	9	6
1	-	-	-	-	-	-	31	92	59	34	52	95	57	81	4	96	77	29
2	-	-	-	-	-	-	-	-	-	-	-	-	12	4	6	12	7	4
Port (i,j)	4						5						6					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
1	11	0	1	5	12	14	-	-	-	-	-	-	-	-	-	-	-	-
2	2	38	6	42	4	24	26	44	33	4	43	52	-	-	-	-	-	-
3	11	7	6	9	3	3	30	55	12	36	23	30	40	13	9	31	21	50
4	-	-	-	-	-	-	8	3	8	8	10	7	26	12	25	0	10	20
5	-	-	-	-	-	-	-	-	-	-	-	-	33	49	3	9	28	29
Port (i,j)	$1'$						$2'$						$3'$					
Weight	30	25	20	15	10	5	30	25	20	15	10	5	30	25	20	15	10	5
4	11	16	5	11	0	18	-	-	-	-	-	-	-	-	-	-	-	-
5	18	23	22	4	9	17	65	9	1	33	51	29	-	-	-	-	-	-
6	34	0	1	99	14	60	13	14	3	11	23	28	26	18	19	8	16	21

TABLE 15. Revised transportation matrix for 40 ft. containers.

Port (i,j)	2			3			4			5		
Weight	15	10	5	15	10	5	15	10	5	15	10	5
1	246	186	294	276	200	212	22	12	52	-	-	-
2	-	-	-	32	36	22	80	96	56	140	74	190
3	-	-	-	-	-	-	36	30	12	170	96	106
4	-	-	-	-	-	-	-	-	-	22	32	34
Port (i,j)	6			$1'$			$2'$			$3'$		
Weight	15	10	5	15	10	5	15	10	5	15	10	5
3	106	80	142	-	-	-	-	-	-	-	-	-
4	76	50	60	54	32	36	-	-	-	-	-	-
5	164	24	114	82	52	52	148	68	160	-	-	-
6	-	-	-	68	200	148	54	28	102	88	54	74

and treats each hold as a bay. Therefore, we were able to produce more realistic solutions, which will constitute inputs for the slot assignment problem. While assigning containers to the slots, the transversal stability of the ship must also be considered. We reserved these steps of the CSP for future studies.

APPENDIX

EXAMPLE PROBLEM INPUT DATA

In this example, we consider a vessel with a capacity of 4000 TEU conducting a round-robin tour of 6 ports. In the literature, the weight of a single TEU is accepted as 15 tons. Therefore, by definition, a 10,000 TEU ship, for example, is a ship that can accommodate up to 10,000 20 ft. containers, whose weights are all 15 tons. In this case, a ship can only transport as many containers as its capacity, provided all containers are 15 tons. In our example and experimental trials, this fact is overseen. For the example vessel, we assume that the ship has 22 bays, $22 \times 4 = 88$ holds, 500 pre-loaded containers, 1000 and 750 units of 20 ft. and 40 ft. containers, respectively. The bay numbers, capacities, and the distances to the LCG points of each bay are presented in Table 10.

However, this representation of the vessel model and bay information will not allow us to include the hatch cover

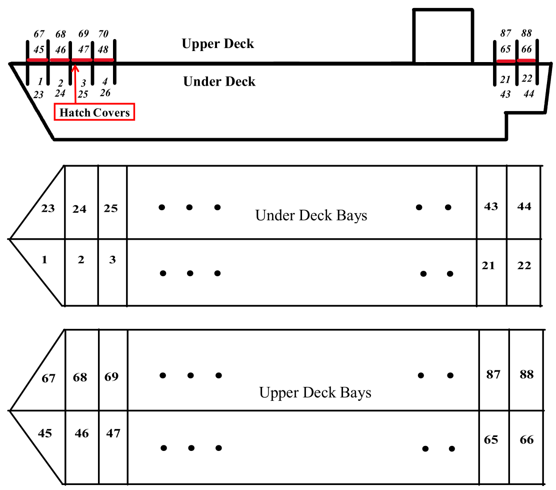

FIGURE 16. Vessel representation and bay numbering.

configuration in our mathematical models. To overcome this difficulty, we slightly modify the vessel model by separating decks into under and upper decks, separating the starboard and port side, and assuming that each hold is a different bay as depicted in Figure 16. Simultaneously, we equally distributed the capacity of each bay between the upper and under decks. The modified bay numbers, capacities, and the distances to the LCG point of each bay are presented in Table 11.

In this example, we assume that each bay can accommodate 20 ft. containers. Additionally, the under deck bay pairs 1–2, 3–4, 5–6,…,19–20, 21–22,…,43–44 and upper deck bay pairs 45–46, 47–48,…,65–66, 67–68,…,87–88 can also accommodate 40 ft. containers. We further classified the containers into six weight groups as follows:

- Group 1 ($k = 1$): 30 ton containers (weights between 27.5 and 32.5 tons)
- Group 2 ($k = 2$): 25 ton containers (weights between 22.5 and 27.5 tons)
- Group 3 ($k = 3$): 20 ton containers (weights between 17.5 and 22.5 tons)
- Group 4 ($k = 4$): 15 ton containers (weights between 12.5 and 17.5 tons)
- Group 5 ($k = 5$): 10 ton containers (weights between 7.5 and 12.5 tons)
- Group 6 ($k = 6$): 5 ton containers (weights between 2.5 and 7.5 tons)

The example transportation matrix given in Table 12 shows the number of containers to be transported from port i (rows) to port j (columns) with respect to their weight groups. Note that, the number of containers loaded at previous ports (containers already loaded before the first port) are given in the third row of the table. The data for the exact number of 20 and 40 ft. containers are given in Tables 13 and 14.

Recall that, according to our assumption, we count each 40 ft. container as two 20 ft. containers. For this reason, the 40 ft. container transportation matrix is revised by doubling the number of 40 ft. containers and distributing them to suitable weight categories as given in Table 15. Table 16 reports the detailed bay distribution of the pre-loaded containers.

TABLE 16. Bay distribution of pre-loaded containers (at Port 0).

Bay #	Bay capacity	# of containers	Port ($i-j$)	Weight group (ton)
1	13	13	0-3	25
2	30	30	0-3	25
3	36	18	0-2	15
3	36	14	0-2	10
3	36	4	0-3	25
4	42	27	0-2	30
4	42	9	0-2	15
9	55	51	0-3	5
13	55	41	0-3	20
14	55	49	0-3	30
16	55	18	0-3	25
16	55	37	0-3	15
21	32	28	0-2	25
29	50	24	0-3	10
37	55	10	0-2	5
41	39	32	0-2	20

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DISCLAIMER

Conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of any affiliated organization or government.

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