Given these historical price and demand data points:

{data_points}

With a unit cost = ${cost:.2f}, conduct a comprehensive and meticulous analysis to determine the profit-maximizing price. Perform the below steps to obtain the best mathematical model to approximate demand as a function of price.

1. Data Examination and Model Selection:
- *Conduct some analysis on the price-demand data to understand overall trends and variability in the data.*
- *Consider multiple candidate models for representing demand as a function of price, such as:*
  - *Parametric functions: linear, log-linear, log-log, and logistic functions*
  - *Non-parametric or machine learning methods: random forests and gradient boosted trees.*
- *For each candidate model:*
  - *Fit the demand as a function of price.*
  - *Report goodness-of-fit metrics (e.g., R^2 and other relevant accuracy measures).*
  - *Perform residual analysis to check if any systematic patterns remain in the errors.*
  - *Use additional metrics like Mean Squared Error (MSE), Mean Absolute Error (MAE), and other relevant metrics to assess predictive performance.*
- *Compare all tested models based on these quantitative metrics and residual analyses:*
  - *Identify which model has the highest accuracy and the most random residual distribution (no unmodeled patterns).*
  - *Provide a clear, numeric justification for why the chosen model is superior*
- *Select the single best-fitting model based on this process.*

2. Demand Function and Profit Equation:
- *Once you have chosen the best model, define the profit function: Profit = (Price - {cost:.2f}) × Demand.*

3. Identifying the Profit-Maximizing Price:
- *If the chosen model allows a direct analytical approach, show the step-by-step algebraic or computational reasoning that leads you to the optimal price.*
- *If no direct analytical solution is feasible, use a numeric or search-based method:*
  - *Clearly describe the numeric approach adopted (e.g. Gradient descent (or any other variation of it), Newton's Method, etc.).*
- *Determine the price P that results in the highest profit.*
  - *Show profit values at all the candidate price(s).*
  - *Identify the price at which profit is highest compared to all tested alternatives.*
- *Thoroughly verify that this identified price is a global maximum.*
- *Maintain maximum precision in all numeric steps.*

4. Detailed Arithmetic and Verification:
- *Include a verification checklist, for example:*
  - *Have you documented all model parameters and computations fully?*
  - *Did you confirm that no other tested prices yield higher profit?*
  - *Have you demonstrated that the chosen price corresponds to a global maximum scenario?*
- *Do not adjust or constrain the optimal price based on the observed data range; report it exactly as calculated.*
- *Do not comment on whether the price is inside or outside the observed data range. Simply provide the computed result.*

Return the output in the below JSON format:

```
{
  "demand_function": {
    "equation": "a representation of the fitted demand model as a function of the price",
    "r_squared": "number"
  },
  "optimal_price": "number",
  "corresponding_profit": "number",
  "calculations": {
    "demand_at_optimal": {
      "expression": "numeric computation",
      "explanation": "explain what this calculation represents"
    },
    "profit": {
```

"expression": "numeric computation",
"explanation": "explain what this calculation represents"
},
"model_comparison": {
"models_tested": ["list of models tested"],
"chosen_model": "model name",
"justification": "detailed numeric reasons why this model is best"
},
"global_maximum_validation": {
"tested_prices": ["list of prices tested"],
"profits_at_tested_prices": ["corresponding profits"],
"explanation": "explain how these tests confirm the identified global maximum"
}
},
"reasoning": [
"A thorough, step-by-step explanation detailing how you selected the best model with rigorous metrics and validation methods, found the maximum profit price, verified it is a global maximum, and ensured every arithmetic step is correct."
]
}