Econ, Problem Set #5, DSGE

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Homework: 1 through 6 at the end of the Chapter 1 on DSGE models

Solutions - Chapter 1

Ch1.1 We guess for the policy function $K_{t+1} = Ae^{z_t}Kt^{\alpha}$ and plug into toe euler equation (60) :

$$\begin{aligned}
\frac{1}{e^{z_t}K_t^{\alpha} - K_{t+1}} &= \beta E_t \left\{ \frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_{t+1}}K_{t+1}^{\alpha} - K_{t+2}} \right\} \\
\frac{1}{e^{z_t}K_t^{\alpha} - Ae^{z_t}K^{\alpha}} &= \beta E_t \left\{ \frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_{t+1}}K_{t+1}^{\alpha} - Ae^{z_{t+1}}K_{t+1}^{\alpha}} \right\} \\
\frac{1}{e^{z_t}K_t^{\alpha} - Ae^{z_t}K^{\alpha}} &= \beta E_t \left\{ \frac{\alpha K_{t+1}^{\alpha-1}}{K_{t+1}^{\alpha} - AK_{t+1}^{\alpha}} \right\} \\
\frac{1}{(1-A)e^{z_t}K_t^{\alpha}} &= \beta \frac{\alpha K_{t+1}^{\alpha-1}}{(1-A)K_{t+1}^{\alpha}} \\
\frac{1}{e^{z_t}K_t^{\alpha}} &= \beta \frac{\alpha K_{t+1}^{\alpha-1}}{K_{t+1}^{\alpha}} \\
\frac{1}{e^{z_t}K_t^{\alpha}} &= \beta \frac{\alpha}{K_{t+1}} \qquad \text{plugging in the guess once more gives} \\
\frac{1}{e^{z_t}K_t^{\alpha}} &= \beta \frac{\alpha}{Ae^{z_t}K_t^{\alpha}} \\
A &= \beta\alpha
\end{aligned}$$

Ch1.2 For the specific functions the equilibrium equations are the following:

$$\begin{aligned}
\frac{1}{c_t} &= \beta E_t \left\{ \frac{1}{c_{t+1}} [(r_{t+1} - \delta)(1-\tau) + 1] \right\} \\
a\frac{1}{1-l_t} &= \frac{w_t(1-\tau)}{c_t} \\
r_t &= \alpha e^{z_t} \left(\frac{K_t}{L_t} \right)^{\alpha-1} \\
w_t &= (1-\alpha)e^{z_t} \left(\frac{K_t}{L_t} \right)^{\alpha}
\end{aligned}$$

Ch1.4 Characterizing equations are as follows:

$$\begin{aligned}
c_t &= (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \\
c_t^{-\gamma} &= \beta E_t[c_t^{-\gamma}((r_{t+1} - \delta)(1-\tau) + 1)] \\
a(1-l_t)^{-\xi} &= c_t^{-\gamma} w_t (1-\tau) \\
r_t &= e^{z_t}\alpha[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1}{\eta}-1} K_t^{\eta-1} \\
w_t &= e^{z_t}(1-\alpha)[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1}{\eta}-1} L_t^{\eta-1} \\
T_t &= \tau[w_t l_t + (r_t - \delta)k_t] \\
z_t &= (1-\rho)\overline{z} + \rho z_{t-1} + \epsilon_t^2
\end{aligned}$$

Ch1.5 Solving for the steady States $\bar{r}$ and $\bar{K}$ and plugging in the parameter values from the exercise gives

$$\begin{aligned}
\bar{r} &= \frac{1-\beta}{\beta(1-\tau)} + \delta = 0.124 \\
\bar{k} &= \left(\bar{r}^{\frac{1}{\alpha-1}}\right)\frac{1}{\alpha} = 7.287
\end{aligned}$$