Math, Problem Set #4, Intro to Optimization

Instructor: Jorge Barro

Due Monday, July 10 at 8:00am

Homework: 1, 5, 6, 11, and 14 at the end of Chapter 6 of Humpherys et al. (2017)

6.1 Restating the maximization as a minimization problem:

$$\begin{aligned} \underset{\{x\}}{min} \quad & -e^{w^T x} \\ subject\,to \quad & w^T x \leq w^T A w - w^T A y + a \\ & y^T w = w^T x + b \end{aligned}$$

6.5

$$\begin{aligned} \underset{\{x_k, x_m\}}{min} \quad & -(0.05 x_k + 0.07 x_m) \\ subject\,to \quad & 100h = l_k + l_m \\ & 240Kg = r_k + r_m \\ & x_k = 60 l_k + \frac{1000}{3} r_k \\ & x_m = 30 l_m + 250 r_m \end{aligned}$$

where x_k and x_m are the produced quantities of knobs amd milk bottles, respectively. l denotes the number of hours of work devoted to production of either of those products and r the needed raw inputs.

6.6 Find all critical points of $f(x,y) = 3x^2 y + 4xy^2 + xy$:

The FONC'S give the system:

$$f_x = 6xy + 4y^2 + y = 0 \tag{1}$$

$$f_y = 3x^2 + 8xy + x = 0 \tag{2}$$

From (1) we get $y_1 = 0$, $y_2 = -\frac{1}{4}(1+6x)$ and from (2) we get $x_1 = 0\,, x_2 = -\frac{1}{3}(1+8y)$. Solving gives the critical points $(0,0)$, $(-\frac{1}{3}, 0)$, $(0, -\frac{1}{4})$ and $(-\frac{1}{9}, -\frac{1}{12})$

6.11 Show that newton method for the function $f(x) = ax^2 + bx + c\, a > 0$ converges in only one iteration for any initial $x_0 \in \mathbb{R}$.

$$\text{in generall we have:} \quad x_{t+1} = x_t - \frac{f'(x_t)}{f''(x_t)}$$

$$\text{In this cas ewe have:} \quad x_1 = x_0 - \frac{2ax_0 + b}{2a} = x_0 - x_0 - \frac{b}{2a} = -\frac{b}{2a}$$

6.14 The solution is in the Jupyter Notebook.