1. $m(x) = \frac{1}{N}\sum_{i=1}^{N} x_i$

$$m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i\right)$$

$$\sum_{i=1}^{N} a = Na \qquad \sum_{i=1}^{N} bx_i = b\sum_{i=1}^{N} x_i$$

$$m(a+bX) = \frac{1}{N}\left(Na + b\sum_{i=1}^{N} x_i\right)$$

$$= a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

$$= a + bm(X)$$

2. $\mathrm{cov}(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - (a+bm(Y)))$$

$$(a+by_i) - (a+bm(Y)) = by_i - bm(Y) = b(y_i - m(Y))$$

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\, b(y_i - m(Y))$$

$$= b\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))\right]$$

$$= b\,\mathrm{cov}(X,Y)$$

3. $\text{cov}(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}[(a+bx_i)-(a+bm(X))]^2$

$$= \frac{1}{N}\sum_{i=1}^{N}[b(x_i-m(X))]^2$$

$$= \frac{b^2}{N}\sum_{i=1}^{N}(x_i-m(X))^2$$

$$\text{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i-m(X))^2$$

$$\boxed{\text{cov}(a+bX, a+bX) = b^2\text{cov}(X,X)}$$

$$s^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i-m(X))^2$$

$$\boxed{\text{cov}(X,X) = s^2}$$

4. Non-decreasing transformation preserves order. If g is non-decreasing $\{X \le M\}$ means $\{g(X) \le g(M)\}$
$\therefore P(g(X) \le g(M)) = P(X \le M) \ge 0.5$ this holds true for any quantile as well. However, this does not apply to range or IQR because they both measure differences – unless g is linear.

5. No because there could be flat spots on the non-decreasing function causing $m(g(X)) = g(m(X))$ to not always be true