

Transformations of the equations in OpenSBLI framework

Satya P Jammy & OpenSBLI developers team

December 14, 2023

The Cartesian coordinates system is (x0,x1,x2)

The Curvilinear coordinate system is vector is (xi0,xi1,xi2)

The transformation parameters provided are curvilinear in x0,curvilinear in x1

The transformation parameters provided are stretched in x0,stretched in x1

$$\frac{d}{dx0}f = D00 \cdot \frac{d}{dxi0}f + D10 \cdot \frac{d}{dxi1}f \quad (1)$$

$$\frac{d}{dx1}f = D01 \cdot \frac{d}{dxi0}f + D11 \cdot \frac{d}{dxi1}f \quad (2)$$

$$\frac{d}{dx2}f = \frac{d}{dxi2}f \quad (3)$$

$$\begin{aligned} \frac{d^2}{dx0^2}f &= (D00)^2 \cdot \frac{d^2}{dxi0^2}f + D00 \cdot D10 \cdot \frac{d^2}{dxi0dxi1}f + D00 \cdot D10 \cdot \frac{d^2}{dxi1dxi0}f \\ &+ D00 \cdot SD000 \cdot \frac{d}{dxi0}f + D00 \cdot SD100 \cdot \frac{d}{dxi1}f + (D10)^2 \cdot \frac{d^2}{dxi1^2}f \\ &+ D10 \cdot SD001 \cdot \frac{d}{dxi0}f + D10 \cdot SD101 \cdot \frac{d}{dxi1}f \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{d^2}{dx0dx1}f &= D00 \cdot D01 \cdot \frac{d^2}{dxi0^2}f + D00 \cdot D11 \cdot \frac{d^2}{dxi0dxi1}f + D01 \cdot D10 \cdot \frac{d^2}{dxi1dxi0}f \\ &+ D01 \cdot SD000 \cdot \frac{d}{dxi0}f + D01 \cdot SD100 \cdot \frac{d}{dxi1}f + D10 \cdot D11 \cdot \frac{d^2}{dxi1^2}f \\ &+ D11 \cdot SD001 \cdot \frac{d}{dxi0}f + D11 \cdot SD101 \cdot \frac{d}{dxi1}f \end{aligned} \quad (5)$$

$$\frac{d^2}{dx0dx2}f = D00 \cdot \frac{d^2}{dxi0dxi2}f + D10 \cdot \frac{d^2}{dxi1dxi2}f \quad (6)$$

$$\begin{aligned}
\frac{d^2}{dx_1 dx_0} f &= D00 \cdot D01 \cdot \frac{d^2}{dxi_0^2} f + D00 \cdot D11 \cdot \frac{d^2}{dxi_1 dxi_0} f + D00 \cdot SD010 \cdot \frac{d}{dxi_0} f \\
&+ D00 \cdot SD110 \cdot \frac{d}{dxi_1} f + D01 \cdot D10 \cdot \frac{d^2}{dxi_0 dxi_1} f \\
&+ D10 \cdot D11 \cdot \frac{d^2}{dxi_1^2} f + D10 \cdot SD011 \cdot \frac{d}{dxi_0} f + D10 \cdot SD111 \cdot \frac{d}{dxi_1} f
\end{aligned} \tag{7}$$

$$\begin{aligned}
\frac{d^2}{dx_1^2} f &= (D01)^2 \cdot \frac{d^2}{dxi_0^2} f + D01 \cdot D11 \cdot \frac{d^2}{dxi_0 dxi_1} f + D01 \cdot D11 \cdot \frac{d^2}{dxi_1 dxi_0} f \\
&+ D01 \cdot SD010 \cdot \frac{d}{dxi_0} f + D01 \cdot SD110 \cdot \frac{d}{dxi_1} f + (D11)^2 \cdot \frac{d^2}{dxi_1^2} f \\
&+ D11 \cdot SD011 \cdot \frac{d}{dxi_0} f + D11 \cdot SD111 \cdot \frac{d}{dxi_1} f
\end{aligned} \tag{8}$$

$$\frac{d^2}{dx_1 dx_2} f = D01 \cdot \frac{d^2}{dxi_0 dxi_2} f + D11 \cdot \frac{d^2}{dxi_1 dxi_2} f \tag{9}$$

$$\frac{d^2}{dx_2 dx_0} f = D00 \cdot \frac{d^2}{dxi_2 dxi_0} f + D10 \cdot \frac{d^2}{dxi_2 dxi_1} f \tag{10}$$

$$\frac{d^2}{dx_2 dx_1} f = D01 \cdot \frac{d^2}{dxi_2 dxi_0} f + D11 \cdot \frac{d^2}{dxi_2 dxi_1} f \tag{11}$$

$$\frac{d^2}{dx_2^2} f = \frac{d^2}{dxi_2^2} f \tag{12}$$