IMPERIAL

Hawkes Processes
Modelling self-exciting and mutually exciting point processes

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Introduction

Hawkes Processes describe a type of inhomogeneous Poisson processes used to model self-exciting and mutually exciting point processes.

They have found practical applications in seismology, finance, epidemiology, and social networks, among other fields. In this poster, we discuss Hawkes processes and provide some examples of their current applications.

What are stochastic point processes?

A stochastic point process is a set of a random number of random variables in a mathematical space. They provide a way to model the number of events and their associated values in a given mathematical space.

The intensity function describes the local rate of events:

$$\lambda(s) = \lim_{\delta \to 0} \frac{N(s, s+\delta)}{\delta} \tag{1}$$

where $N(s, s+\delta)$ is the counting measure on the interval $(s, s+\delta)$.

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Univariate case

In the univariate case [2], the intensity function is defined as:

$$\lambda(t) = \mu + \sum_{t_i<t} \phi(t - t_i) \tag{2}$$

where

- $\lambda(t)$ is the intensity at time t
- μ is the baseline intensity
- $\phi(t - t_i)$ is the kernel or exciting function

Marked point processes

The mark m_i of an event x_i refers to the associated information of the event that affects the intensity. For example, it could be the demographic of the population in the context of modeling the spread of diseases [2]. We can thus generalize the kernel function to contain the mark, such that we have

$$\lambda(t) = \mu + \sum_{t_i<t} \phi(t - t_i, m_i) \tag{3}$$

Generalized marked multivariate Hawkes Process

Finally, by considering both mutually exciting terms (i.e. $i \neq j$) and self-exciting terms, and by including the marks of each event, we have the conditional intensity of the marked multivariate Hawkes Process

$$\lambda_i(t|\mathcal{H}_t) = \mu_i + \sum_{j=1}^{D} \sum_{t_i<t} \int_{M_j} \phi_{ij}(t - t_i, m)\, N_j(dm) \tag{4}$$

where

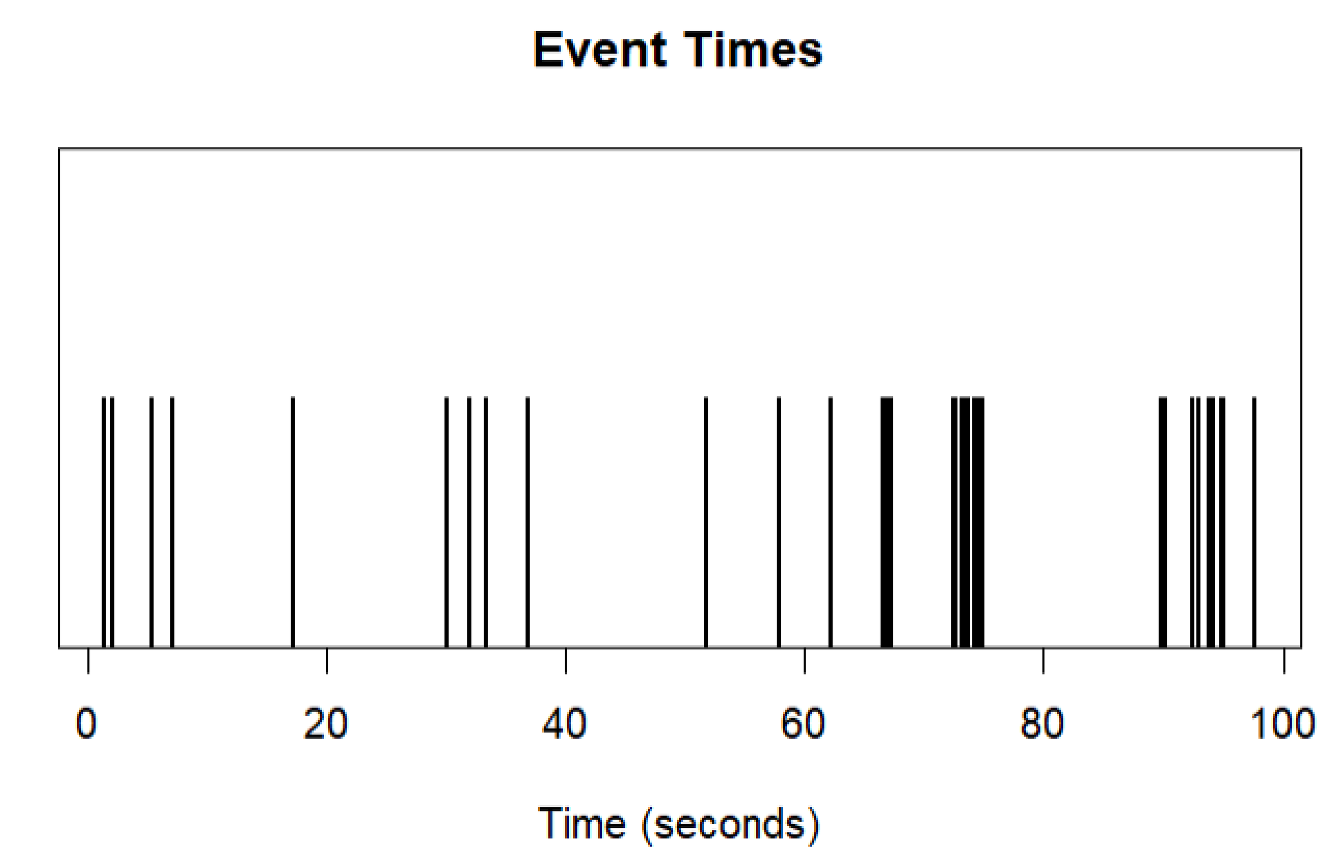
- $\mathcal{H}_t$ is the history up to time t
- D is the number of dimensions
- M_i is the mark space

Simulation and Application

By simulating Hawkes processes, we can not only verify the validity of using a Hawkes model for the events we intend to investigate, but we are also able to study the effect that each event has (such as on the size of clusters formed or on the time between events) so as to better plan for such events in the future.

Simulating using R

We can create simple simulations of Hawkes processes in R using the `{hawkes}` package.



Current applications

Perhaps the most prominent application is the Epidermic Type Aftershock Sequence (ETAS) model, which has now extended to a spatio-temporal model that uses a range of kernels - such as exponential decay and power-law functions - to model earthquakes and their aftershocks. [3]

In the financial context, Aït-Sahalia et al. (2015) proposed an adapted Hawkes process as a way to model financial contagion, referred to as a Hawkes jump-diffusion model. It included a Brownian component to account for stochastic volatility and was used to model asset markets in different countries, through which they found strong evidence for self-excitation and mutual excitation in the markets. [1]

References

[1] Yacine Aït-Sahalia, Julio Cacho-Diaz, and Roger J.A. Laeven. Modeling financial contagion using mutually exciting jump processes. Journal of Financial Economics, 117(3):585–606, 2015.

[2] Rafael Lima. Hawkes processes modeling, inference and control: An overview. arXiv preprint arXiv:2005.11839, 2020.

[3] Alex Reinhart. A review of self-exciting spatio-temporal point processes and their applications. Statistical Science, 33(3):299–318, 2018.