SOME SURFACE-DIAGRAMMATIC PROOFS

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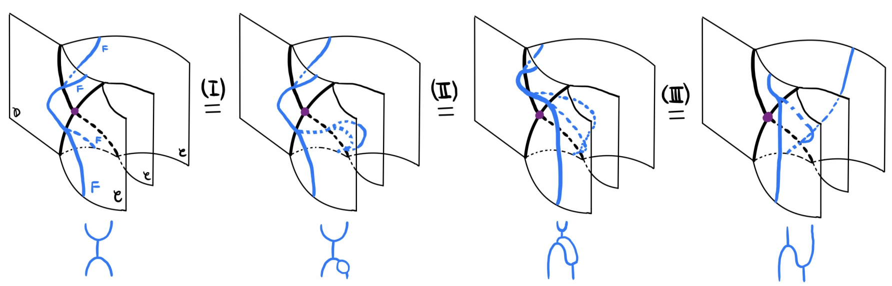
Abstract. This is a collection of surface-diagrammatic proofs that have not made it into my master's thesis.

First up, the next result is mentioned in Remark 2.16. of my master's thesis. Day and Pastro prove the result using commutative diagrams [DP08, Prop. 3].

Proposition 0.0.1. *Let $\mathcal{C}$ and $\mathcal{D}$ be monoidal categories. Any strong monoidal functor $F\colon \mathcal{C} \to \mathcal{D}$ is a Frobenius monoidal functor.*

Proof. We show that $F\colon \mathcal{C} \to \mathcal{D}$ satisfies the Frobenius relation (F1):



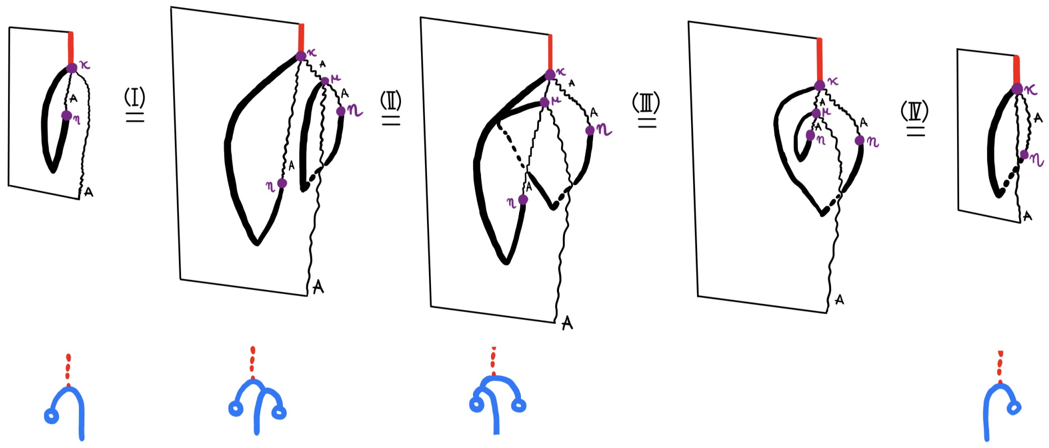
Equalities (I) and (III) hold since the monoidal structure on F is strong, while equality (II) follows from the associativity axiom of the (lax) monoidal structure. The proof of Frobenius relation (F2) is obtained from the above proof by reflecting each surface diagram along the side face. ☐

The following result appears in Remark 4.31. in my master's thesis:

Proposition 0.0.2. *Let (A, μ, η) be an algebra in a linearly distributive category. An invariant LD-pairing $\kappa\colon A \otimes A \to K$ on A is compatible with the unit η, i.e. we have*

$$\kappa \circ (\eta \otimes A) \circ l_A = \kappa \circ (A \otimes \eta) \circ r_A.$$

Proof. The string-diagrammatic proof appears below our surface-diagrammatic proof:



Equations (I) and (IV) hold by the unitality of the multiplication μ, while equation (II) follows from the associativity of the multiplication μ. Equation (III) holds by Mac Lane's coherence theorem for the $\otimes$-monoidal structure.

Since no coherence morphism involving the ⅋-monoidal structure appears in the above surface-diagrammatic proof, the string-diagrammatic proof above is a perfectly valid proof of the proposition, i.e. surface diagrams are not necessary for the proof. □

References

[DP08] Brian Day and Craig Pastro. Note on Frobenius monoidal functors. *New York J. Math.*, 14:733–742, 2008.

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