

1.

(a) $9n + 77 = O(n)$, $C < 50$

$$T(n) \leq 86n, n \geq 1$$

$$T(n) \leq 43n, n \geq 2$$

$$C = 43, k = 2$$

(b) $10n^2 + 80 = O(n^2)$, $C < 40$

$$T(n^2) \leq 90n^2, n \geq 1$$

$$T(n^2) \leq 45n^2, n \geq 2$$

$$T(n^2) \leq 30n^2, n \geq 3$$

$$T(n^2) \leq 18n^2, n \geq 5$$

$$T(n^2) \leq 15n^2, n \geq 6$$

$$T(n^2) \leq 10n^2, n \geq 9$$

$$C = 10, k = 9$$

(c) $7n^2 + 200n = O(n^2)$, $C < 100$

$$T(n^2) \leq 207n^2, n \geq 1$$

$$T(n^2) \leq 69n^2, n \geq 3$$

$$T(n^2) \leq 23n^2, n \geq 9$$

$$C = 69, k = 3$$

(d) $11n^2 + 75n + 84 = O(n^2)$, $C < 40$

$$T(n^2) \leq 170n^2, n \geq 1$$

$$T(n^2) \leq 85n^2, n \geq 2$$

$$T(n^2) \leq 34n^2, n \geq 5$$

$$T(n^2) \leq 17n^2, n \geq 10$$

$$C = 34, k = 5$$

2.

(a)

$$T(n) : 1 + 3 + 5 + \dots + 2n - 1 = n^2, \quad n \geq 1$$

$$T(n+1) : 1 + 3 + 5 + \dots + 2n - 1 + 2(n+1) - 1 = (n+1)^2, \quad n \geq 1$$

$$1 + 3 + 5 + \dots + 2n - 1 + 2n + 1 = n^2 + 2n + 1$$

$$T(n) + 2n + 1 = n^2 + 2n + 1$$

$$T(n) = n^2$$

(b)

$$T(n) : 1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \dots + n \cdot 2^{n-1} = (n-1) \cdot 2^n + 1, \quad n \geq 1$$

$$T(n+1) : 1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \dots + n \cdot 2^{n-1} + (n+1) \cdot 2^{n+1} - 1 = ((n+1)-1) \cdot 2^n + 1 - 1$$

$$1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \dots + n \cdot 2^{n-1} + (n+1) \cdot 2^n = n \cdot 2^{n+1} + 1$$

$$1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \dots + n \cdot 2^{n-1} + 2^n \cdot n + 2^n = n \cdot 2^{n+1} + 1$$

$$T(n) + 2^n \cdot n + 2^n = 2^{n+1} \cdot n + 1$$

$$T(n) = 2^{n+1} \cdot n - 2^n \cdot n - 2^n + 1$$

$$T(n) = 2^n(2n - n - 1) + 1$$

$$T(n) = 2^n(n - 1) + 1$$

$$T(n) = (n - 1) \cdot 2^n + 1$$