

Research Motivation

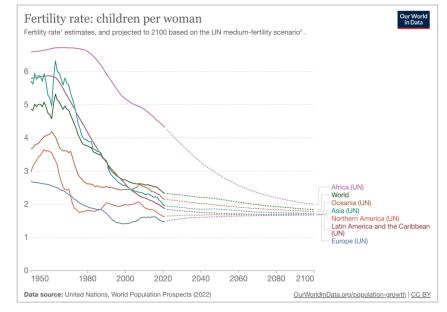
Fertility Crisis

> Semi-Endogenous Growth (SEG) theory:

Population growth is the sole driver of sustained economic growth (Jones, 2022)

$$g_{y} = \sigma g_{A} = \frac{\sigma n}{\beta}$$

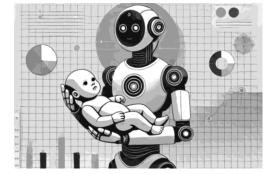
> Fertility rates are falling worldwide



The Role of Technology

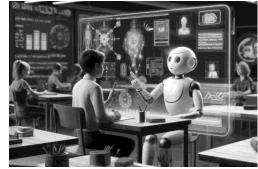
- Recent popularisation of AI has changed the common assumption that automation could only replace routine tasks
- Proof of the feasibility of automation in childcare.
 - > Robotisation of elder care clinics





AI as an aid to education





EC331: Assignment 2

Literature Review & Contribution

BASELINE MODELS

Jones (2022)

SEG Model

Key features:

- Economy growth rate depends on the growth rate of ideas.
- > Decreasing returns of idea production.

Final Good

 $Y_t = A_t^{\sigma} L_{Y,t}$

Idea

 $\frac{\dot{A}_t}{A_t} = z L_{A,t} A_t^{-\beta}$

Pop. Dynamics

$$L_{Y,t} + L_{A,t} = L_t ; \frac{\dot{L}_t}{L_t} = n$$

Shortcomings:

Exogenous Pop. Growth.

Doepke et al. (2023)

Endogenous Fertility

Key features:

- Households' childbearing is determined by maximising the utility derived from consumption (c) and number of children (n).
- Boosts to fertility: $\uparrow \delta$; $\downarrow \phi$; $\uparrow s$; $\downarrow \psi$; $\uparrow w_m$.

Utility

$$u = \log(c) + \delta \log(n)$$

B.C.

$$c + \psi n = w_m + [1 - (1 - s)n\phi]w_f$$

F.O.C.

$$n = \frac{\delta}{1+\delta} \frac{w_m + w_f}{\psi + (1-s)\phi w_f}$$

Shortcomings:

> Technology has no impact on pop. Growth.

Modelling Automation

Zeira (1998)

Task-Based Approach

Main Mechanism:

$$Y = \sum_{i=1}^{N} X_{i}^{\frac{1}{N}} \qquad X_{i} = \begin{cases} L_{i} & \text{if } i > N_{A} \\ K_{i} & \text{if } i \leq N_{A} \end{cases}$$

Referenced in:

Acemoglu & Restrepo (2018), Aghion (2019), Jones (2022)...

> Endogenously models the replacement of labour with capital.

Context of its Application:

- > Most referenced framework to model automation
- > Originally focuses on labour market
- > Repurposed into SEG frameworks

Dissertation contribution

- > Literature: Mainly focuses on the application of the Task-Based Approach into the SEG model's idea production function.
- > Gap: If AI can perform high-skill non-routine tasks, automation could impact virtually any industry (not just research). There's no literature discussing automation's potential in solving the fertility crisis.

Model Building

Final Good & Idea Production

Final Good Production

$$Y_t = A_t^{\sigma_y} L_{Y,t}$$

Idea Production

$$\frac{\dot{A}_t}{A_t} = zL_{A,t}A_t^{-\beta}$$

Allocation of labour

$$rac{L_A}{L_Y} = \Lambda$$
 , $L_{A,t} + L_{Y,t} = L_t$

Population Growth

$$\frac{\dot{L}_{\rm t}}{\rm L_{\rm t}} = \frac{\dot{n}_t}{2n_t} - m_t$$

Follows Jones (2022)

Childbearing Decision

Utility

$$u(c,n,E) = \log(c) + \delta \log(n) + \delta \gamma \log(E)$$

Constraints

Time:

$$l_t = 1 - n_t \sum_{i=N_{A,t}}^{N} \phi_i$$

Budget:

$$wl_t = c + n_t \sum_{i=1}^{N_{A,t}} \psi_i$$

Maximisation Problem

$$\max_{c_t, n_t, E_t} \log(c_t) + \delta \log(n_t) + \delta \gamma \log(E_t)$$

Education

$$\mathbf{E_{t}} = \mathbf{E_{t-1}} + \sum_{i=1}^{N} X_{i,t-1}^{\frac{1}{N}}$$

Where: X_i is the output of task $i \in [1, N]$ raising the education level of a child.

$$\boldsymbol{X_{i,t}} = \begin{cases} \boldsymbol{\phi} & if \ i > N_{A,t} \ (\text{not automated}) \\ \boldsymbol{\psi} & if \ i \leq N_{A,t} \ (\text{automated}) \end{cases}$$

Automation Boundary

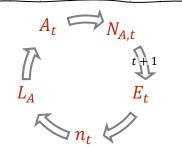
$$\dot{N}_{A,t} = A_t^{\sigma_e} \left(1 - \frac{\dot{N}_{A,t}}{\Delta N} \right) \left(1 - e^{\kappa (A_{crit} - A_t)} \right)$$

$$\max_{c_t, n_t, E_t} \log(c_t) + \delta \log(n_t) + \delta \gamma \log(E_t) \quad s.t. \quad w \left(1 - n_t \sum_{i=N_{A,t}}^{N} \phi_i\right) = c + n_t \sum_{i=1}^{N_{A,t}} \psi_i$$

Combines Doepke et al. (2023) & Zeira (1998)

Model Intuition

The level of technology A_t affects the automation boundary N_{At} , which affects education level E_t and the budget & time constraints of the fertility maximisation problem (with a time-lag), all of which have an impact on population growth through n which affects the level of technology A_t .



Issues Encountered

- Difficulty deciding where to include A_t in the fertility maximisation problem
- \rightarrow Trouble finding a closed-form solution for n, due to the mathematical complexity of the derivation of $\frac{\partial E_t}{\partial n}$.
- Can't yet test/analyse the final predictions of the model

Model Testing

Dual Approach

Model Simulations

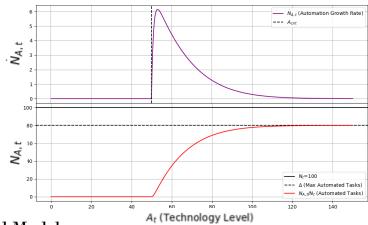
(Testing the Theoretical Mechanisms)

Mostly used in the process of building the model, to confirm its mechanisms work as expected.

1. Function Specific

Isolating each function of the model and coding simulations to confirm that the observed variables behave as expected

Ex: Automation boundary $(N_{A,t})$ as a function of technology (A_t)



2. Solved Model

Solving for a closed-form solution of n, running comparative statics simulations.

Find the model's balanced growth path (BGP) and simulate shocks to different parameters to test the model's sensitivity.

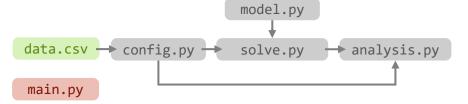
Main uses of the approach:

- (1) Graphics allow for better mechanism illustration in the final assignment
- (2) Trouble-shooting model inconsistencies

Dynamic Algorithm (Testing on Real-World Data)

To prove the accuracy of the model

Modular Python Program:



data.csv

Input / Calibration

Empirical or estimated real-world values for every parameter and every variable at every period t

model.py

Model Mechanisms

Holds the model functions used in solve.py.

analysis.py

Analyses Predictions

Stores model predictions and plots them against real world data.

config.py

Configurates Data

Splits data.csv into 3 data-frames:

- (1) Parameters
- (2) Initial Conditions (n_0, A_0, Y_0)
- (3) Variables: (n_t, A_t, Y_t)

solve.py

Model Predictions

Solves the model iteratively for each period t

main.py

Process

Calls all the different scripts (outline of the process)

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Next Steps...

1. Solving the Model

- > Solve for a closed-form solution for n:
- 3 routes to explore: (1) math-heavy derivation (2) use python solvers (3) simplify the maximisation problem.
- > Determine under which conditions the economy reaches a BGP
- Find the closed-form solutions for the growth rate of Ideas (g_A) and the growth rate of Output (g_Y) along the BGP.

3. Testing

- > Finish coding the dynamic python algorithm
- > Initiate variable values at t = 0 (e.g. Year: 1925)
- > Use the historical data of the parameters to simulate the model predictions from t=0 to now.
- > Compare to historical variable values and assess model accuracy

2. Data & Calibration

 $Find\ enough\ data\ to\ calibrate\ the\ model\ and\ population\ {\tt data.csv}$

Main Difficulty

Scarcity or lack of readily available historical data for parameters: σ_Y , β , Λ , σ_E , A_{crit} , ϕ , ψ , δ , γ , Δ , κ

Potential Solutions:

- (1) Search the literature for estimations of the parameters
- (2) Make my own estimations if needed (time-intensive)
- (3) Calibrate parameters left over to match the data as best as possible

4. Fine-Tuning

If model accuracy is unsatisfactory, use function-specific simulations and compare with historical data to identify which part of the model needs adjustment.