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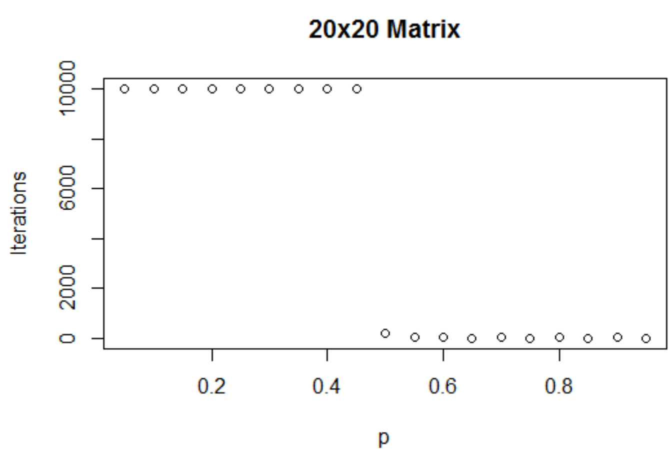
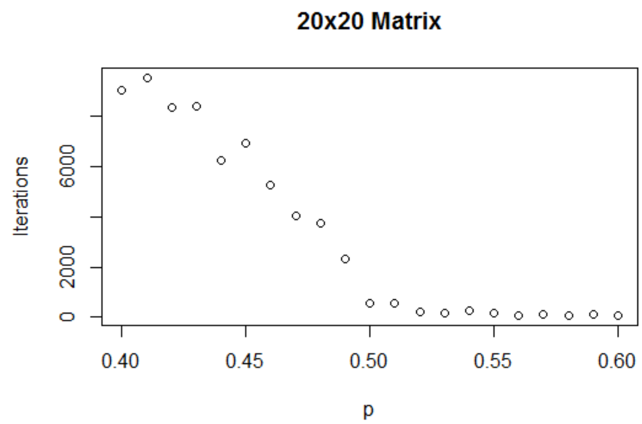
Traffic Simulation Model

For our simulation we look at various sizes of the matrix M and different densities p. To test whether a system is free flowing or jammed at the limit, we run each simulation for 10,000 iterations before we make any conclusions. If a system jams, we log the number of iterations before all movement ceased. For each simulation we can pick a range of lengths, widths, a range of denisities, and how many times to iterate over each step. This lets us try many different matrix sizes, densities, and allows for a better understanding of the transition from free flowing to jammed traffic. We store our results in a data frame which holds our input parameters and the number of iterations reached. Repitition is key here since we can often get different results depending on the starting matrix. This is because our matrices are always random and hence we choose to run the simulation for several different matrices rather than running it on only one matrix for significantly more iterations. Ultimately we avoid "bad rolls" on our matrices which could lead to misinformation.

We begin our simulations with a basic square matrix to try to understand where the model transitions from free flowing to being completely jammed. The initial resuls looked like this:

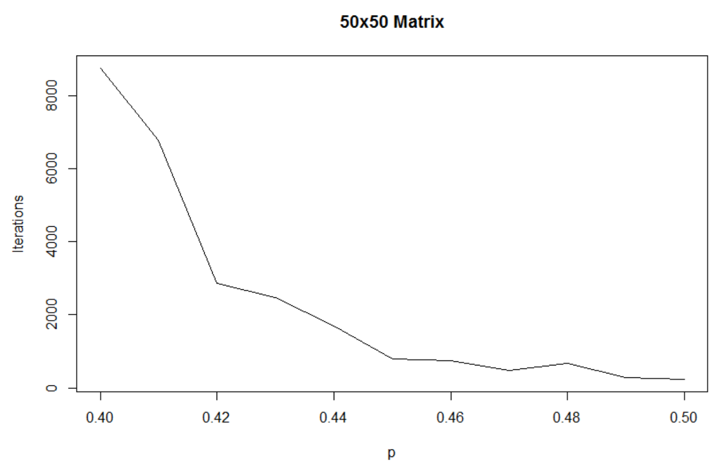


We can see that the break happens somewhere between p = 0.4 and p = 0.6. Since this simulation was only over rather large steps of p, we run a second sumulation but only in this smaller window and with smaller steps. We also average the results of 10 simulations over each step to help smooth out our results.

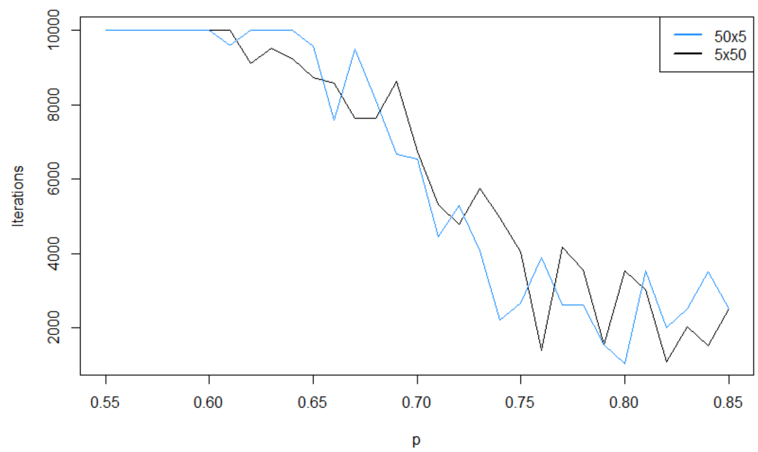


Here we can see that the transition actually happens between p = 0.4 and p = 0.5 and that it is gradual. Repeating the simulation ten times at every p value has femoved some of the noise and our results are clearer. Next we look at different shapes for the matrix to see if they behave differently by running our very first simulation but over matrices ranging from 5x5 to 5x20 to 20x5. The initial results are hard to visualize well, but it seems that rectangular matrices can tolerate higher densities before they jam. To check this we run simulations on two extremes: a 5x50 and a 50x5 matrix and compare.

We run the 50x50 matrix on p values ranging from 0.4 to 0.5 where we discovered the transition previously and we run 20 simulations on each point.



From this plot we can see that there is a sharp transition happening between p = 0.40 and p = 0.42 where the matrix goes from almost entirely free flowing to stuck for a rather small change in density. Let’s compare there results to a 5x50 and 50x5 matrix. We should note that the two are not necessarily equal since red cars moving east move first and this could make the system behave differently for a wide versus a narrow matrix.



Clearly we can see that rectangular matrices can handle higher densities before jamming. But is this really the whole story? My original hypothesis was that a matrix like a 2x50 would jam for very low p values because 2 cars alone can jam all traffic. However what I found after running simulations over very streched matrices was the opposite. These matrices appeared to be free flowing up to and beyond p = 0.8! Looking at the matrices step by step reveals that these are actually states where most of the matrix is jammed by one car is free to move indefinitely. This happens because with a length or width of 50 there is bound to be one place where there is only one car in a row or column that is unobstructed. The longer or wider the matrix the more likely this is to happen. As a result we can conclude that this is what was most likely happening in our 5x50 and 50x5 matrix simulation which is what allowed them to appear to have a higher point of jamming.

To conclude, we saw that square matrices transition from free flowing states to jammed states very sharply between p = 0.40 and p = 0.42. We also saw that very wide or long matrices tend to reach mixed states where some of the cars are moving and the rest are stopped which gives the illusion of a free flowing state.