Asset Pricing: Assignment 1

Due on September 13, 2016

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Efficient Portfolios and the CAPM

Selected portfolios

1) We selected a market cross-section of portfolios, with assets split on the market equity (ME) and the book-to-market ratio (BM) (5 size categories $\times$ 5 book-to-market ratio categories). Both properties are factors in the Fama&French 3-factor model thus we expect that these portfolios together form a fair approximation of the market portfolio. The data on these portfolios has been obtained from this site[1].
2) The average means depicted in Table 1a show that the stylized facts on the BM and ME largely hold: The average return increases with a higher BM while it decreases with an increasing ME. All the while Table 1b shows that a more average BM and a higher ME results in lower volatility.

Table 1: ME1..ME5 depict the market equity and BM1..BM5 depict the book-to-market ratio. Both show 20-percentiles in ascending order.

(a) Heatmap of the average returns of the different portfolios.

	ME1	ME2	ME3	ME4	ME5
BM1	1.06	1.29	1.33	1.42	1.28
BM2	1.61	1.53	1.58	1.39	1.34
BM3	1.62	1.73	1.59	1.52	1.29
BM4	1.79	1.74	1.67	1.64	1.36
BM5	1.91	1.78	1.84	1.65	1.42

(b) Heatmap of the average monthly realised variance of the different portfolios.

	ME1	ME2	ME3	ME4	ME5
BM1	62.98	51.33	43.72	34.51	21.64
BM2	46.93	35.37	29.90	26.60	19.56
BM3	35.75	29.38	25.04	25.51	18.82
BM4	31.77	27.97	24.08	22.99	18.90
BM5	36.68	35.92	30.05	29.92	24.93

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/tw_5_ports.html

The correlation of one portfolio with any other is on average between .73 and .88, with the smallest correlation per portfolio ranging from .53 to .78. This shows a strong comovement of the different portfolios suggesting the presence of a common driving factor such as the market portfolio return.
3) Using the historical returns of the selected portfolios and the market portfolio, we construct the efficient frontier assuming that no riskless asset is available to invest in. The efficient frontier gives the lowest obtainable volatility for a certain return of a portfolio and hence is obtained by solving the following optimization problem:

$$\min_{\pi} \frac{1}{2}\pi'\Sigma\pi \quad \text{subject to} \quad \mu'\pi = \mu_p \quad \text{and} \quad 1'\pi = 1$$

where π is the vector of portfolio weights, Σ is the co-variance matrix and μ is equal to the vector of average returns. This gives us the global minimum variance portfolio π_1 and its corresponding portfolio π_u. Table 2 contains the average returns and volatility of these portfolios.

Table 2: Characteristics of the returns (in percentages) of the different portfolios.

Portfolio	Mean	Volatility
Minimum Variance Portfolio	1,249	3.387
Other Portfolio	2.525	4.816
Market Portfolio	0.911	4.427

We see that the global minimum variance portfolio achieves a higher expected return than the market portfolio but at the same time also has a lower expected volatility. This can be explained by the fact that the global minimum variance portfolio lies on the efficient frontier while the market portfolio, in this case, does not. This can be shown more clearly as we draw the efficient frontier. Given the fact that the two obtained portfolios are MV efficient, we can use them to generate all other portfolio's on the frontier as any combination of them is also MV efficient. This yields Figure 1 below,

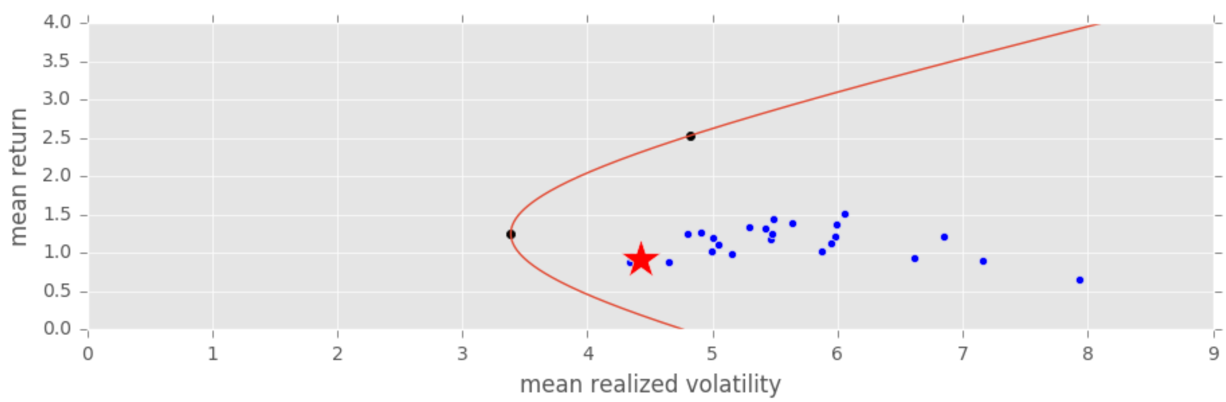

Figure 1: Plot of the efficient frontier and the return/volatility trade-off of the different portfolios. The market portfolio is shown as a red star, the tangency and minimum-variance portfolios are black. The selected portfolios are blue.

The world of excess returns

4) We now transform the returns of the original 25 portfolios and the market return to excess returns by subtracting the risk-free rate obtained from the same dataset. Again we are interested in the minimum volatility obtainable given a certain portfolio return. We now do not have the restriction that all weights on

the portfolios must sum to 1 as we have an additional asset, the riskless asset, to invest in (or borrow from). Hence the minimization problem boils down to:

$$\min_{\pi} \frac{1}{2}\pi'\Sigma\pi \quad \text{subject to} \quad \mu'\pi = \mu_p^e$$

Where all variables are defined as before but the subscript e denotes that we use excess returns. Solving the above yields a portfolio π_* called the tangency portfolio. This portfolio only contains investments in the risky portfolios.

5) As an investable riskless asset we take the average of the riskfree rate over the entire sample period as one of the assumptions of the CAPM is stationary returns. The characteristics of the portfolios and the riskless asset can be found in table 3

Table 3: Characteristics of the excess returns (in percentages) of the different portfolios

Portfolio	Mean	Volatility	Sharpe-ratio
Market Portfolio	0.508	4.441	0.114
Tangency Portfolio	2.737	6.100	0.448

We see that the tangency portfolio outperforms the market portfolio by a great margin. It achieves both a higher return and volatility resulting in a Sharpe-ratio ($E[r_p - r_f]/\sigma$) multiple of that of the market portfolio. Using the two-fund separation, with the tangency portfolio and the riskless asset as funds, we can calculate the efficient frontier: every portfolio on the frontier is a linear combination of the riskless asset and the tangency portfolio. Figure 2 shows the efficient frontier without a riskless asset (black), with a riskless asset (blue) and the return/volatility trade-off of the different portfolios.

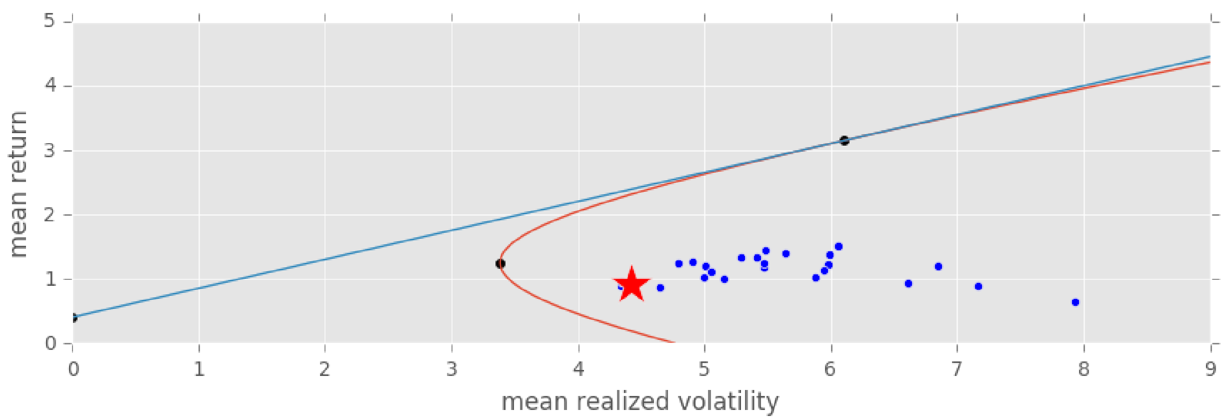

Figure 2: Plot of the efficient frontier and the return/volatility trade-off of the different portfolios. The market portfolio is shown as a red star, the tangency and minimum-variance portfolios are black. The selected portfolios are blue.

The GRS test

6) We now regress the excess returns of each of our original 25 portfolios i on the excess return of the market and a constant:

$$R_i, t - R_t^f = \alpha_i + \beta_i(R_i^m, t - R_t^f) + \epsilon_i, t \tag{1}$$

Applying this regression yields the coefficients and standard errors as shown in table 4. The first thing to notice is that all β_i coefficients are positive and significantly different from zero. Overall we see that small stocks with a low market capitalization (ME1) are more sensitive to swings in the market returns than their larger counterparts. This indicated by their larger β coefficient. The α coefficient are almost all significantly different from zero. Some are negative indicating that on average they earn a lower return than one would expect given the sensitivity to the market.

Table 4: Regression results of regression (1).

Portfolio	α	β	se_α	se_β	Portfolio	α	β	se_α	se_β
BM1-ME1	-0.472	1.424	0.193	0.043	BM4-ME3	0.376	0.960	0.098	0.022
BM2-ME1	0.185	1.228	0.167	0.037	BM5-ME3	0.512	1.028	0.122	0.027
BM3-ME1	0.254	1.099	0.139	0.031	BM1-ME4	-0.006	1.222	0.092	0.021
BM4-ME1	0.472	1.017	0.136	0.030	BM2-ME4	0.038	1.085	0.076	0.017
BM5-ME1	0.556	1.077	0.150	0.034	BM3-ME4	0.187	1.032	0.086	0.019
BM1-ME2	-0.222	1.397	0.145	0.032	BM4-ME4	0.353	0.954	0.091	0.020
BM2-ME2	0.133	1.170	0.117	0.026	BM5-ME4	0.312	1.039	0.118	0.026
BM3-ME2	0.382	1.059	0.109	0.024	BM1-ME5	-0.026	0.987	0.064	0.014
BM4-ME2	0.414	1.016	0.111	0.025	BM2-ME5	0.057	0.933	0.063	0.014
BM5-ME2	0.407	1.112	0.137	0.031	BM3-ME5	0.038	0.872	0.079	0.018
BM1-ME3	-0.154	1.329	0.121	0.027	BM4-ME5	0.130	0.828	0.093	0.021
BM2-ME3	0.204	1.123	0.091	0.020	BM5-ME5	0.167	0.887	0.123	0.028
BM3-ME3	0.279	1.004	0.092	0.021					

7) We perform the GRS test to determine how well the CAPM framework explains the excess returns of the portfolios under scrutiny. The GRS test statistic follows an F-distribution with $(25, 599)$ degrees of freedom. With $\alpha = .05$, the rejection region is $[1.525, \infty)$. The computed test statistic is 65.24 with a p-value of 0 up to machine precision. This implies that exposure to volatility leaves a significant part of the excess return unexplained and we should look for other factors to describe the excess return of these portfolios.

8) The GRS test shows that Jensen's α explains a significant part of the excess returns of the portfolio's. According to the CAPM one is only rewarded for the exposure to the market factor and hence α should be equal to zero. Given the fact that the α's are jointly significant the market factor cannot explain all the excess returns of the portfolios. The Sharpe ratios of the portfolios are also higher than we might expect based on the returns: 70% of the Sharpe ratios of the portfolios are higher than the Sharpe ratio of the market portfolio, clearly indicating that the market portfolio is not efficient. Table 5 shows that the higher Sharpe ratios are concentrated around average ME and high BM values.

Table 5: The Sharpe ratios of the portfolios. The market Sharpe ratio is 0.115.

	ME1	ME2	ME3	ME4	ME5
BM1	0.032	0.068	0.079	0.104	0.102
BM2	0.118	0.122	0.141	0.114	0.120
BM3	0.136	0.169	0.157	0.140	0.111
BM4	0.175	0.176	0.176	0.174	0.127
BM5	0.182	0.162	0.188	0.153	0.124

9) The market is certainly not efficient under the assumptions of the CAPM. Further research should be performed to investigate which assumptions are violated and whether other factors play a role in the mean/-variance trade-off decision of investors. Well known factors are off course the book-to-market ratio and the size factor. Adding these to the CAPM model and thus transforming it into a 3-factor model may take away the explanatory power of the α's.