

# Asset Pricing: Assignment 1

Due on September 13, 2016

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## Efficient Portfolios and the CAPM

This report will analyze a set of different portfolios, test whether they outperform the market and use the results to create a new factor model.

### Selected portfolios

We selected a market cross-section of portfolios, with assets split on the market equity (ME) and the book-to-market ratio (BM) (5 size categories  $\times$  5 book-to-market ratio categories). Both properties are factors in the Fama&French 3-factor model thus we expect that these portfolios together form a fair approximation of the market portfolio. We also expect that because these portfolios are based on the well known risk factors (ME and BM) the CAPM model with only the market portfolio as a risk factor does not hold. The data on these portfolios has been obtained from [this site](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/tw_5_ports.html)<sup>1</sup>.

The average means depicted in Table 1a show that the stylized facts on the BM and ME largely hold: The average return increases with a higher BM while it decreases with an increasing ME. All the while Table 1b shows that a

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<sup>1</sup>[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data\\_Library/tw\\_5\\_ports.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/tw_5_ports.html)

more average BM and a higher ME results in lower volatility. The correlation of one portfolio with any other is on average between .73 and .88, with the smallest correlation per portfolio ranging from .53 to .78. This shows a strong comovement of the different portfolios suggesting the presence of a common driving factor such as the market portfolio return.

Table 1: ME1..ME5 depict the market equity and BM1..BM5 depict the book-to-market ratio. Both show 20-percentiles in ascending order.

(a) Heatmap of the average returns of the different portfolios.

	ME1	ME2	ME3	ME4	ME5
BM1	1.06	1.29	1.33	1.42	1.28
BM2	1.61	1.53	1.58	1.39	1.34
BM3	1.62	1.73	1.59	1.52	1.29
BM4	1.79	1.74	1.67	1.64	1.36
BM5	1.91	1.78	1.84	1.65	1.42

(b) Heatmap of the average monthly realised variance of the different portfolios.

	ME1	ME2	ME3	ME4	ME5
BM1	62.98	51.33	43.72	34.51	21.64
BM2	46.93	35.37	29.90	26.60	19.56
BM3	35.75	29.38	25.04	25.51	18.82
BM4	31.77	27.97	24.08	22.99	18.90
BM5	36.68	35.92	30.05	29.92	24.93

Using the historical returns of the selected portfolios and the market portfolio, we construct the efficient frontier assuming that no riskless asset is available to invest in. The efficient frontier gives the lowest obtainable volatility for a certain return of a portfolio and hence is obtained by solving the following optimization problem:

$$\min_{\pi} \frac{1}{2} \pi' \Sigma \pi \quad \text{subject to} \quad \mu' \pi = \mu_p \quad \text{and} \quad 1' \pi = 1$$

where  $\pi$  is the vector of portfolio weights,  $\Sigma$  is the co-variance matrix and  $\mu$  is equal to the vector of average returns. This gives us the global minimum variance portfolio  $\pi_1$  and its corresponding portfolio  $\pi_u$ . Table 2 contains the average returns and volatility of these portfolios.

Table 2: Characteristics of the returns (in percentages) of the different portfolios.

Portfolio	Mean	Volatility
Minimum Variance Portfolio	1.249	3.387
Other Portfolio	2.525	4.816
Market Portfolio	0.911	4.427

We see that the global minimum variance portfolio achieves a higher expected return than the market portfolio but at the same time also has a lower expected volatility. This can be explained by the fact that the global minimum variance portfolio lies on the efficient frontier while the market portfolio, in this case, does not. This can be shown more clearly as we draw the efficient frontier. Given the fact that the two obtained portfolios are MV efficient, we can use them to generate all other portfolio's on the frontier as any combination of them is also MV efficient. This yields Figure 1 below,

## The world of excess returns

We now transform the returns of the original 25 portfolios and the market return to excess returns by subtracting the risk-free rate obtained from the same dataset. Again we are interested in the minimum volatility obtainable given a certain portfolio return. We now do not have the restriction that all weights on the portfolios must sum to 1 as we have an additional asset, the riskless asset, to invest in (or borrow from). Hence the minimization problem boils down to:

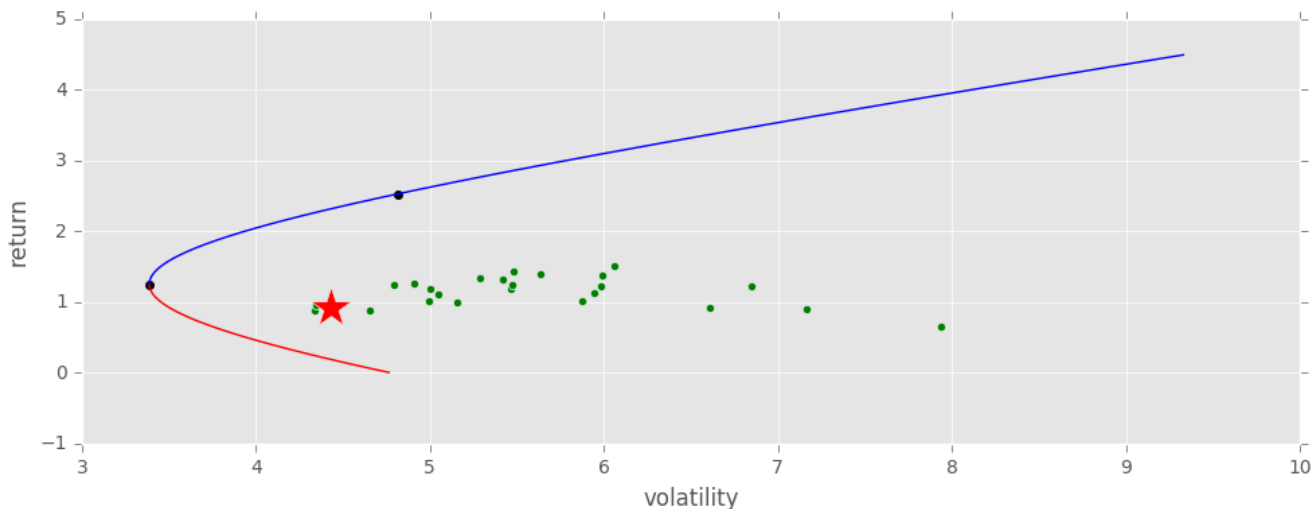


Figure 1: Plot of the efficient frontier and the return/volatility trade-off of the different portfolios. The inefficient part of the frontier is shown in red. The market portfolio is shown as a red star, the tangency and minimum-variance portfolios are black. The selected portfolios are blue.

$$\min_{\pi} \frac{1}{2} \pi' \Sigma \pi \quad \text{subject to} \quad \mu' \pi = \mu_p^e$$

Where all variables are defined as before but the subscript e denotes that we use excess returns. Solving the above yields a portfolio  $\pi_*$  called the tangency portfolio. This portfolio only contains investments in the risky portfolios. We also have an asset that has no risk. For this investable riskless asset we take the average of the riskfree rate over the entire sample period, as one of the assumptions of the CAPM is stationary returns. The characteristics of the portfolios and the riskless asset can be found in table 3.

Table 3: Characteristics of the excess returns (in percentages) of the different portfolios.

Portfolio	Mean	Volatility	Sharpe-ratio
Market Portfolio	0.508	4.441	0.114
Tangency Portfolio	2.737	6.100	0.448
Riskless Asset	0.403	-	-

We see that the tangency portfolio outperforms the market portfolio by a great margin. It achieves both a higher return and a lower volatility resulting in a Sharpe-ratio ( $E[r_p - r_f]/\sigma$ ) multiple of that of the market portfolio.

Using the two-fund separation, with the tangency portfolio and the riskless asset as funds, we can calculate the efficient frontier: every portfolio on the frontier is a linear combination of the riskless asset and the tangency portfolio. Figure 2 shows the efficient frontier without a riskless asset (black), with a riskless asset (blue) and the return/volatility trade-off of the different portfolios.

## The GRS test

We now regress the excess returns of each of our original 25 portfolios  $i$  on the excess return of the market and a constant:

$$R_{i,t} - R_t^f = \alpha_i + \beta_i(R_t^m - R_t^f) + \epsilon_{i,t} \quad (1)$$

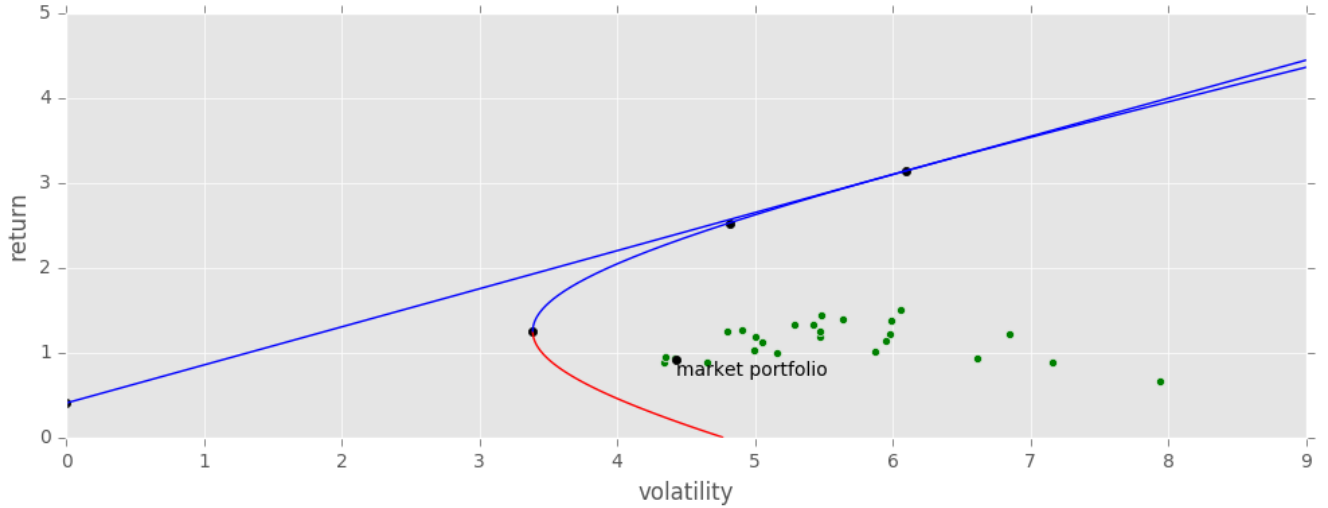


Figure 2: Plot of the efficient frontier and the return/volatility trade-off of the different portfolios. The inefficient part of the frontier is shown in red. The market portfolio is shown as a red star, the tangency and minimum-variance portfolios are black. The selected portfolios are blue.

Applying this regression yields the coefficients and standard errors as shown in table 4. The first thing to notice is that all  $\beta_i$  coefficients are positive and significantly different from zero, meaning that the excess market return has significant explanatory power over the excess portfolio returns. Overall we see that small stocks with a low market capitalization (ME1) are more sensitive to swings in the market returns than their larger counterparts. This indicated by their larger  $\beta$  coefficient. The  $\alpha$  coefficient are almost all significantly different from zero. Some are negative indicating that on average they earn a lower return than one would expect given the sensitivity to the market.

Table 4: Regression results of regression (1). \*: not significant for  $\alpha < 5\%$ . \*\*: not significant for  $\alpha < 1\%$ .

Portfolio	$\alpha$	$\beta$	$se_\alpha$	$se_\beta$	Portfolio	$\alpha$	$\beta$	$se_\alpha$	$se_\beta$
BM1-ME1	** -0.472	1.424	0.193	0.043	BM4-ME3	0.376	0.960	0.098	0.022
BM2-ME1	* 0.185	1.228	0.167	0.037	BM5-ME3	0.512	1.028	0.122	0.027
BM3-ME1	* 0.254	1.099	0.139	0.031	BM1-ME4	* -0.006	1.222	0.092	0.021
BM4-ME1	0.472	1.017	0.136	0.030	BM2-ME4	* 0.038	1.085	0.076	0.017
BM5-ME1	0.556	1.077	0.150	0.034	BM3-ME4	** 0.187	1.032	0.086	0.019
BM1-ME2	* -0.222	1.397	0.145	0.032	BM4-ME4	0.353	0.954	0.091	0.020
BM2-ME2	* 0.133	1.170	0.117	0.026	BM5-ME4	0.312	1.039	0.118	0.026
BM3-ME2	0.382	1.059	0.109	0.024	BM1-ME5	* -0.026	0.987	0.064	0.014
BM4-ME2	0.414	1.016	0.111	0.025	BM2-ME5	* 0.057	0.933	0.063	0.014
BM5-ME2	0.407	1.112	0.137	0.031	BM3-ME5	* 0.038	0.872	0.079	0.018
BM1-ME3	* -0.154	1.329	0.121	0.027	BM4-ME5	* 0.130	0.828	0.093	0.021
BM2-ME3	** 0.204	1.123	0.091	0.020	BM5-ME5	* 0.167	0.887	0.123	0.028
BM3-ME3	0.279	1.004	0.092	0.021					

We perform the GRS test to determine how well the CAPM framework explains the excess returns of the portfolios under scrutiny. The GRS test statistic follows an  $F$ -distribution with (25, 599) degrees of freedom. With  $\alpha = .05$ , the rejection region is  $[1.525, \infty)$ . The computed test statistic is 65.24 with a p-value of 0 up to machine precision. This implies that exposure to volatility leaves a significant part of the excess return unexplained and we should look for other factors to describe the excess return of these portfolios. These results of the GRS test show that Jensen's  $\alpha$  explains a significant part of the excess returns of the portfolio's. According to the CAPM one is only rewarded for the exposure to the market factor and hence  $\alpha$  should be equal to zero. Given the fact that the  $\alpha$ 's are jointly

significant the market factor cannot explain all the excess returns of the portfolios. The Sharpe ratios of the portfolios are also higher than we might expect based on the returns: 70% of the Sharpe ratios of the portfolios are higher than the Sharpe ratio of the market portfolio, clearly indicating that the market portfolio is not efficient. Table 5 shows that there is a profound increase in Sharpe ratios as the Book-to-Market ratio increases. For the Market Equity we see that the Sharpe ratio only increases with smaller size for the highest 3 BM groups. For the group with the lowest BM smaller ME also means lower Sharp ratios.

Table 5: The Sharpe ratios of the portfolios. The market Sharpe ratio is 0.115.

	ME1	ME2	ME3	ME4	ME5
BM1	0.032	0.068	0.079	0.104	0.102
BM2	0.118	0.122	0.141	0.114	0.120
BM3	0.136	0.169	0.157	0.140	0.111
BM4	0.175	0.176	0.176	0.174	0.127
BM5	0.182	0.162	0.188	0.153	0.124

Hence, the market is certainly not efficient under the assumptions of the CAPM. Further research should be performed to investigate which assumptions are violated and whether other factors play a role in the mean/variance trade-off decision of investors. Well known factors are off course the book-to-market ratio and the size factor. Adding these to the CAPM model and thus transforming it into a 3-factor model may take away the explanatory power of the  $\alpha$ 's.

## Extending the CAPM with hedge portfolios

In the previous section we concluded that the CAPM does not describe the entire relation between volatility and return for our portfolio selection. Tables 1a and 1b clearly show that return and volatility are clustered for different combinations of market equity and book-to-market ratio. This suggests that portfolios describing the book-to-market ratio- and market equity- effect on the mean-volatility trade-off can be used to hedge part of the trade-off that is not explained by the CAPM. Hence we construct two hedge portfolios with a small variation to [Fama and French(1993)]:

**SMB** We construct the classical ‘small-minus-big’ factor as follows:  $SMB = (\overline{ME1} - \overline{ME5})$ , where  $\overline{MEi} = \sum_{k=1}^5 (BMk, MEi)$ .

**HML** For the ‘high minus low’ portfolio we weigh the different size portfolios equally, not by value. We did so because the effect of the BM is much more present in portfolio ranges with a smaller ME, and this effect would largely be mitigated if the high-ME portfolios are given larger weights. HML is constructed as:  $HML = (\overline{BM5} - \overline{BM1})$ , where analogously  $\overline{BMi} = \sum_{k=1}^5 (BMi, MEk)$ .

## Properties of the hedge portfolios

The SMB and HML both have slightly positive means. The histograms in Figure 3 show skewed and fat-tailed distributions. This is confirmed by the Jarque-Bera test, which rejects normality of the SMB, HML and markets series with p-values of 0 up to machine precision. The SMB and HML time series in Figure 3 also show heterogeneity. When we regress SMB resp. HML on the excess market returns and a constant both regressions have poor explanatory power with an  $R^2$  of 0.073 resp. 0.095. The constant is not significant in explaining SMB with a p-value of .46, suggesting that there is no consistently positive effect on market returns that is explained by SMB but not by the excess market returns. The market is significant (p-val:6e-12) with coefficient 0.25, indicating that SMB and excess market returns have a small but significant co-movement.

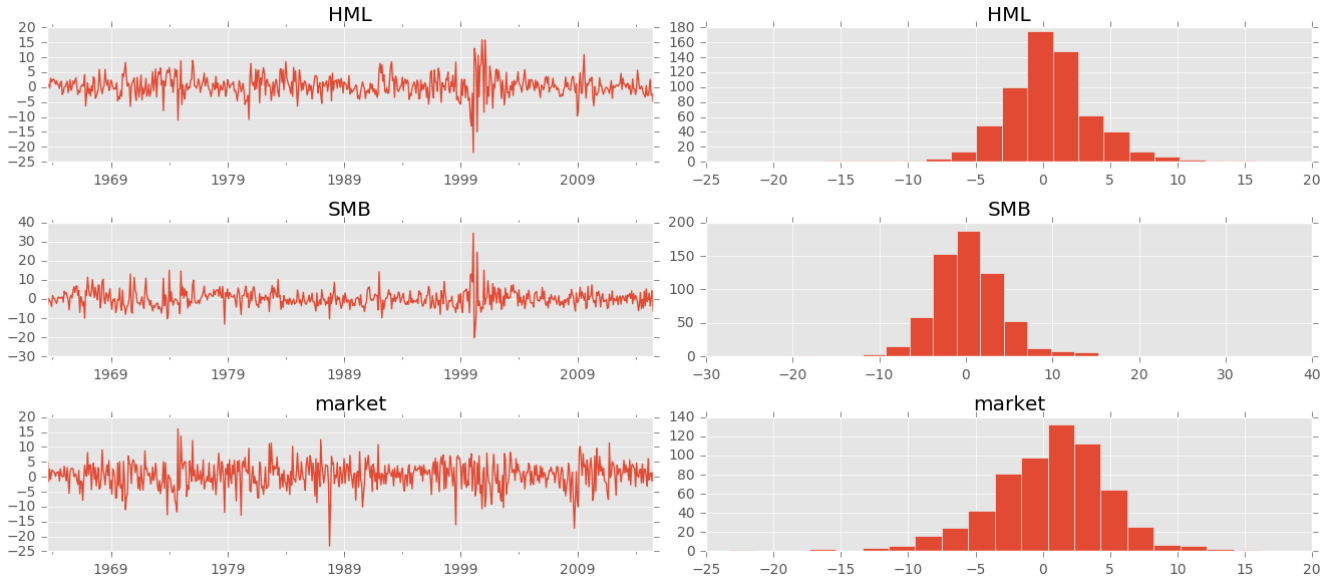


Figure 3: Time series and histograms of the SMB, HML and market factors.

Table 6: Statistical properties of the factors making up the hedge portfolios. The Jarque-Bera (JB) statistic rejects normality with .1% type-I error risk if  $JB > 13.82$ . The regression results report the standard error in parentheses.

	SMB	HML	market
mean	0,262	0,443	0,508
std	4,399	3,511	4,441
min	-20,156	-21,946	-23,24
max	34,364	15,852	16,1
skew	1,106	-0,223	-0,536
kurt	8,101	4,733	1,988
JB	1804	577	130

$$\text{SMB} \sim 0.126(0.166) + 0.268(0.043) \times \text{market} \quad (2)$$

$$\text{HML} \sim 0.567(0.135) - 0.243(0.041) \times \text{market} \quad (3)$$

The constant has significant explanatory power (p-val:3e-5) on the HML with coefficient .50, hence HML describes a positive effect on market returns that is not captured by the excess market returns. The excess market returns themselves explain a negative effect (-0.25, p-val:3e-15) on HML.

## Relevance of the hedge portfolios

To judge the relevance of the hedge portfolios in explaining the portfolio returns we regressed the portfolios on the excess market returns, SMB, HML and a constant as shown in equation (4). The SMB and HML portfolios show a significant explanatory power on almost all portfolios. The exceptions are (ME5,BM5) where the effect of SMB is insignificant at  $\alpha = 5\%$  and (ME1,BM2) where HML is insignificant at  $\alpha = 1\%$ . Furthermore the SMB and HML factors strongly correlate with the portfolio properties ME and BM while the portfolio properties show little correlation with the sensitivity to the market and the constant term, as is visually apparent in table 7.

$$\text{BMi MEj} = \alpha_{ij} + \beta_{0ij} \times \text{HML} + \beta_{1ij} \times \text{SMB} + \beta_{2ij} \times \text{Market} \quad (4)$$

Table 7: Heatmaps of the regression coefficients of regression (4).

	HML					SMB					const					Mrkt				
	ME1	ME2	ME3	ME4	ME5	ME1	ME2	ME3	ME4	ME5	ME1	ME2	ME3	ME4	ME5	ME1	ME2	ME3	ME4	ME5
BM1	-.253	-.336	-.388	-.382	-.276	.995	.611	.426	.202	-.172	-.454	-.108	.013	.185	.152	1.096	1.152	1.121	1.075	.966
BM2	.055	.122	.163	.178	.094	.943	.571	.323	.124	-.162	.035	-.008	.071	-.079	.024	.99	1.046	1.076	1.095	.999
BM3	.267	.336	.371	.369	.217	.809	.509	.282	.11	-.156	.001	.128	.033	-.036	-.065	.947	1.005	1.019	1.092	.966
BM4	.402	.459	.501	.453	.444	.778	.492	.265	.161	-.123	.146	.093	.059	.076	-.106	.906	.996	1.011	1.021	.969
BM5	.626	.703	.695	.721	.619	.838	.62	.408	.22	-.023	.095	-.069	.067	-.124	-.181	1.005	1.117	1.088	1.155	1.044

Table 8: Coeffients and standard errors for 25 regressions explaining the selection of portfolios through the \*: not significant for  $\alpha \leq 5\%$ . \*\*: not significant for  $\alpha \leq 1\%$ .

	HML	SMB	const	market		HML	SMB	const	market
ME1 BM1	-.253(.028)	.995(.023)	-.454(.068)	1.096(.018)	ME3 BM4	.501(.032)	.265(.023)	*.059(.072)	1.011(.019)
ME1 BM2	** .055(.028)	.943(.022)	*.035(.054)	.990(.018)	ME3 BM5	.695(.028)	.408(.021)	*.067(.074)	1.088(.019)
ME1 BM3	.267(.022)	.809(.016)	*.001(.045)	.947(.012)	ME4 BM1	-.382(.029)	.202(.026)	.185(.064)	1.075(.017)
ME1 BM4	.402(.021)	.778(.014)	.146(.046)	.906(.014)	ME4 BM2	.178(.034)	.124(.023)	*.079(.073)	1.095(.022)
ME1 BM5	.626(.022)	.838(.021)	** .095(.046)	1.005(.014)	ME4 BM3	.369(.034)	.110(.026)	*.036(.073)	1.092(.024)
ME2 BM1	-.336(.034)	.611(.030)	*.108(.077)	1.152(.019)	ME4 BM4	.453(.032)	.161(.023)	*.076(.072)	1.021(.021)
ME2 BM2	.122(.034)	.571(.025)	*.008(.072)	1.046(.019)	ME4 BM5	.721(.028)	.220(.022)	*.124(.073)	1.155(.019)
ME2 BM3	.336(.032)	.509(.025)	*.128(.068)	1.005(.019)	ME5 BM1	-.276(.019)	-.172(.016)	.152(.051)	.966(.014)
ME2 BM4	.459(.027)	.492(.020)	*.093(.067)	.996(.017)	ME5 BM2	.094(.024)	-.162(.018)	*.024(.055)	.999(.016)
ME2 BM5	.703(.026)	.620(.019)	*.069(.069)	1.117(.018)	ME5 BM3	.217(.030)	-.156(.021)	*.065(.068)	.966(.019)
ME3 BM1	-.388(.028)	.426(.025)	*.013(.068)	1.121(.019)	ME5 BM4	.444(.026)	-.123(.017)	*.106(.064)	.969(.018)
ME3 BM2	.163(.035)	.323(.029)	*.071(.075)	1.076(.020)	ME5 BM5	.619(.039)	*.023(.032)	**-.181(.090)	1.044(.027)
ME3 BM3	.371(.032)	.282(.023)	*.033(.072)	1.019(.020)					

## Results of the extended model

To test the performance of the extended model (4) we perform a GRS test to test the joint significance of all intercepts. The GRS statistic has value 0.0063. This statistic follows an F distribution with (25, 599) degrees of freedom such that the GRS test fails with a p-value of 1 up to machine precision to reject the hypothesis that the intercepts are jointly significant. Table 8 shows that virtually all factor coefficients are significant while the greater part of the intercepts are insignificant, which is in line with the GRS test.

From the preceding we conclude that the CAPM extended with the HML and SMB factors explains the returns on the portfolios that this report has considered. Moreover the HML and SMB factors are indispensable in explaining the returns of these portfolios.

## The influence of momentum

We now look at whether momentum has a significant influence on the returns of the portfolios. To do so, we construct a momentum factor by taking the cumulative returns over the period  $t - 12$  to  $t - 2$  (so not using the previous month). We then regress, for each month, the observed portfolio returns on the cumulative returns of the portfolios. This yields 613 separate regressions. The resulting slope coefficients of these regressions are plotted in figure 4a.

The slope coefficients seem to be both positive and negative with a mean around zero. If high (low) previous returns would also mean higher (lower) current returns then we would expect the slope coefficients to be generally positive. On the other hand if periods of high (low) returns are followed by a reversal and hence lower (higher) returns we would expect negative slope coefficients. Given the fact that coefficients in figure 4a do not show a clear cut sign, momentum does not seem to be an important effect. To formally test this we take the average of all the obtained slope coefficients and see if this average is significantly different from zero. With mean of 0.022 and a sample standard deviation of 0.128 a t-test, with a corresponding p value of 0.432, cannot reject the null hypothesis that the average of the slope coefficients is different from zero. There seems to be no momentum effect in our data.

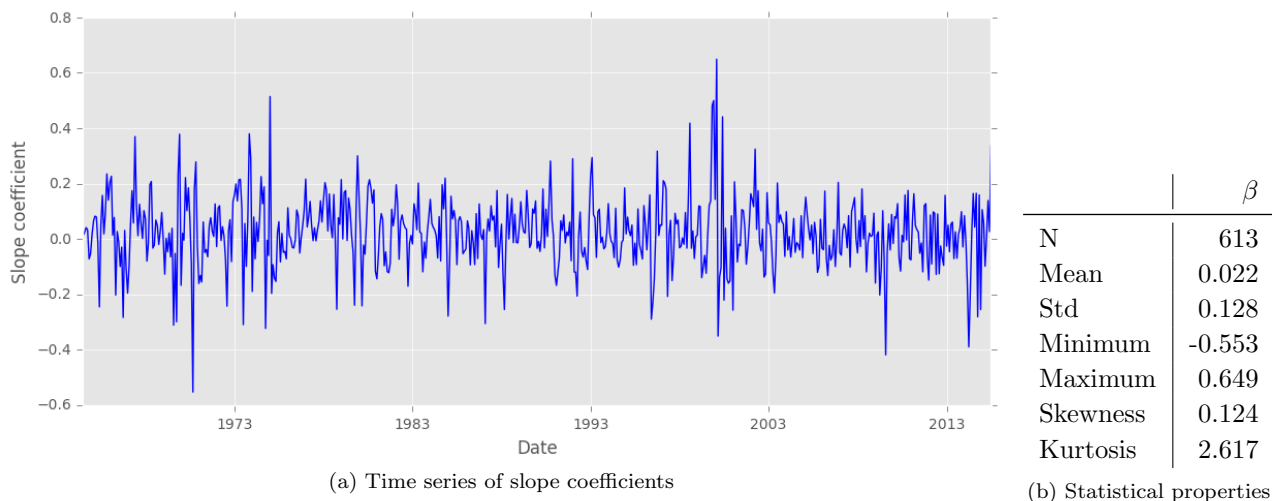


Figure 4: Statistical properties and time series of the slope coefficients from the first step of the Fama-MacBeth procedure.

## Implementation of feedback

### Are the steps of the analysis clearly described?

*In step 2, they left out to describe the market return and variance, which is crucial for the analysis. In step 3, they call the other portfolio besides the GMV the tangency, but that is just another portfolio without any worth mentioning meaning. Step 4 clearly describes the procedure, but doesn't provide the actually results op the weights of the tangency portfolio, which is a quite odd. Also, they don't describe its properties in step 4, but in step 5, which is a bit confusing. They also don't discuss the gross risk-free rate in step 5, so we are wondering if they used one rate for all data points or that they just picked a number for the gross risk-free rate and don't mention it because they perhaps can't motivate their choice. Also, they forgot to draw the inefficient part of the frontier with a riskless asset. In step 7, the procedure of the GRS test is not described at all. The rest of this report is clearly described. One last remark overall, is that the steps are numbered, which is not the right thing to do. According to the assignment, you can only use these steps to structure it, not really use them in the actual report.*

The properties of the market portfolio were described in table 2. Note that the assignment also asks for how the 'other' portfolio looks and hence it was included in table 2. We have not included the weights of the portfolio as with 25 assets this would results in very large, and in our eyes uninformative, tables. We cleary mention that we use the average of the riskfree rate over the entire sample period. We have removed the numbering as we agree that without it the report flows better. We feel that how the GRS test works is beyond the scope of this report. However if we have enough space to work with in the end we will elaborate further on how it works. We improved the figures to show the inefficient part of the frontier.

### Are the steps of the analysis clearly motivated? Do you agree?

*The motivation of the choice portfolios is a bit unclear. We don't understand why they chose the factors size and bm instead of for example also momentum. In step 3, the motivation why the GMV performs better than the market portfolio is on the other side great. They say in step 6 that all the beta's are significantly different from zero, but we don't say anywhere prove of that in a form of a test. Furthermore, everything else is motivated in a clear way.*

The t statistic of the  $\beta$  coefficient is equal to the coefficient divided by its standard error, thus can be easily derived



from the table. For the readers' convenience we have included significance markers at two levels of significance.

### **Are the results correct and clearly reported?**

*In table 1a, the average returns for the different portfolios do look normal. That is, when looking at the portfolios sorted on size, the historical relation between smaller firms and higher returns is satisfied. Also, a low book-to-market ratio may indicate growth opportunities and in this table the growth opportunities may be symbolized by the low returns. Therefore, no strange results are present in table 1a. In table 1b then, higher realized variances are obtained for smaller firms and this is in accordance with observations from the past. This result could be reasonable, as the financial balance of smaller firms may be less stable than that of bigger firms. From figure 1, there can be concluded that the Global Minimum Variance (GMV) portfolio and the other portfolio, which are obtained from minimizing the variance given a desired return, a budget constraint and the absence of a risk free asset, are correctly calculated. Namely, the GMV portfolio lies on the outer left point of the efficient frontier and the other portfolio lies on the efficient frontier. Also, the market portfolio lies in the efficient frontier, which is correct. Also, it is clearly explained why the GMV portfolio has a lower volatility and a higher return than the market portfolio. Then, in figure 2 the frontiers with and without a risk-free asset are drawn. It contains the tangency portfolio instead of the drawn GMV and the other portfolio in figure 1. The results of the portfolios are clearly reported in table 3. Then, in table 4 the estimated betas of the GRS test are all roughly somewhat more than 1 in value which indicates the portfolios are somewhat higher estimated in return than the market portfolio and the values of beta differ to some extent for each portfolio and this is clearly explained. The standard errors of the alphas and betas do look correct and the GRS test seems to be clearly reported by including the rejection region and p-value. The conclusion of the GRS test is clearly reported by reasoning using the Sharpe ratios of the market portfolio and the market portfolio with test assets. As a last note, from table 5 the conclusion was drawn that portfolios with higher book-to-market ratios and an average size implicitly means on average higher sharpe ratios. There could be more elaborated why this is the case for this set of portfolios.*

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Elaborated a bit on the Sharpe Ratios.

### **Do the implications and conclusions follow from the results?**

*On page 2 there is a logical explanation for the high correlations between different portfolios, namely, due to the presence of a common driving factor. Next, from figure 1 there is a logical explanation about why the return and volatility of the market portfolio are respectively lower and higher than the return and volatility of the GMV portfolio. Namely, they argued that the GMV lies on the efficient frontier, instead of the market portfolio. However, in table 3 sharpe ratios of the tangency portfolio and the market portfolio are given, but it could be mentioned that the tangency portfolio significantly outperforms the market portfolio. Nothing is said about this. Then, in table 4 there is talked about the values of the betas and that they are significant, but it is not clearly explained what the implication of the significant betas is. Subsequently, there is a good conclusion about the result of the GRS test. Namely, the CAPM is not able to predict the portfolios well given the assumptions of the CAPM and the sharpe ratio of the market portfolio including test assets outperforms the sharpe ratio of the market portfolio. However, it is again not clearly mentioned that this outperformance is significant. At last, the authors argue that some other factors could be included to reduce the mentioned outperformance, which is a logical solution.*

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We already clearly mentioned the fact that the tangency portfolio achieves higher returns for a lower volatility, and hence a higher Sharpe ratio, than the market portfolio.

### **Is the use of graphs and tables good? Is the use of language good?**

*When looking at the graphs, they are just fine, meaning that the graphs contain all necessary information that is needed to answer the questions about the efficient frontier. The tables are also just fine and standard, except the heat*

*map, a nice feature as the reader can quickly determine how big or small a particular value is. The use of language does not deviate much from the use of language in a standard report.*

## References

[Fama and French(1993)] Eugene F. Fama and Kenneth R. French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3 – 56, 1993. ISSN 0304-405X. doi: [http://dx.doi.org/10.1016/0304-405X\(93\)90023-5](http://dx.doi.org/10.1016/0304-405X(93)90023-5). URL <http://www.sciencedirect.com/science/article/pii/0304405X93900235>.