Hand-in intermediate versions - Part 1 (BB 52221352)

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S.H.L.C.G. VERMEULEN'S PEERMARK REVIEW OF ANONYMOUS'S PAPER (0% COMPLETED)

ASSIGNED QUESTIONS

1. Are the steps of the analysis clearly described?
2. Are the steps of the analysis clearly motivated? Do you agree?
3. Are the results correct and clearly reported?
4. Do the implications and conclusions follow from the results?
5. Is the use of graphs and tables good? Is the use of language good?

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No comments added

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1. Introduction

All investors face the trade-off of return and volatility. They aim to maximize their return for a given volatility. To do this, investors try to outperform the market portfolio. But what does it mean if it is possible for those investors to succeed, especially from an asset pricing point of view? In this report an analysis of different portfolios compared to the market portfolio will be discussed.

2. Data

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3. Market Efficiency Analysis

3.1 Mean-Variance Efficiency Analysis

In this section we evaluate whether the market portfolio is mean-variance efficient for two cases with and without the presence of a riskless asset.

Firstly, we construct the efficient frontier based on the chosen 6 portfolios and the market portfolio and without a riskless asset. This efficient frontier in the mean-volatility space contains all efficient portfolios in this setting. That also means all portfolios are either on or inside this frontier. We will check whether the market portfolio is on the efficient part of the frontier and is consequently an efficient portfolio.

To calculate the variance of a portfolio on the efficient frontier, denoted by σ_p^2, we use the formula

$$\sigma_p^2 = \frac{A - 2B\mu_p + C\mu_p^2}{AC - B^2},$$

which depends on the desired return, μ_p, and where $A = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$, $B = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\mathbf{1}$ and $C = \mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}$ with $\boldsymbol{\mu}$ denoting the mean and $\boldsymbol{\Sigma}$ the covariance matrix of the portfolio returns and $\mathbf{1}$ representing a $(6+1)$ dimensional vector of ones. Figure 1 shows a plot of the resulting volatilities against the desired returns (dashed line). We observe that none of the chosen portfolios lays on the upper part of the frontier. Thus none of them is efficient in this setting. However, we can construct two efficient portfolios with the corresponding portfolio weight vectors π_1 and π_μ, where π_1 corresponds to the global minimum variance (GMV) portfolio. Table 1 shows the weights of how much is invested in the base portfolios to obtain these two constructed portfolios and their corresponding expected monthly return and volatility. The GMV portfolio has an expected monthly return of 1.15% with a corresponding monthly volatility of 3.86.

Table 1: Constructed efficient portfolios including global minimum variance and tangency portfolio

	Expected	*Expected*	*Portfolio weights*						
	Return	***Volatility***	Small growth	Small neutral	Small value	Big growth	Big neutral	Big value	Market
π_1	1.15%	3.86%	-1.04	1.12	-0.25	-0.18	-0.25	-0.20	1.81
π_μ	2.02%	5.10%	-2.14	2.38	0.69	1.30	-0.63	-0.47	-0.13
π_*	2.49%	6.46%	-2.75	3.07	1.21	2.11	-0.84	-0.62	-1.19

Secondly, we construct the efficient frontier based on the chosen 6 portfolios and the market portfolio again, but now with the presence of a riskless asset, which has an average monthly return of R_f and a volatility of 0. This assumption enables investors to borrow and lend against the riskfree rate. Thus every investor will only invest in 2 portfolios: the riskfree asset and an efficient portfolio, such that the combination has the maximal expected return possible for a given volatility. That means one will choose the efficient portfolio on the hyperbolic efficient frontier that creates a tangent with the riskfree asset. We call this portfolio tangency portfolio. The created tangent represents all possible combinations an investor would invest in and symbolizes the efficient frontier in this setting.

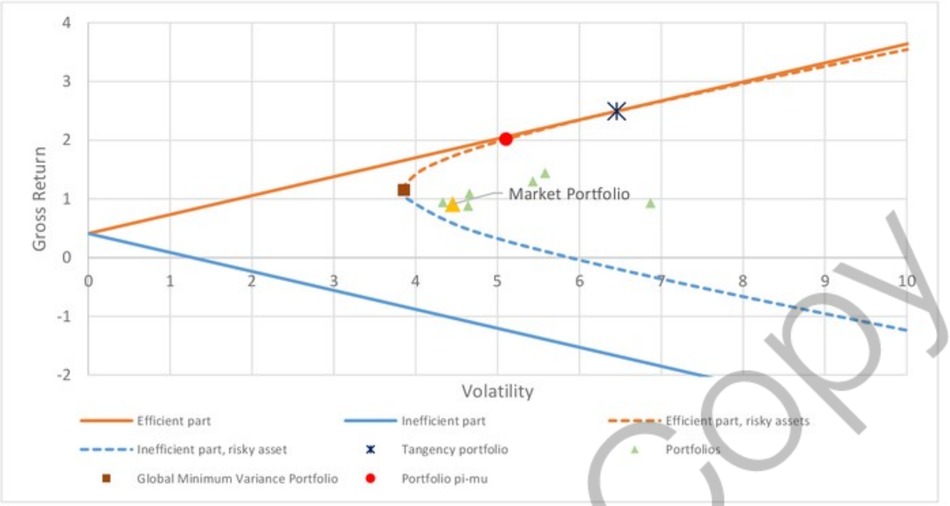

Figure 1: Efficient frontiers (with and without riskless asset)

We calculate the tangency portfolio by using the following equation:

$$\boldsymbol{\pi}_* = \frac{1}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}^e}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}^e.$$

Here $\boldsymbol{\mu}^e = \boldsymbol{\mu} - R_f\mathbf{1}$ denotes the excess returns of the set of portfolios and the vector $\boldsymbol{\pi}_*$ contains the weights of how much has to be invested in the base portfolios to obtain the tangency portfolio. These weights and the corresponding expected return and volatility are shown in Table 1. Figure 1 shows the resulting efficient frontier. We see that the tangency indeed creates a tangent on the hyperbolic efficient frontier. Furthermore, we observe that for a given volatility the returns in the setting with a riskless asset are higher than in the setting without – only for the tangency portfolio we obtain the same return.

From this mean-variance analysis we conclude that the market portfolio is not mean-variance efficient. This results from the fact that it is not on the efficient frontier. That means we could achieve the same return with a smaller risk, which is expressed by the volatility. We observed a sign for this in the descriptive statistics in **Error! Reference source not found.**, as the big neutral portfolio gives a higher return than the market portfolio with a smaller corresponding volatility.

3.2 Testing of Capital Asset Pricing Model

To be able to test whether the Capital Asset Pricing Model (CAPM) is able to explain the cross-sectional differences of the portfolios, we use the GRS-test. This means we start by performing the regression

$$r_{i,t} = \alpha_i + \beta_i r_t^m + \varepsilon_{i,t}\,,$$

where $r_{i,t}$ is the excess return on portfolio i, α_i is the intercept, β_i the coefficient for the market excess return, r_t^m the market excess return itself and $\varepsilon_{i,t}$ an error term that has an expectation of zero. From the results of the regression, the sample variance of the error term, $\widetilde{\boldsymbol{\Sigma}}$, and the Sharpe ratio of the market portfolio, $Sh[r^m]$, we can calculate the test statistic

$$\mathrm{z} = \frac{\mathrm{T}-n-1}{n}\,(1 + Sh[r^m]^2)^{-1}\,\widehat{\boldsymbol{\alpha}}'\,\widetilde{\boldsymbol{\Sigma}}^{-1}\widehat{\boldsymbol{\alpha}}\,,$$

where T is the number of observations and n the number of portfolios. Using the F-distribution with degrees of freedom of n and T-n-1, we will calculate the p-value. A small p-value means that the portfolios can significantly improve the Sharpe ratio of the market portfolio. Finally, to see how much the Sharpe ratio can be improved, the Sharpe ratio of the efficient portfolio of the test portfolios and the market portfolio is calculated by the formula

$$Sh[(\boldsymbol{r}', r^m)'] = \sqrt{\widehat{\boldsymbol{\alpha}}' \, \widetilde{\boldsymbol{\Sigma}}^{-1} \widehat{\boldsymbol{\alpha}} + Sh[r^m]^2} \, .$$

To be able to do this finite sample GRS test, we need to make a few assumptions, which are listed below.

1) The error terms are independently identically distributed and follow a normal distribution.
2) The market premium is exactly equal to its sample average.
3) The expectations (moments) are constant over time.

In addition to the finite sample version of the GRS test, the asymptotic GRS test is calculated. For this test, part of the first assumption, the assumption of normality of the error terms, can be relaxed. The test statistic for asymptotic GRS testing is defined as

$$\mathrm{z} = \frac{\mathrm{T}}{1 + Sh[r^m]^2} \, \widehat{\boldsymbol{\alpha}}' \, \widetilde{\boldsymbol{\Sigma}}^{-1} \widehat{\boldsymbol{\alpha}} \, .$$

The corresponding p-value can be found using the chi-squared distribution with the number of portfolios (n) as the degrees of freedom.

Performing the regression mentioned above, we obtain regression coefficients and standard errors for the different portfolios, gathered in the following table.

Table 2: Regression Coefficients

	Small growth	*Small neutral*	*Small value*	*Big growth*	*Big neutral*	*Big value*
α_i	-0.152	0.354	0.504	-0.0343	0.0833	0.227
s.e. α_i	0.137	0.105	0.122	0.0462	0.0655	0.0925

Only the coefficients of the big growth and the big neutral portfolios come relatively close to zero. Compared to its standard error the big neutral portfolio is not even that close to zero. Therefore Table 2 indicates that the pricing errors are not equal to zero, which means that the CAPM probably does not hold.

Using the data of the six portfolios, we obtain the following GRS-test results.

Table 3: GRS Results

	GRS (Finite Sample)	*GRS (Asymptotic)*
Value of Test Statistic	9.118	55.34
P-Value	1.447E-09	3.95E-10

As the p-values in both cases are very small, the hypothesis of pricing errors being equal to zero should be rejected at any reasonable significance level. Thus we conclude that the CAPM does not hold.

The GRS also shows how much the market portfolio can be improved by means of Sharpe ratios. Table 4 shows the calculated Sharpe ratios. According to these results, the possible significant improvement is equal to 0.21. This means the Sharpe ratio of the market portfolio should be able to increase to almost three times of its initial value. That seems to be very high and therefore unrealistic. This could be a result of the made assumptions, which might not hold in reality. In particular, the assumption of homoscedastic error terms may be doubted.

Table 4: Sharpe Ratios

$Sh[r^m]$	0.11
$Sh[(\boldsymbol{r}', \boldsymbol{r}^m)']$	0.32
$Sh[(\boldsymbol{r}', \boldsymbol{r}^m)'] - Sh[\boldsymbol{r}^m]$	0.21

To summarize the market efficiency analysis, we conclude that the market portfolio is not mean-variance efficient as shown by the mean-variance efficiency analysis using efficient frontiers and the GRS-test results. The Sharpe ratio of market portfolio can be improved significantly and using the set portfolios we can construct mean-variance efficient portfolios that perform significantly better than the market portfolio itself.