# A Decision-Theoretic Model of Optimal Penalty Strategies in Rugby Union


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# 1 Introduction

This project examines the central strategic decision in rugby union: whether a team awarded a penalty should kick at goal for three points or kick to touch to pursue a try. Although this choice has long been debated and often decides close matches, it remains guided largely by intuition rather than quantitative evidence. Despite its strategic importance, no existing framework evaluates penalty decisions in a way that incorporates both the current match situation and the broader incentives created by league structures. This project develops the first integrated, data-driven model capable of identifying the optimal penalty action for any game state and for any relevant objective.

The analysis focuses on three objectives that teams may prioritise at different stages of a season: maximising the probability of winning the match, maximising expected league points under the bonus-point system, and pursuing longer-run goals such as finishing high enough to be promoted or avoiding positions associated with relegation. These objectives are evaluated independently rather than combined into a single utility function. Although related, short-term and long-term incentives can differ markedly: a decision that increases the chance of winning may not improve a team's league position, while decisions that reduce relegation risk may not be optimal for winning on the day.

Methodologically, the project combines three modelling components. First, in-game set-piece models return distributions over possible next game states after kicking at goal or kicking to touch. Second, a scoring model maps these states into distributions over match outcomes, which can be converted into win probabilities and league-points pairs. Third, a league-simulation model evaluates each league-points pair by simulating remaining fixtures in the season. The empirical application uses data from the All-Ireland League, but the framework is designed to be adaptable to any level of the sport.

# 2 Proposal

## 2.1 Objectives & Central Research Questions

The central aim of this project is to construct a quantitative, decision-theoretic framework that identifies the optimal penalty action in rugby union—kick at goal or kick to touch—for any given match state and team objective. Formally, the project seeks to determine the action that maximises a chosen outcome measure (win probability, expected league points, promotion probability, or relegation safety) conditional on time remaining, score difference, field position, and the relative strengths of the competing teams. The output of the model is a conditional policy function: for any match state and any user-specified objective, the framework maps each penalty option to its expected consequences and selects the action that maximises the chosen objective.

Although these objectives are evaluated separately rather than combined into a single utility function, each captures a fundamentally different incentive structure. A decision that maximises win probability may not maximise expected league points, and league points themselves are only instrumental to longer-run goals such as promotion or relegation survival. These distinctions make it essential to analyse all three objectives, as they can imply genuinely different optimal actions in identical match states.

*All three objectives are evaluated within a single unified modelling framework, so the project remains conceptually focused while illuminating how different incentive structures lead to different optimal decisions.*

To address this aim, the project pursues the following specific objectives:

- **Develop and estimate a Weibull competing-hazards model of rugby scoring.** This model captures the timing and type of scoring events and will be compared with Poisson and negative binomial alternatives. It provides the mechanism for translating simulated match trajectories into win probabilities and league-points outcomes.

- **Construct a penalty transition function that maps each action into a distribution over subsequent game states.** This function incorporates dedicated in-game models for goal-kicking success, territorial outcomes from touch-finders, and try-scoring probabilities from lineouts.

- **Simulate full match score distributions using the integrated in-game and pre-game scoring models.** These simulations allow the evaluation of how each penalty option affects match outcomes and league-points pairs.

- **Quantify the season-level consequences of alternative penalty decisions through Monte Carlo league simulations.** By simulating all remaining fixtures in the season, the project evaluates how different league-points outcomes change a team's promotion and relegation probabilities.

## 2.2 Motivation

Penalty decisions are among the most influential moments in rugby union, frequently shaping the outcome of tight matches. Yet they remain largely guided by intuition, experience, or perceived momentum rather than by formal quantitative evidence. The choice between kicking at goal for three points or kicking to touch in pursuit of a try involves balancing immediate scoring prospects against potential territorial gains, time remaining, and the strengths of both teams. These factors interact in complex and often counterintuitive ways, making it difficult for coaches and captains to reliably identify the option that best serves their team's interests in a given situation.

Beyond their effect on individual matches, penalty decisions have broader implications within league structures. Modern competitions, including the All-Ireland League, employ bonus-point systems and promotion–relegation mechanisms that create incentives extending well beyond the immediate match result. A decision that maximises the probability of winning may not maximise expected league points, and a strategy that improves expected league points may not enhance a team's chances of finishing in a promotion position. Conversely, teams facing relegation threats may prefer actions that increase the likelihood of securing losing bonus points, even if this reduces their probability of winning on the day. These competing incentives turn penalty decisions into an applied economics problem involving risk, intertemporal objectives, and institutional design. The results also speak to league incentive design, showing how bonus-point systems and promotion–relegation rules shape strategic behaviour and potentially distort match-level decision-making.

Despite the strategic and economic significance of these decisions, there is currently no modelling framework that links set-piece choices to match outcomes and season-level consequences. Existing rugby analytics tools—such as expected-points models—do not account for dynamic game states or league objectives. This project fills that gap by developing a unified framework that, for each penalty option and objective, produces probability distributions over match and league outcomes, providing a transparent and evidence-based basis for in-game decision-making.

## 2.3 Literature and How It Informs the Proposal

The existing literature on rugby analytics is limited, particularly in relation to in-game decision-making. The closest contribution is Fitzpatrick and Nolan (2024), who introduce an expected-points (xP) framework for valuing kicks at goal and possessions. Their model demonstrates how historical states can be used to estimate expected outcomes, in parallel with the NFL's Expected Points and soccer's expected-goals methodologies. However, the xP framework does not model state transitions, does not incorporate match time or score as dynamic variables, and does not link decisions to league incentives. This project builds on their intuition but extends it substantially by modelling full state transitions and evaluating actions under multiple objectives.

A separate body of literature examines scoring dynamics in football using renewal and hazard-based approaches. Boshnakov, Kharrat, and McHale show that Poisson models fit football scoring poorly, with Weibull count models providing superior in-sample and out-of-sample performance. Their results support the use of non-Poisson renewal models when scoring hazards vary over time. Related work on bivariate Weibull count models further demonstrates that inter-arrival time modelling can capture dependence between competing teams. These studies directly inform the methodological choice to adopt a Weibull competing-hazards model for rugby scoring.

Monte Carlo simulation is another widely used tool in sports forecasting, particularly for evaluating end-of-season outcomes. Lahvička (2012) models the remainder of an English Premier League season repeatedly to estimate how match results affect final standings—quantifying the importance of each match in terms of its impact on promotion and relegation probabilities. His approach validates the use of league-wide simulation to capture how individual game outcomes propagate into long-run incentives. This directly supports this project's use of Monte Carlo methods to evaluate how different penalty decisions shift a team's chances of promotion or relegation.

Other sports have also developed decision-theoretic tools for evaluating in-game strategy. Romer (2006), for example, uses dynamic programming and play-by-play data to assess NFL fourth-down decisions. He finds that coaches frequently deviate from the win-maximising action, suggesting a gap between intuition and optimal play. This strengthens the motivation for a data-driven decision framework in rugby, particularly in high-pressure situations with long-term consequences.

A closely related methodological contribution comes from basketball. Cervone et al. (2016) develop a multiresolution stochastic-process model that computes the expected possession value (EPV) by combining continuous-time hazard modelling with a discrete state-transition structure. Their framework demonstrates how micro-level event models, transition kernels, and dynamic value functions can be integrated to evaluate the consequences of alternative actions in real time. This provides a strong methodological analogue for the present project, which similarly links state transitions from set-piece actions to downstream scoring and match outcomes.

This project advances the literature by integrating set-piece outcome models, hazard-based scoring processes, and league simulations into a unified framework for evaluating penalty decisions under different strategic objectives. By drawing on tools from rugby, football, American football, *and basketball*, it addresses a fundamental question in applied sports economics: how should teams act under uncertainty when incentives are dynamic, high-stakes, and season-dependent?

## 2.4 Contribution

This project makes three main contributions to the literature. First, it introduces the first decision-theoretic framework for penalty choices in rugby union. Unlike existing expected-points models, this framework is fully decision-theoretic: it evaluates alternative penalty actions under explicit objectives using probabilistic state transitions. Existing rugby analytics tools do not model such transitions or account for season-level incentives. This project extends the literature by formally valuing each penalty option under multiple objectives—win probability, expected league points, promotion probability, and relegation risk—within a unified probabilistic framework. The approach is closely aligned with decision models in other sports (e.g., Romer's fourth-down analysis in the NFL), where coaches have been shown to systematically deviate from the win-maximising choice, highlighting the relevance of formal decision tools.

Second, it provides a methodological contribution through the development of the first Weibull competing-hazards scoring model for rugby. This approach captures time-varying scoring rates across scoring types and integrates naturally with the set-piece transition models, offering a more flexible and realistic representation of match scoring than standard Poisson or negative binomial alternatives.

Third, the project links micro-level set-piece outcomes to macro-level league consequences via Monte Carlo simulation of the remaining fixtures in a season. This connection between in-game decisions and season-long incentives has not been explored in prior rugby analytics, and the resulting framework is adaptable to other leagues and levels of the sport. All components—set-piece models, hazard-based scoring model, and league simulations—are computationally lightweight, modular, and fully feasible using the available IRFU and Opta datasets.

## 2.5 Research Design and Methodology

The project integrates set-piece transition models, a competing-hazards scoring model, and a league-level Monte Carlo simulation to evaluate how alternative penalty actions influence match and season-level outcomes.

### 2.5.1 Match State and Data Sources

A match state summarises the current score (points, tries, and time since last score for each team), elapsed time, home–away status, and team attributes (attack, defence, kicker ability, lineout strength, and 22-entry efficiency). Full formal notation is deferred to the dissertation.

Two datasets underpin the analysis. A long-run IRFU dataset of roughly 7,000 AIL fixtures (2002–2025) is used to estimate long-run scoring intensities and to construct team–season attack and defence ratings, with a light recency weight applied to recent matches. A second dataset of around 150 Opta-coded matches provides timestamped events, territorial outcomes, lineout performance, and kick difficulty, enabling construction of the set-piece models and kicker ratings.

### 2.5.2 Set-Piece Transition Models

Each penalty action (kick at goal or kick to touch) is modelled as a transition from the current state to a weighted collection of successor states.

**Goal-Kick Model.** A logistic regression predicts the probability of scoring three points based on kick difficulty and kicker rating. It yields two successor states, corresponding to a made or missed kick, each with updated time and probability weight.

**Lineout Transition Model (Kick to Touch).** A territory model returns a distribution over touch-entry locations; a small weight is retained for failing to find touch. Conditional on a successful kick, a lineout model generates successor states with $\Delta S \in \{0, 5, 7\}$ points added and a distribution of sequence durations. Field position is not carried forward, as the subsequent scoring model does not condition on spatial information.

**Penalty Transition Function.** Together, these components define a penalty transition function that maps each action to a probability distribution over future match states. These successor states act as inputs to the scoring model.

### 2.5.3 Scoring Model and League Simulation

Subsequent scoring is modelled using a Weibull competing-hazards specification for tries and penalties. A Weibull model is used because rugby scoring exhibits strong duration dependence: the longer a team goes without scoring, the higher the instantaneous scoring hazard. This violates the constant-rate assumption of Poisson models, making the Weibull specification more appropriate. Shape parameters are estimated using the timestamped Opta data to capture temporal dynamics in scoring hazards, while scale parameters are estimated using the long-run IRFU dataset to anchor overall scoring rates. In pre-game mode, the model generates distributions over full-time scores and league-points pairs; in in-game mode, it conditions on successor states produced by the set-piece models.

For season-level incentives, the scoring model is embedded in a Monte Carlo simulation of the remaining fixtures, using fixed recency-weighted team–season ratings. The simulation generates a large set of possible end-of-season league tables. For each league-points pair $(i, j)$ produced by a penalty action, these simulated tables are adjusted by adding $i$ points to the hero team and $j$ to the opponent, yielding a distribution of finishing positions conditional on that outcome. Promotion and relegation probabilities then follow from the law of total probability. The procedure is computationally light and will be implemented in Python.

**Milestones and Deliverables.** The work plan is organised into four sequential phases.

(i) *Data preparation (early stage):* cleaning the IRFU scoreline dataset, constructing team–season attack and defence ratings, and processing the Opta-coded matches to extract set-piece, territorial, and time-based features. Deliverable: completed match-state dataset and inputs for the set-piece models.

(ii) *Model estimation (mid stage):* estimation of the goal-kick, territory, and lineout models, followed by estimation of the Weibull competing-hazards scoring model, with the shape parameter identified from Opta timestamps and the scale parameter from IRFU scoreline data. Deliverable: validated set-piece models and scoring model.

(iii) *Simulation and analysis (late stage):* implementation of the Penalty Transition Function (PTF), in-game scoring simulations, and Monte Carlo league simulations. Deliverable: optimal penalty decisions under win probability, expected league points, and promotion/relegation objectives.

(iv) *Writing and refinement (final stage):* drafting empirical results, interpretation, and the completed thesis.

**Risks.** Risks include delays in processing Opta data, longer-than-expected model estimation, or computational constraints during simulation. A further risk is model mis-specification if scoring patterns differ across eras.

**Contingency Plans.** If delays arise, the project can prioritise a reduced model scope—for example, simplifying the lineout transition model or reducing simulation repetitions. Parallel computing will be used if necessary. Robustness checks using Poisson and negative binomial models mitigate concerns over model mis-specification.

# 3 References

# References


[1] Boshnakov, Georgi N., Tarak Kharrat, and Ian G. McHale. (2017). "A Bivariate Weibull Count Model for Forecasting Association Football Scores." *International Journal of Forecasting* 33(2): 458–466. Available at: https://drive.google.com/file/d/1X5BX7y38XJm261RgSisqu6IyovSLZO1d/view?usp=sharing.

[2] Boshnakov, Georgi N., Tarak Kharrat, and Ian G. McHale. (2016). "Are Goals Poisson Distributed?" University of Manchester / University of Salford Working Paper. Available at: https://drive.google.com/file/d/1XGQgPE3s84pqwjzNGdS4dqydzOiiHf7T/view?usp=sharing.

[3] Cervone, Daniel, Alex D'Amour, Luke Bornn, and Kirk Goldsberry. (2016). "A Multiresolution Stochastic Process Model for Predicting Basketball Possession Outcomes." *Journal of the American Statistical Association* 111(514): 585–599. Available at: https://drive.google.com/file/d/1OPZ4fOTDKsxxI8ylvEsyMYFGwwFICcAi/view?usp=sharing.

[4] Fitzpatrick, Brian, and David Nolan. (2024). *The Application of Expected Points in Rugby Union: Proposal of a Novel Framework*. SportRiv Preprint. Available at: https://drive.google.com/file/d/1N9K2ZQ4TBT-PgLk3eglvCgbL9ebnxwGd/view?usp=sharing.

[5] Lahvička, Jiří. (2012). "Using Monte Carlo Simulation to Calculate Match Importance: The Case of English Premier League." MPRA Working Paper No. 40998. Available at: https://drive.google.com/file/d/1rD7jN4uH1BvgJv5JWvRk2UZhJRfcZGcJ/view?usp=sharing.

[6] Romer, David. (2006). "Do Firms Maximize? Evidence from Professional Football." *Journal of Political Economy* 114(2): 340–365. Available at: https://drive.google.com/file/d/11eLuFjS79qOQfCqeFxMW56QgYDuCAxNF/view?usp=sharing.

[7] Irish Rugby Football Union (IRFU). (2002–2025). *All-Ireland League Results and Match Data, 2002–2025*. Dublin: IRFU. Available at: https://www.irishrugby.ie/club/ail/.

[8] Opta Sports. (2024–2025). *Opta Rugby Union Event-Level Dataset (Timestamped Match Events), Version 1.0*. London: Stats Perform / Opta Sports. Available at: https://optaplayerstats.statsperform.com/en_GB/soccer.