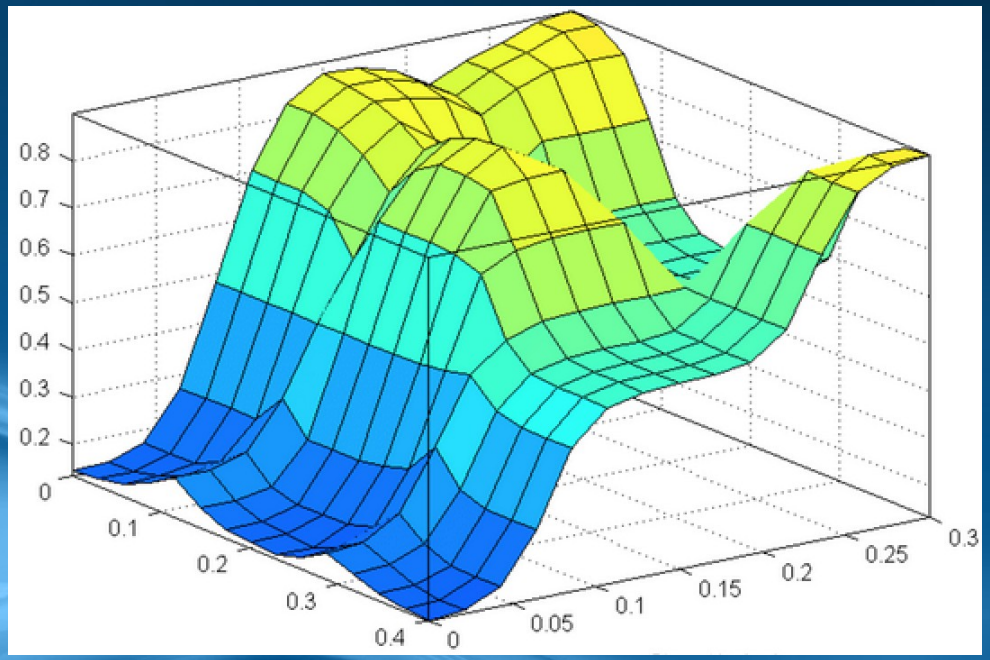

Maxeler Apps
Fuzzy Logic

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Sep 2015

Fuzzy Logic

- Fuzzy logic extends boolean logic with probability
- Instead of just 1 or 0 every truth value may be any real number between 0 and 1
- Numbers are often replaced by linguistic terms
  - For example the variable age may be young or old
- Fuzzy logic is often applied to
  - control theory
  - artificial intelligence
  - neural language processing

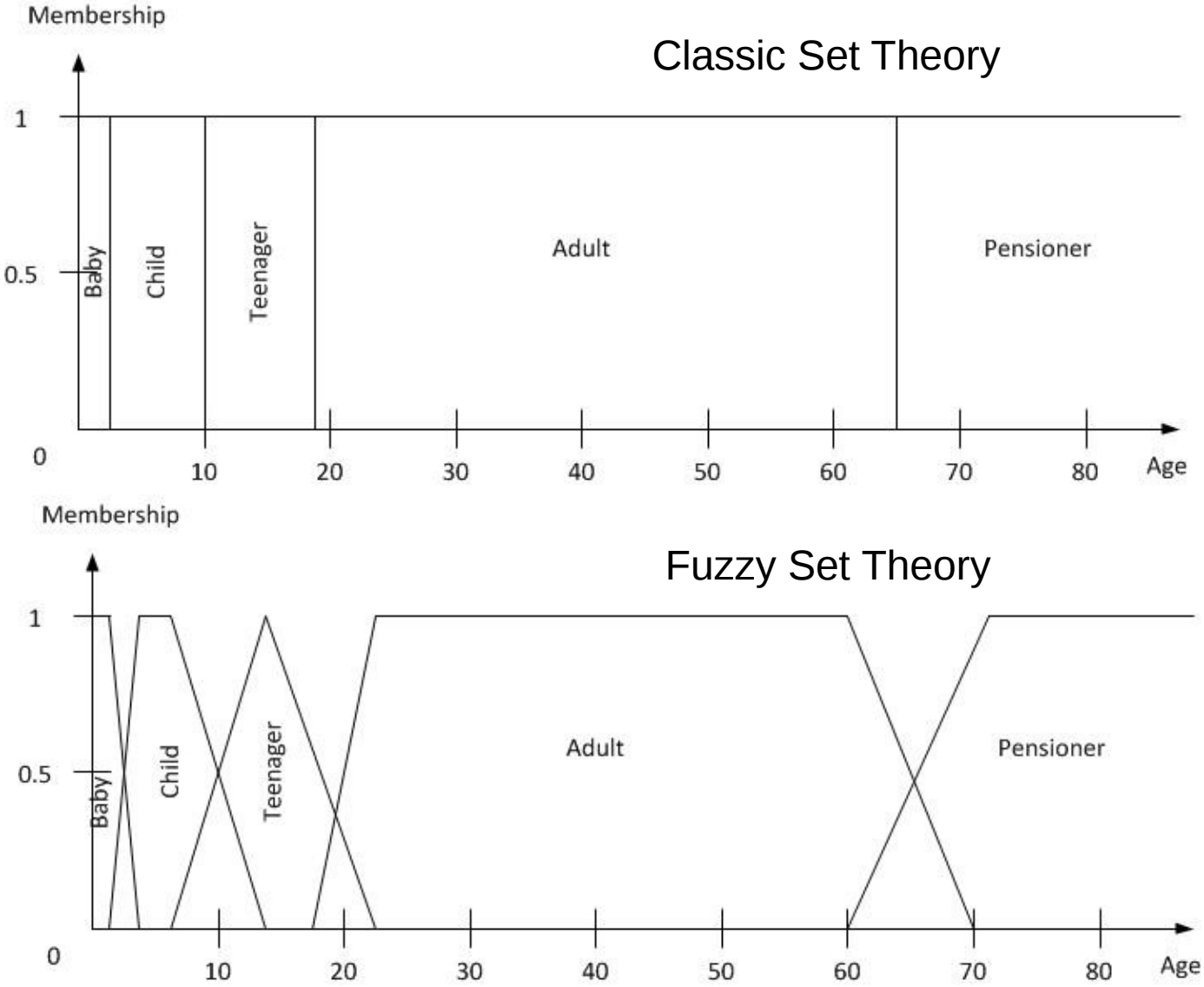
Fuzzy Logic (cont)

- Typical fuzzy logic based application has three phases
  1. **Fuzzification:** Transformation of input values into fuzzy logic
     - e.g. Transform a given age to a value between 0 and 1 for young and old
  2. **Evaluation** of rules and logic operations to calculate the output values of interest
  3. **Defuzzification:** Transformation of fuzzy logic values back into the targeted result domain

Fuzzy Sets

- In contrast to classical set theory the membership of a value to a fuzzy set is not binary
- An example is given on the next slide
  - The classification of different ages to different stages of life is not absolute
  - This can easily be expressed with fuzzy sets

Fuzzy Sets – Age example



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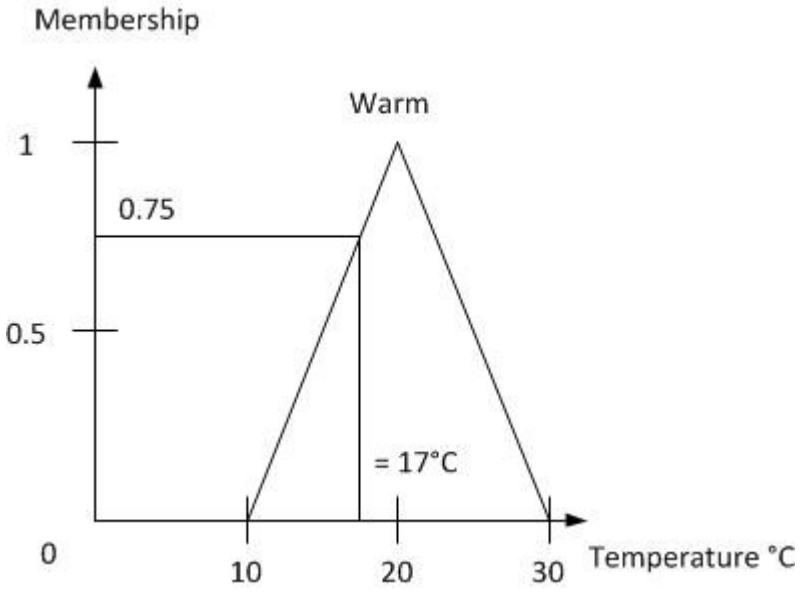
Fuzzification

- To transform a value into fuzzy logic one only needs the description of all fuzzy sets belonging to the variable of interest
- Then for every set a transform like below is done



Evaluation

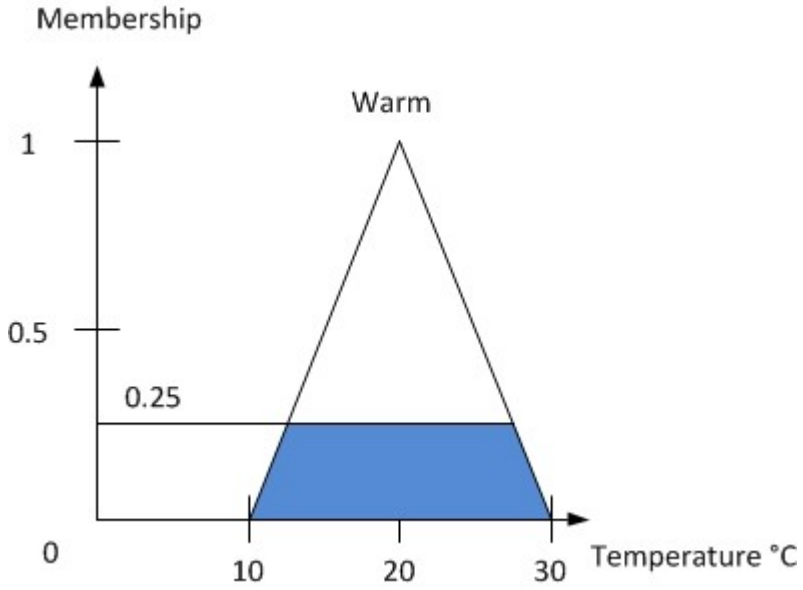
- The AND, OR, and NOT operators exist in fuzzy logic
  - Usually defined as minimum, maximum and complement
- Evaluation rules can be described as IF-THEN rules
  - IF temperature IS hot THEN speed up fan
    - In this example
      - temperature and fan are variables
      - hot and speed up are fuzzy sets
      - On Evaluation the membership degree of hot would be assigned to the membership degree of speed up

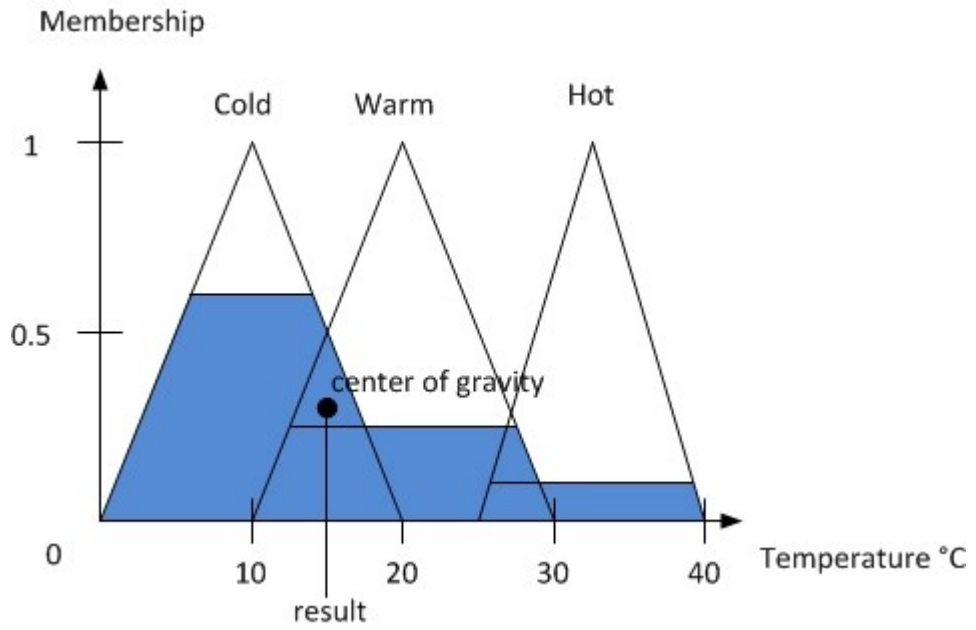
Defuzzification

- In most cases after the evaluation several fuzzy sets have a certain degree of membership
- To get a quantifiable result defuzzification has to be applied
- There are many different methods
  - The simplest method is to choose the fuzzy set with the highest degree of membership
    - This obviously introduces a large inaccuracy since all other results are discarded
  - Another more useful defuzzification technique is called *center of gravity*
    - An example is shown on the next slide

Defuzzification - Example

- First trapezoids are calculated for every fuzzy set based on the degree of membership
- Afterwards the center of gravity of all trapezoids combined is calculated
- Simple assignment to “cold” would provide a different (poor) result





Implementation

- This C++ Implementation provides an easy to use and modify tool to generate a DFE based fuzzy logic app
- The input is a simple description of all variables and rules
  - See: http://www.fuzzylite.com/fll-fld/
  - Only a few of the available options get used
  - A minimal example is shown on the next slide

Fuzzy Logic Description

```
Engine: SimpleDimmer
InputVariable: Ambient
  range: 0.000 1.000
  term: DARK Triangle 0.000 0.250 0.500
  term: MEDIUM Triangle 0.250 0.500 0.750
  term: BRIGHT Triangle 0.500 0.750 1.000
OutputVariable: Power
  range: 0.000 1.000
  term: LOW Triangle 0.000 0.250 0.500
  term: MEDIUM Triangle 0.250 0.500 0.750
  term: HIGH Triangle 0.500 0.750 1.000
RuleBlock:
  rule: if Ambient is DARK then Power is HIGH
  rule: if Ambient is MEDIUM then Power is MEDIUM
  rule: if Ambient is BRIGHT then Power is LOW
```

Implementation (cont)

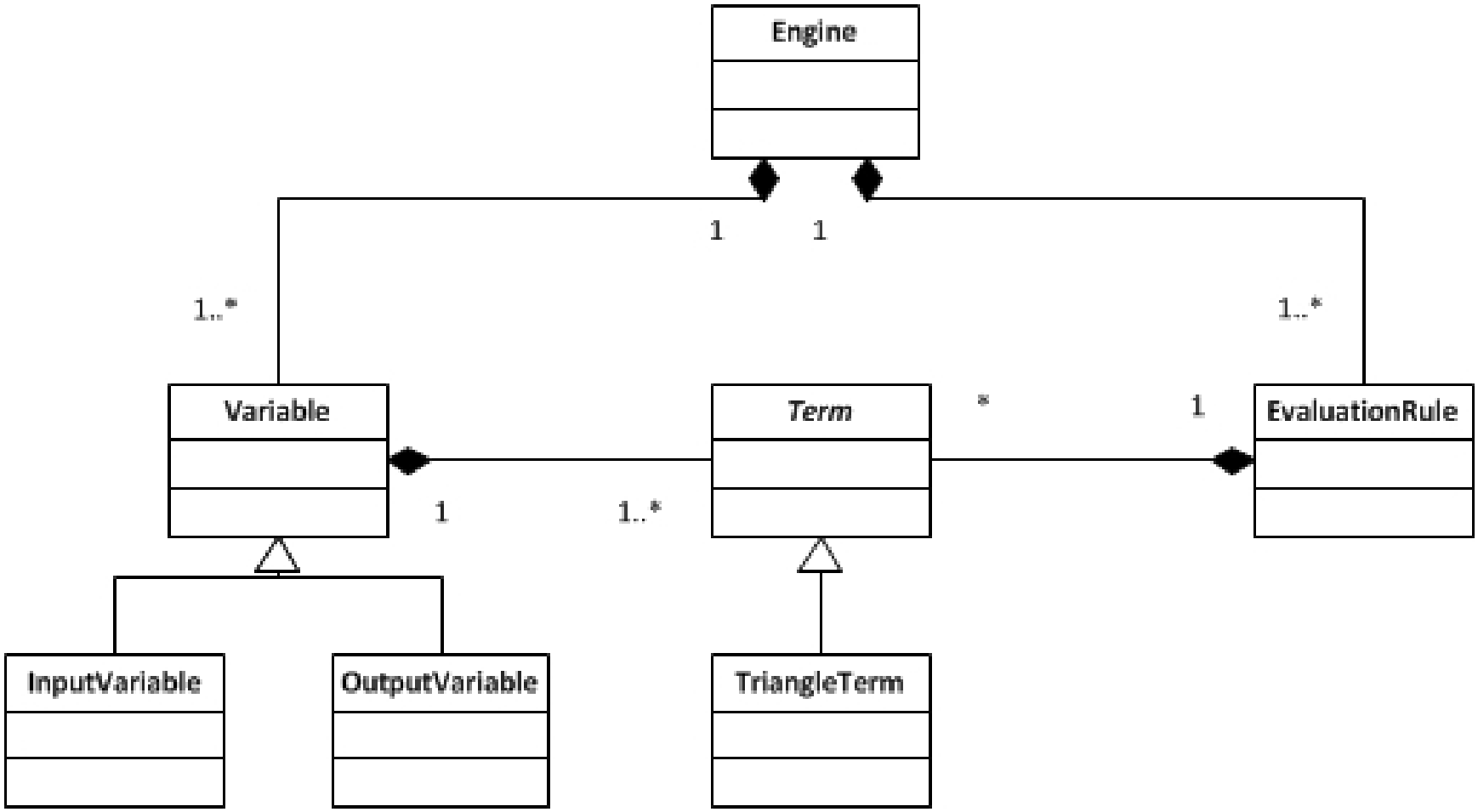
- In this implementation only fuzzy sets that form a triangle are implemented
  - Other geometric shapes can be added
- The rule evaluation is straightforward
  - AND, OR and NOT are simple to implement
- The most challenging is to find a good way to do defuzzification

Implementation - Defuzzification

- Finding a defuzzification method is always a trade-off between complexity and precision
- The chosen method is a weighted sum of the centroids of all trapezoids that contribute to the final result
  - First the centroid C_{xi} and Area A_i of every Trapezoid is calculated
  - Then the weighted sum is calculated:

$$result = \frac{\sum_i A_i \times C_{xi}}{\sum_i A_i}$$

Implementation – Architecture



Implementation – Adding new Fuzzy Set Shapes

- To add a new fuzzy set shape three things have to be done
  1. A new class inheriting *Term* has to be created
     - This mainly involves changing some strings and creating functions for (de)fuzzification
  2. The rule parser in the *Engine* class has to be adjusted
  3. Both static functions and the enum in the *Term* class need to be modified

Summary

- Fuzzy logic has many possible applications
- This implementation makes it easy to use fuzzy logic on an DFE
- If many rules should be evaluated with various involved variables and fuzzy sets a DFE based implementation might be necessary to meet timing constraints