Lattice Boltzmann on DFE

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Lattice Boltzman

- The lattice Boltzmann method (LBM) is a finite difference type algorithm for fluid dynamics at the mesoscopic level.
- Instead of treating a fluid as a macroscopic field, probabilistic distributions of fluid particles are modelled.
- Distributions of particles flow and collide and the macroscopic fluid variables are observed as emergent behaviour.

The Boltzmann Equation

- The Boltzmann gas equation model the behaviour of a distribution of particles as a pair of processes, free flight and some collision integral.

$$\partial_t f + \mathbf{v} \cdot \partial_{\mathbf{x}} f = Q(f)$$

- There are many models for a collision integral but the simplest is relaxation to a local equilibrium.

$$Q(f) = -\frac{1}{\tau}(f - f^{\text{eq}}).$$

- Navier-Stokes equations may be recovered from the Boltzmann equation via integration.

$$\rho = \int f \; \mathrm{d}\mathbf{v}, \qquad \rho\mathbf{u} = \int \mathbf{v}f \; \mathrm{d}\mathbf{v}, \qquad \frac{1}{2}\rho\mathbf{u}^2 + \rho D\Theta = \frac{1}{2}\int \mathbf{v}^2 f \; \mathrm{d}\mathbf{v}.$$

Discretization

- The Boltzmann equation may be relatively simply discretized onto a regular grid, giving the lattice Boltzmann Method.

$$f_i(\mathbf{x} + \epsilon \mathbf{v}_i, t + \epsilon) = f_i(\mathbf{x}, t) + \omega(f_i^{\mathrm{eq}}(\mathbf{x}, t) - f_i(\mathbf{x}, t))$$

- Navier-Stokes equations is now recovered by summation (with some discretization errors).

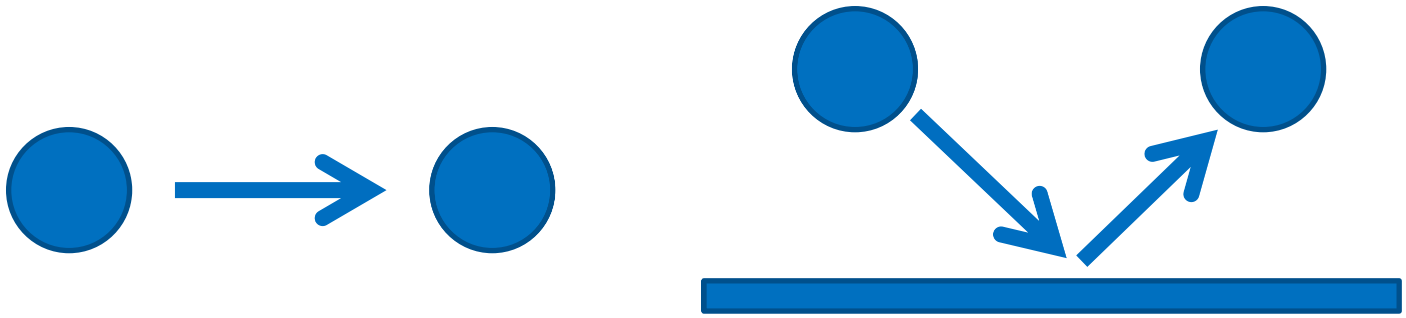
$$\rho = \sum_{i=1}^{n} f_i, \qquad \rho \mathbf{u} = \sum_{i=1}^{n} \mathbf{v}_i f_i, \qquad \frac{1}{2}\left(\rho \mathbf{u}^2 + \rho D \Theta\right) = \frac{1}{2} \sum_{i=1}^{n} \mathbf{v}_i^2 f_i.$$

- Many variations exist to generalize the physics or improve stability.

$$f_i(\mathbf{x} + \epsilon \mathbf{v}_i, t + \epsilon) = f_i(\mathbf{x}, t) + \omega(f_i^{\mathrm{eq}}(\mathbf{x}, t) - f_i(\mathbf{x}, t)) - (\omega_N - \omega) w_i g_i N(\mathbf{x}, t)/4.$$

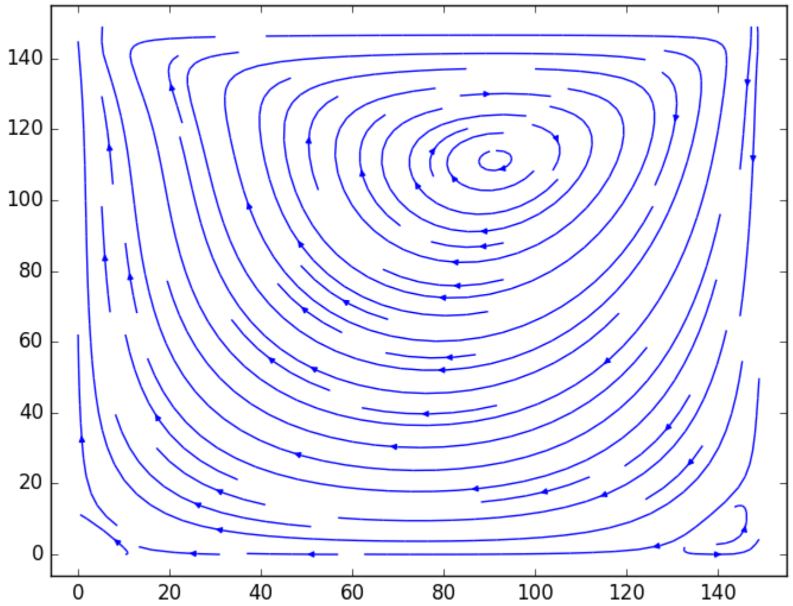
Offsets and muxes in lattice Boltzmann on DFE

- In lattice Boltzmann particles fly from one lattice site to another, this is achieved on DFE by stream offsets.
- Near walls particles may come from different directions (they bounce off walls).
- The DFE pipeline must mux between all the possible direction a particle came from



Lid Driven Cavity

- To demonstrate lattice Boltzmann on DFE we can use the simple lid driven cavity example.
- A 2D fluid in a box has its motion driven by one boundary (the lid).
- This causes the fluid to spin inside the box and at low viscosities generates a turbulent flow where vortices are shed.



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