Hand in 6

FYSC20

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Problem: The centers of two circular conducting loops A and B, of radii a and $b \gg a$, respectively, are located at the origin of a Cartesian coordinate system. At time $t = 0$ both loops lie on the xy-plane. While the larger loop remains at rest, the smaller loop, having a resistance R, rotates about one of its diameters lying on the x axis with constant angular velocity ω. A constant current I circulates in the larger loop.

(a) Evaluate the current I_A induced in loop A, neglecting self-inductance effects. You can look up the magnetic field.

(b) Evaluate the power dissipated in loop A due to Joule heating.

(c) Now consider the case when loop A is at rest on the xy plane, with a constant current I circulating in it, while loop B rotates around the x-axis with constant angular velocity ω. Evaluate the electromotive force induced in B, neglecting self-inductance effects.

Solution (a): We start by drawing a sketch of the problem

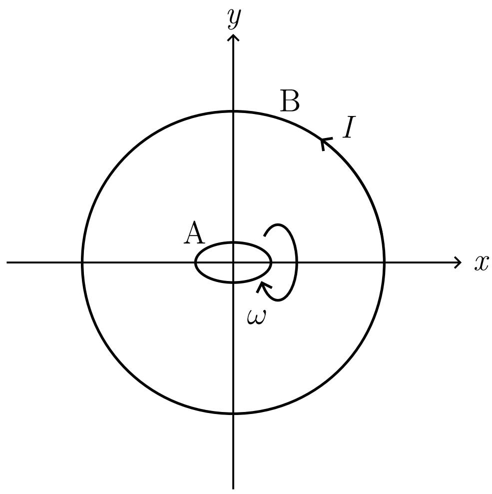

Figure 1: Sketch of the problem.

We know that since $b \gg a$ the magnetic field is

$$\mathbf{B}_B = \frac{\mu_0 I}{2b}\hat{\mathbf{z}}. \tag{1}$$

The flux through loop A is then

$$\Phi_A = \int \mathbf{B}_B \cdot \mathrm{d}\mathbf{a} \tag{2}$$

where $\mathrm{d}\mathbf{a} = (\hat{\mathbf{z}} \cos \omega t + \hat{\mathbf{y}} \sin \omega t)\mathrm{d}A$ due to the rotation. Thus

$$\Phi_A = \int \frac{\mu_0 I}{2b} \hat{\mathbf{z}} \cdot (\hat{\mathbf{z}} \cos \omega t + \hat{\mathbf{y}} \sin \omega t) = \frac{\mu_0 I}{2b} \cos \omega t \int \mathrm{d}A = \frac{\mu_0 I \pi a^2}{2b} \cos \omega t. \tag{3}$$

Then the electromotive force is

$$\mathcal{E}_A = -\frac{\mathrm{d}\Phi_A}{\mathrm{d}t} = \frac{\mu_0 I \pi a^2 \omega}{2b} \sin \omega t. \tag{4}$$

Using Ohm's law we get

$$I_A = \frac{\mathcal{E}_A}{R} = \frac{\mu_0 I \pi a^2 \omega}{2bR} \sin \omega t. \tag{5}$$

Answer (a): The enduced current in loop A is

$$I_A = \frac{\mu_0 I \pi a^2 \omega}{2bR} \sin \omega t. \tag{6}$$

Solution (b): We obtain the formula for Joule heatin by combining $P = I\Delta V$ and $\Delta V = RI$ thus we have

$$P_A = I_A^2 R = \frac{\mu_0^2 I^2 \pi^2 a^4 \omega^2}{4b^2 R} \sin^2 \omega t. \tag{7}$$

Answer (b): The power dissipated in loop A from Joule heating is

$$P_A = \frac{\mu_0^2 I^2 \pi^2 a^4 \omega^2}{4b^2 R} \sin^2 \omega t. \tag{8}$$

Solution (c): Since we have two loops we get mutual inductance. By the Neumann formula we get that $M_{AB} = M_{BA}$ and since we have the equation

$$\Phi_A = M_{AB} I_B \tag{9}$$

and we have calculated that

$$\Phi_A = \frac{\mu_0 I \pi a^2}{2b} \cos \omega t \tag{10}$$

Since $I = I_B$ it follows that

$$M_{AB} = \frac{\mu_0 \pi a^2}{2b} \cos \omega t. \tag{11}$$

We have that $I_A = I$ thus

$$\Phi_B = M_{BA} I_A = M_{AB} I = \frac{\mu_0 I \pi a^2}{2b} \cos \omega t \tag{12}$$

and the electromotive force becomes

$$\mathcal{E}_B = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = \frac{\mu_0 I \pi a^2 \omega}{2b} \sin \omega t. \tag{13}$$

Answer (c): The electromotive force induced in loop B is

$$\mathcal{E}_B = \frac{\mu_0 I \pi a^2 \omega}{2b} \sin \omega t. \tag{14}$$