Problem Sheet 2

FYSC22

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5 Activity reminder

(a)

Solution: Assuming the person is standing facing the radioactive source. The area of the person that is in line of sight of the source can be estimated to $1\,\mathrm{m}^2$. The source is radiating omnidirectionally thus the area of the sphere with radius x is $A = 4\pi x^2$ and the effective dosage is proportional to the area of the human to the area of the sphere. Thus the ratio of total radiation hitting the person is $R = 1/(4\pi x^2)$. The mass of the person can be estimated to be $70\,\mathrm{kg}$. Converting $2\,\mu\mathrm{Ci} = 74\,\mathrm{kBq}$, $0.5\,\mathrm{MeV} = 8.01 \times 10^{-14}\,\mathrm{J}$ and $8\,\mathrm{h} = 28\,800\,\mathrm{s}$, we can then calculate the dose to

$$D = \frac{74\,\mathrm{kBq} \times 28\,800\,\mathrm{s} \times 8.01 \times 10^{-14}\,\mathrm{J}}{70\,\mathrm{kg} \times 4\pi x^2} \tag{1}$$

for $x_1 = 1\,\mathrm{m}$ we get $D_1 = 194\,\mathrm{nGy}$ and for $x_2 = 4\,\mathrm{m}$ we get $D_4 = 12.1\,\mathrm{nGy}$. For γ the quality factor is 1. thus we get $DE_1 = 194\,\mathrm{nSv}$ and $DE_4 = 12.1\,\mathrm{nSv}$.

Answer: We get the dose to $D_1 = 194\,\mathrm{nGy}$ and $D_4 = 12.1\,\mathrm{nGy}$. And the equivalent dose to $DE_1 = 194\,\mathrm{nSv}$ and $DE_4 = 12.1\,\mathrm{nSv}$.

(b)

Table 1: Natural abundance and half-life of relevant isotopes.

Isotope	**Natural Abundance**	**Half-Life**
$^{14}\mathrm{C}$	10^{-12}	$5730(40)\,\mathrm{yr}$
$^{40}\mathrm{K}$	0.000117	$1.277(8) \times 10^9\,\mathrm{yr}$

Solution: Assuming we have the same person at $70\,\mathrm{kg}$, the mass of carbon is $14\,\mathrm{kg}$ and the mass of potassium is $0.175\,\mathrm{kg}$. The number of $^{14}\mathrm{C}$ in the body is $N(^{14}\mathrm{C}) = (14\,\mathrm{kg} \times 10^{-12})/(14\,\mathrm{u} \times 1.66 \times 10^{-27}\,\mathrm{kg\,u^{-1}}) = 6.02 \times 10^{14}$. Thus the activity is $A(^{14}\mathrm{C}) = N \ln 2/T_{1/2} = 2308(16)\,\mathrm{Bq}$. Similarly for $^{40}\mathrm{K}$ the total number of nuclides in the body is $N(^{40}\text{K}) = (0.175\,\text{kg} \times 0.000117)/(40\,\text{u} \times 1.66 \times 10^{-27}\,\text{kg}\,\text{u}^{-1}) = 3.08 \times 10^{20}$. The total activity for ^{40}K is then $A(^{40}\text{K}) = N \ln 2/T_{1/2} = 5298(33)\,\text{Bq}$. Assuming that the excited ^{40}Ar state decays instantly we get that $A(^{40}\text{Ar}) = 0.11 \times A(^{40}\text{K}) = 583(4)\,\text{Bq}$. The total activity is $A = A(^{14}\text{C}) + A(^{40}\text{K}) + A(^{40}\text{Ar}) = 8190(40)\,\text{Bq}$

Answer: The total intrinsic activity is $A = 8190(40)\,\text{Bq}$

(c)

Solution: Since the half-life is much longer than a year and since we eat and therefore replenish the supply of ^{14}C and ^{40}K we can assume that the activity is constant and that all radiation is absorbed by the person. The we can calculate the dose as $D =$ Absorbed energy/mass, which gives us $D(^{14}\text{C}) = 26.67(19)\,\mu\text{Gy}$, $D(^{40}\text{K} \to {}^{40}\text{Ca}) = 442.7(28)\,\mu\text{Gy}$, and $D(^{40}\text{K} \to {}^{40}\text{Ar}) = 61.5(4)\,\mu\text{Gy}$. The total dose is therefore $D = 530.8(32)\,\mu\text{Gy}$. Since all radiation is either β^- or γ the quality factor is 1 for all radiation, and thus the dose equivalent is $DE = 530.8(32)\,\mu\text{Sv}$.

Answer: The total dose absorbed during one year from intrinsic radiation is $D = 530.8(32)\,\mu\text{Gy}$ and the total dose equivalent is $DE = 530.8(32)\,\mu\text{Sv}$.

6 Alpha-decay and shell-model rules

(a)

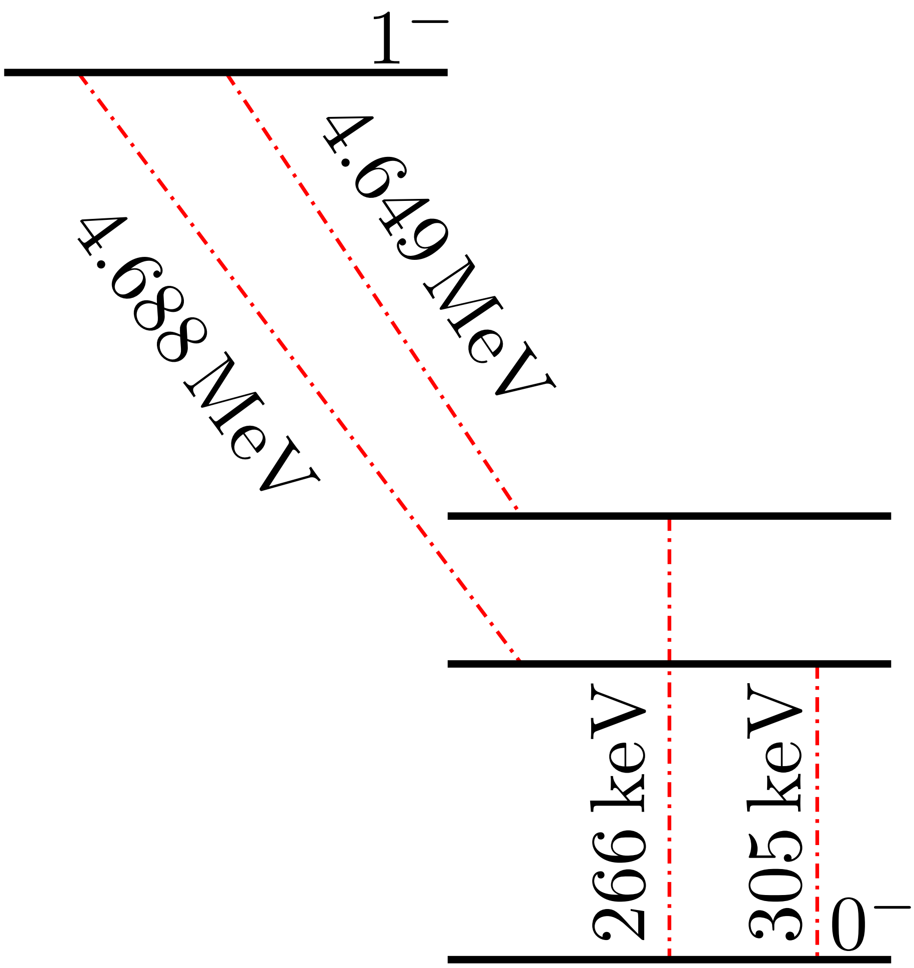

Figure 1: Decay scheme of the alpha decay.

(b)

Both are odd-odd since the ground state of the daughter has integer spin, which indicates that it is not even-odd, and since even-even have even parity in the ground state the daughter must be odd-odd. The parent state differs by two neutrons and two protons so it must also be odd-odd.

(c)

The selection rules tells us that parity must be conserved. Thus we have to go from odd spin, odd parity, to even spin, even parity. However to the ground state is even spin, odd parity, which then isn't allowed.

(d)

Allowed transfers, as said in (c), are from the odd spin, odd parity, to e.g. even spin, even parity. The most probable transition for an alpha is to not change spin parity between the initial and final state. Thus the most probable spin parity for the excited states is 1^-. So both could have the same spin parity but with different configurations in the orbitals.

7 β decay from ^{137}Cs

8 The deuteron — a strange fellow

The radial schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}u(r) + V(r)u(r) = Eu(r), \tag{2}$$

where $E = -B$ is the binding energy. Between $r = 0\,\text{fm}$ and $r = R = 2.1\,\text{fm}$ we have $V(r) = -V_0$ and thus

$$\frac{\partial^2}{\partial r^2}u_{\text{I}}(r) = -\frac{2m(V_0 - B)}{\hbar^2}u_{\text{I}}(r), \tag{3}$$

letting $k = \sqrt{2m(V_0 - B)/\hbar^2}$ we get

$$\frac{\partial^2}{\partial r^2}u_{\text{I}}(r) = -k^2 u_{\text{I}}(r) \iff u_{\text{I}}(r) = A\sin kr + B\cos kr. \tag{4}$$

For $r > R = 2.1\,\text{fm}$ we have $V(r) = 0$ and thus

$$\frac{\partial^2}{\partial r^2}u_{\text{II}}(r) = \frac{2mB}{\hbar^2}u_{\text{II}}(r), \tag{5}$$

letting $q = \sqrt{2mB/\hbar^2}$ we get

$$\frac{\partial^2}{\partial r^2}u_{\text{II}}(r) = q^2 u_{\text{II}}(r) \iff u_{\text{II}}(r) = Ce^{-qr}. \tag{6}$$

For $r = 0\,\text{fm}$ the wavefunction must be zero, thus

$$B = 0 \implies u_{\text{I}}(r) = A\sin kr \tag{7}$$

Then at $r = R = 2.1\,\text{fm}$ we must have $u_{\text{I}}(R) = u_{\text{II}}(R)$ which is

$$A\sin kR = Ce^{-qR} \iff C = \frac{\sin kR}{e^{-qR}}A. \tag{8}$$

The wavefunction also must be normalised and thus

$$1 = \int_0^\infty |u(r)|^2 \mathrm{d}r = |A|^2 \int_0^R |\sin kr|^2 \mathrm{d}r + |C|^2 \int_R^\infty \left|e^{-qr}\right|^2 \mathrm{d}r \tag{9}$$

$$= |A|^2 \left[\int_0^R \sin^2 kr \mathrm{d}r + \frac{\sin^2 kR}{e^{-2qR}} \int_R^\infty e^{-2qr} \mathrm{d}r \right] \tag{10}$$

$$= |A|^2 \left[\left(\frac{r}{2} - \frac{\sin 2kr}{4k} \right) \Bigg|_0^R + \frac{\sin^2 kR}{e^{-2qR}} \times \left(-\frac{e^{-2qr}}{2q} \right) \Bigg|_R^\infty \right] \tag{11}$$

$$= |A|^2 \left[\frac{R}{2} - \frac{\sin 2kR}{4k} + \frac{\sin^2 kR}{2q} \right] \tag{12}$$

$$\iff A = \sqrt{\frac{1}{\frac{R}{2} - \frac{\sin 2kR}{4k} + \frac{\sin^2 kR}{2q}}} \tag{13}$$

Calculating this numerically with python gives $A = 0.56$ and $C = 0.87$. By integrating $|u(r)|^2$ we can find the probability of the distance being greater than 2.1 fm. That is we want to calculate

$$P(r > 2.1\,\mathrm{fm}) = \int_{2.1\,\mathrm{fm}}^\infty |u(r)|^2 \mathrm{d}r. \tag{14}$$

Using python we get a probability of $P(r > 2.1\,\mathrm{fm}) = 62.2\,\%$.

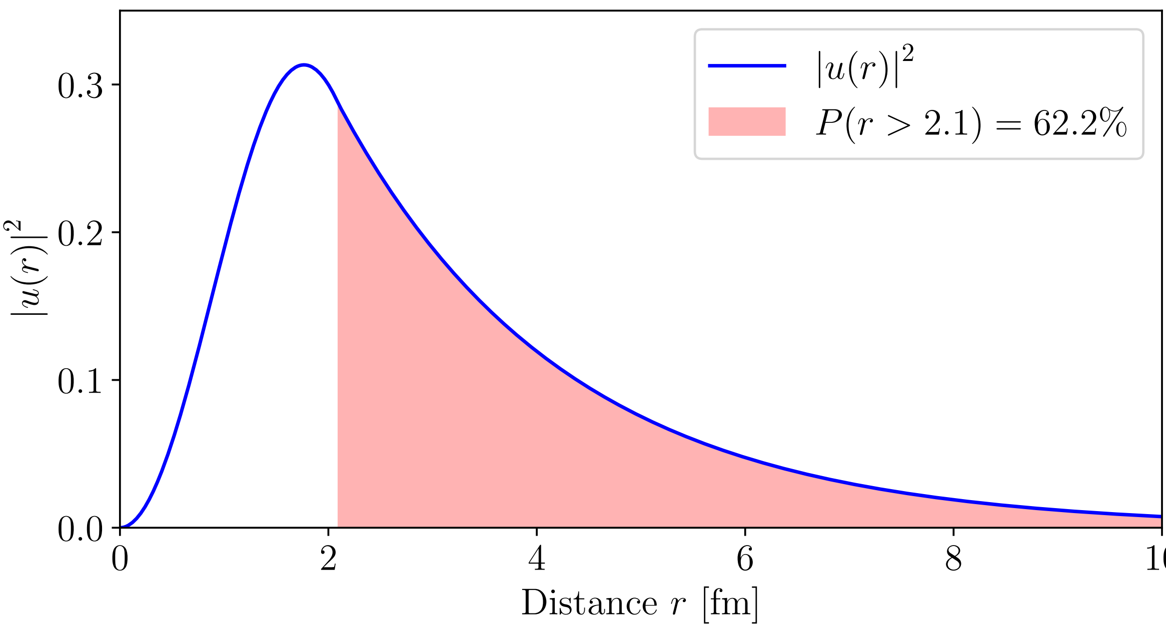

Figure 2: The probability distribution of the deuteron.

Answer: The deuteron spend 62.2 % of the time beyond their nuclear force range.

A Code

Activity reminder

```python
#%%
from uncertainties import ufloat
import math as m

#%%
# (b)
def activity(N: float, half_life: float):
  """
  parameters:
    N, the number of nuclides
    half_life, the half life in years
  returns:
    A, the activity
  """
  year_to_sec = 31556926
  T = half_life*year_to_sec
  A = N * m.log(2) / T
  return A

A_C14 = activity(6.02e14, ufloat(5730, 40))
A_K40 = activity(3.08e20, ufloat(1.277e9, 0.008e9))
A_Ar40 = 0.11*A_K40
print(A_C14)
print(A_K40)
print(A_Ar40)
print(A_C14 + A_K40 + A_Ar40)
#%%
# (c)
def dose_yr(A: float, E: float, m: float = 70):
  """
  parameters:
    A, the activity in Bq
    E, the energy per decay in MeV
    m, the mass of the person in kg
  returns:
    D, the dose in one year
  """
  year = 31556926
  energy = E * 1.602e-13
  N_decays = A * year
  D = N_decays * energy / m
  return D

D_C14 = dose_yr(A_C14, 0.16)
D_K40_Ca40 = dose_yr(0.89*A_K40, 1.3)
D_K40_Ar40 = dose_yr(0.11*A_K40, 1.461)

print(D_C14)
print(D_K40_Ca40)
print(D_K40_Ar40)
print(D_C14 + D_K40_Ca40 + D_K40_Ar40)
```

The deuteron — a strange fellow

```python
#%%
import numpy as np
from scipy.integrate import quad
import matplotlib.pyplot as plt
from matplotlib import rc

rc('font',**{'family':'serif','serif':['Computer Modern'], 'size':'16'})
rc('text', usetex=True)

#%%
k = np.sqrt( 938.9 * (35 - 2.2) / 197.3**2 )
q = np.sqrt(938.9 * 2.2 / 197.3**2)
R = 2.1

denom = R/2 - np.sin(2*k*R)/(4*k) + np.sin(k*R)**2 / (2*q)
A = np.sqrt(1 / denom)
C = np.sin(k*R)/(np.exp(-q*R)) * A

print(round(A,2), round(C,2))
#%%
r = np.linspace(0, 10, 500)
def u(r):
  return np.piecewise(r, [r <= 2.1, r > 2.1], [lambda x: A*np.sin(k*x), lambda x: C*np.exp(-q*x)])

def P(r):
  return np.abs(u(r))**2

I_outside = quad(P, 2.1, np.inf)[0]
time_outside = round(I_outside*100, 1)

fig, ax = plt.subplots(1,1)
fig.set_figwidth(8)
fig.set_figheight(4)


ax.plot(r, P(r), color='b', label=r'$\left| u(r) \right|^2$')
fill_x = np.linspace(2.1, 10, 300)
ax.fill_between(fill_x, P(fill_x), color='r', alpha=0.3, label=rf'$P(r>2.1) = {time_outside} \%$')
ax.legend()

ax.set_xlabel(r'Distance $r$ [fm]')
ax.set_ylabel(r'$\left| u(r) \right|^2$')

ax.set_xlim(0, 10)
ax.set_ylim(0, 0.35)

plt.savefig('Exercise4.pdf', bbox_inches='tight')
```