# Hand In 4

## FYST85


*Author*
Max Eriksson
maxerikss@gmail.com

Lund University
Department of Physics


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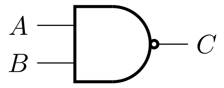
# 1 First Exercise

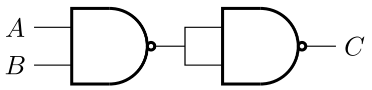
Firstly the NAND-gate looks like




| $A$ | $B$ | $C$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Figure 1: NAND-gate and the corresponding truth table.

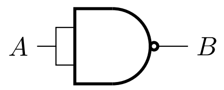
Now we can easily make the AND-gate




| $A$ | $B$ | $C$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Figure 2: AND-gate and the corresponding truth table.

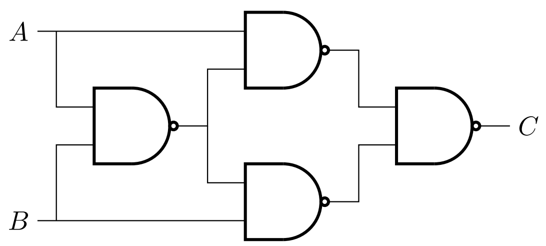
As well as the NOT-gate




| $A$ | $B$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

Figure 3: AND-gate and the corresponding truth table.

And finally the XOR-gate




| $A$ | $B$ | $C$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Figure 4: XOR-gate and the corresponding truth table.

# 2 Second Exercise

Firstly we have that

$$R_k = \begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{bmatrix} \tag{1}$$

We know from Nielsen & Chuang that we can decompose a controlled $U$-operation as $U = e^{i\alpha}AXBXC$ where $ABC = \mathbb{1}$. See Fig. 5 for the circuit decomposition.

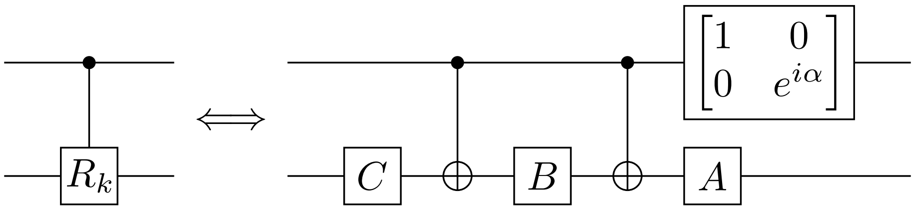



Figure 5: Decomposition of $R_k$-operation.

Then we have

$$R_z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \tag{2}$$

and the identity $XR_z(\theta)X = R_z(-\theta)$. Thus choosing $\alpha = \pi/2^k$, $A = \mathbb{1}$, $B = R_z(-\alpha)$ and $C = R_z(\alpha)$ we get

$$ABC = \mathbb{1}R_z(-\alpha)R_z(\alpha) = R_z(\alpha - \alpha) = R_z(0) = \mathbb{1} \tag{3}$$

and

$$AXBXC = \mathbb{1}XR_z(-\alpha)XR_z(\alpha) = R_z(\alpha)R_z(\alpha) = R_z(2\alpha) \tag{4}$$

and finally

$$e^{i\alpha}R_z(2\alpha) = e^{i\alpha}\begin{bmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{2\alpha i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{bmatrix} = R_k \tag{5}$$