Hand In 3
FYST85

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1 First Exercise

The matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a & 0 & 0 & c \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & b & 0 & 0 & d \end{pmatrix} \tag{1}$$

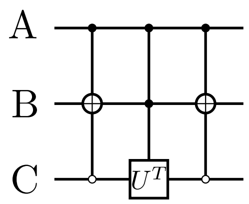
acts non-trivially on the states $|100\rangle$ and $|111\rangle$. Thus we can write a Gray code

$$\begin{matrix} A & B & C \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{matrix} \tag{2}$$

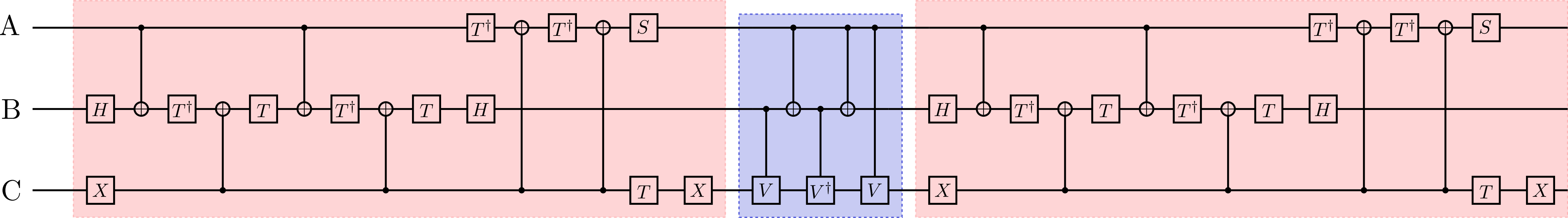
That is, we need a three-qubit operation switching the second qubit before applying the unitary operator U^T where

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{3}$$

which would look like



However, we want to show this with only single-qubit operations and two-qubit operations. This is shown in the next page. It is very wide so can be a bit hard to see. The red parts are the CCNOT gates while the blue part is the double controlled U^T part, where V is a unitary operator such that $V^2 = U^T$


A
B
C
H
T†
T
H
X
T†
T†
S
T
X
V
V†
V
H
T†
T
T†
T
H
X
T†
T†
S
T
X

2 Second Exercise

We have the identities

$$X = HZH, \quad Y = iXZ. \tag{4}$$

Then we have the hamiltonian

$$\mathcal{H} = X_1 \otimes Y_2 \otimes Z_3 \tag{5}$$

rewriting the Hamiltonian using the identities above we get

$$\mathcal{H} = (H_1 Z_1 H_1) \otimes (iX_2 Z_2) \otimes Z_3 \tag{6}$$

which can be factored as

$$\mathcal{H} = (H_1 \otimes iX_2 \otimes \mathbb{1})(Z_1 \otimes Z_2 \otimes Z_3)(H_1 \otimes \mathbb{1} \otimes \mathbb{1}). \tag{7}$$

From Nielsen & Chuang we know that $\mathcal{H} = Z_1 \otimes Z_2 \otimes Z_3$ corresponds to Figure 4.19 in the book. Thus we need to apply the corresponding gates before and after this and we get