1 Vectors and Distance

Vector Addition

$$u = a\hat{i} + b\hat{j}, v = c\hat{i} + d\hat{j}$$

$$u + v = (a+c)\hat{i} + (b+d)\hat{j}$$

$$\tan\theta = \frac{u_y}{u_x} = \frac{b}{a}$$

Distance

$$r_{ab} = \sqrt{(a_x - b_x)^2 + (a_y - b_y)^2}$$

2 Coulomb's Law

$$\mathbf{F}_{12}(\hat{r}) = k\frac{q_1 q_2}{r^2}\hat{r}$$

$$\mathbf{F}_{\text{net}} = \sum_{j=1}^{N} \mathbf{F_{ij}}$$

3 Electric Field

$$\mathbf{F} = q\mathbf{E} \implies \mathbf{E} = \frac{\mathbf{F}}{q}$$

For a point charge

$$\mathbf{E} = k\frac{q}{r^2}$$

$$\mathbf{E}_{\text{net}} = \sum_{i=1}^{N} \mathbf{E}_i$$

Electric field points towards negative charges, away from positive charges

4 Continuous Charge Distributions

	Line	Area	Volume
Density	$Q = \lambda L$	$Q = \eta A$	$Q = \rho V$
Units	Cm^{-1}	Cm^{-2}	Cm^{-3}
Cont.	$dq = \lambda dL$	$dq = \eta dA$	$dq = \rho dV$

5 Common Electric Fields

Unif. inf. charged wire: $\frac{\lambda}{2r\pi\varepsilon_0}$
Unif. inf. charged plane: $\frac{\eta}{2\varepsilon_0}$
Unif. charged sphere ($r \geq R$): $k\frac{Q}{r^2}$
Unif. charged sphere ($r \leq R$): $k\frac{Qr}{R^3}$

6 Flux

"A measure of the number of field lines which pass through a surface"
Uniform field and plane:

$$\Phi = \mathbf{E} \cdot \mathbf{A} = EA\cos\theta$$

Flux depends on the strength E of the field, the area A of the surface, and the angle θ between them.
Variable field/surface:

$$d\phi = \mathbf{E} \cdot d\mathbf{A} \implies \phi = \int \mathbf{E} \cdot d\mathbf{A}$$

7 Gauss' Law

The flux through a *closed* surface only depends on the charge *inside* the surface, it doesn't depend on the size or shape of the surface.
Flux doesn't depend on charge *distribution* inside a gaussian surface, it depends only on the *total enclosed charge*.
The flux of external field through a closed surface is always 0.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

Example: Consider an infinite wire with linear charge density $\lambda C \cdot m^{-1}$. What is the electric field r meters away from the wire?

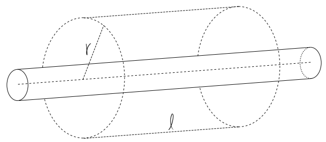

Figure 1: Gaussian surface around wire

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{\lambda \cdot \ell}{\varepsilon_0}$$

$$E \cdot 2r\pi\ell = \frac{\lambda \cdot \ell}{\varepsilon_0} \implies E = \frac{\lambda}{2r\pi\varepsilon_0}$$

8 Conductors

Electric field inside a conductor is 0 as it has reached equilibrium.
Charge inside a conductor is always zero, otherwise a field would be inside.
If a conductor is charged, all of its charges are on its surface.
Electric field right outside a conductor is perpendicular to its surface and has magnitude $\frac{\eta}{\varepsilon_0}$.

8.1 Cavities

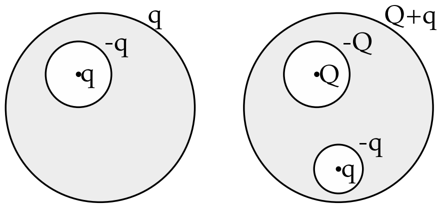

Figure 2: Charge results from spherical charged cavities inside conducting sphere

9 Electric Potential Energy

Electric fields store *potential energy*.
Field accelerates charge $\equiv$ Field does work on charge
Recall $W = \int_A^B \mathbf{F} \cdot d\mathbf{s}$. For conservative forces $W = K_f - K - i = U_i - U_f$.
The change in potential energy is $\Delta U = U_f - U - i = -W$
Constant force:

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{s} = \mathbf{F} \cdot \int_A^B d\mathbf{s} = F\Delta r\cos\theta$$

9.1 Uniform Electric Field

A positive charge will speed up as it "falls" to the negative side in the field.
There is a constant force $F = qE$ in the direction of displacement. The work done is

$$W_{\text{elec}} = qEs_i - qEs_f$$

The change in *electric potential energy* is

$$\Delta U_{\text{elec}} = -W_{\text{elec}}$$

A *negatively charged* particle in the same field has negative potential energy.

9.2 Potential Energy of Point Charges

Consider two like charges q_1 and q_2. The electric field of q_1 pushes q_2. The work done is

$$W_{\text{elec}} = \int_{x_i}^{x_f} F_{12}dx = \int_{x_i}^{x_f} k\frac{q_1 q_2}{x^2}dx$$

$$\frac{-kq_1q_2}{x}\Big|_{x_i}^{x_f} = -kq_1q_2(\frac{1}{x_f} - \frac{1}{x_i})$$

The change in electric potential energy is

$$\Delta U_{\text{elec}} = -W_{\text{elec}} = kq_1q_2(\frac{1}{x_f} - \frac{1}{x_i})$$

Now assume $x_i = \infty$. Then the electric potential energy of two point charges becomes

$$U_{\text{elec}} = \frac{kq_1q_2}{x}$$

Electric field forces are conservative; their work done doesn't depend on path.
For a set of charges, potential energy is

$$U = k\sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

10 Electric Potential Difference

Electric potential difference: Change in potential energy upon displacement a unit charge in an electric field

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q}, \text{measured in volts:} V = \frac{J}{C}$$

$$\Delta V_{AB} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Electrical potential in constant field:

$$\Delta V_{AB} = V_B - V_A = -\frac{W_{AB}}{q}$$

$$= -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -Es\cos\theta$$

Electric potential inside capacitor:

$$\Delta V_C = V_+ - V_- = Ed$$

Electric potential for point charges: The potential of a point charge q at a distance r is

$$V(r) = -\int_\infty^r \mathbf{E} \cdot d\mathbf{r} = -\int_\infty^r \frac{kq}{r^2}dr = \frac{kq}{r}$$

Superposition applies:

$$V_net = k\sum_{i=1}^{N} \frac{q_i}{r_i}$$

Electric potential for a continuous charge distribution:

$$dV = k\frac{dq}{r} V = k\int \frac{dq}{r}$$

10.1 Potential from Electric Field

$$V(b) - V(a) = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

Example: Consider a capacitor

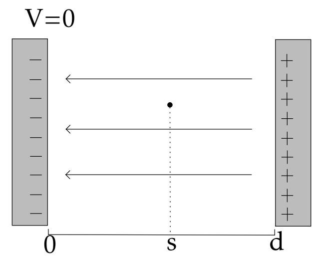

Figure 3: Capacitor

Let $V(0)$ be at the negative plate, so $V(0) = 0$. Then

$$V(s) - V(0) = V(s) = -\int_0^s \mathbf{E} \cdot d\mathbf{x}$$

$$E = -\frac{Q}{\varepsilon_0 A}, \mathbf{E} \cdot d\mathbf{x} = -Edx$$

$$\therefore V(s) = -(-\frac{Q}{\varepsilon_0 A})\int_0^s dx = Es$$

10.2 Electric Field from Potential

$$E_x = -\frac{dV}{dx}, E_y = -\frac{dV}{dy}, \ldots$$

$$\vec{E} = -\nabla V = -(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k})$$